

Integrable Deformations for $\mathcal{N} = 4$ SYM and ABJM Amplitudes

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Based on 1407.4449 with

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Graßmannian Geometry of Scattering Amplitudes

Walter Burke Institute Workshop, California Institute of Technology

Motivation

On-shell integrand for planar $\mathcal{N} = 4$ and ABJM well understood

How to integrate?

Regularization breaks conformal symmetry

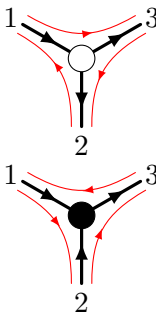
But even for finite ratio function: No practical way to integrate

⇒ Try to deform integrand, preserving as much symmetry as possible

Deformed On-Shell Diagrams in $\mathcal{N} = 4$ SYM (I)

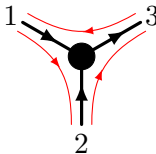
Deformed three-point amplitudes:

[Ferro, Łukowski, Meneghelli]
[Plefka, Staudacher, 2012]



$$= \int \frac{d\alpha_2}{\alpha_2^{1+a_2}} \frac{d\alpha_3}{\alpha_3^{1+a_3}} \delta^{4|4}(C_\circ \cdot \mathcal{W}) \simeq \frac{\delta^4(P) \delta^4(\tilde{Q})}{[12]^{1+a_3} [23]^{1-a_1} [31]^{1+a_2}}$$

$$c_1 = a_1 \equiv a_2 + a_3, \quad c_2 = -a_2, \quad c_3 = -a_3$$



$$= \int \frac{d\alpha_1}{\alpha_1^{1+a_1}} \frac{d\alpha_2}{\alpha_2^{1+a_2}} \delta^{8|8}(C_\bullet \cdot \mathcal{W}) \simeq \frac{\delta^4(P) \delta^8(Q)}{\langle 12 \rangle^{1-a_3} \langle 23 \rangle^{1+a_1} \langle 31 \rangle^{1+a_2}}$$

$$c_1 = a_1, \quad c_2 = a_2, \quad c_3 = -a_3 \equiv -a_1 - a_2$$

Yangian: $\hat{\mathcal{J}}^a \in \mathfrak{psu}(2, 2|4)$, $\hat{\mathcal{J}}^a = f^a_{bc} \sum_{1 \leq i < j \leq n} \hat{\mathcal{J}}^b_i \hat{\mathcal{J}}^c_j + \sum_{i=1}^n \mathbf{u}_i \hat{\mathcal{J}}^a_i$

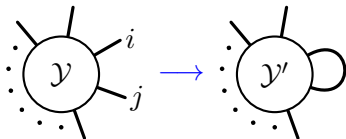
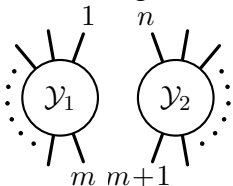
Invariance conditions: $u_j^+ = u_{\sigma(j)}^-$, $u_j^\pm = u_j \pm c_j$

[Beisert, Broedel]
[Rosso, 2014]

Deformed helicities: $\mathfrak{h}_j = 1 - \mathfrak{C}_j$, $h_j = 1 - c_j$

Deformed On-Shell Diagrams in $\mathcal{N} = 4$ SYM (II)

Bigger on-shell diagrams can be obtained by gluing: Products & Fusion



Reduced diagram $\simeq$ permutation σ

Invariance conditions from gluing: $u_j^+ = u_{\sigma(j)}^-$ $u_j^\pm = u_j \pm c_j$

General deformed on-shell diagram:

[Beisert, Broedel]
Rosso, 2014]

$$\mathcal{Y}(\mathcal{W}_i, a_i) = \int \prod_{j=1}^{n_F-1} \frac{d\alpha_j}{\alpha_j^{1+a_j}} \delta^{4k|4k}(C \cdot \mathcal{W}),$$

Edge deformation parameters α_i equal central charges on internal lines, fully determined by external u_i, c_i via left-right paths

Deformations in ABJM: Four-Vertex

The Basic diagram in
ABJM is the four-vertex:

$$\mathcal{A}_4 = \text{four-vertex diagram} = \frac{\delta^3(P) \delta^6(Q)}{\langle 12 \rangle \langle 23 \rangle}$$

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$$\hat{\mathcal{A}}_4(z) = \text{diagram of a four-vertex with a red dot and parameter } z \text{ on the right arm} = \frac{\delta^3(P) \delta^6(Q)}{\langle 12 \rangle^{1-z} \langle 23 \rangle^{1+z}}$$

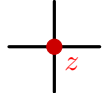
Admits a deformation, parameter z

Can show invariance under level-zero $\mathfrak{osp}(6|4)$

[TB, Huang, Loebbert]
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Level-one generators:
$$\hat{\mathfrak{J}}^a = f^a_{bc} \sum_{1 \leq i < j \leq n} \mathfrak{J}_i^b \mathfrak{J}_j^c + \sum_{i=1}^n u_i \mathfrak{J}_i^a$$

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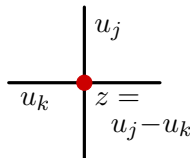
Level-one generators:
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$$\hat{\mathfrak{P}}^{\alpha\beta} = \sum_{1 \leq j < k \leq n} \frac{1}{2} \left[(\mathfrak{L}_{j\gamma}^{(\alpha} + \delta_\gamma^{(\alpha} \mathfrak{D}_{j)} \mathfrak{P}_k^{\gamma\beta)} - \mathfrak{Q}_j^{(\alpha A} \mathfrak{Q}_k^{\beta) A} - (j \leftrightarrow k) \right] + \sum_k u_k \mathfrak{P}_k^{\alpha\beta}$$

Invariance under $\hat{\mathfrak{P}}$ implies full $Y(\mathfrak{osp}(6|4))$ invariance.

Constraints:

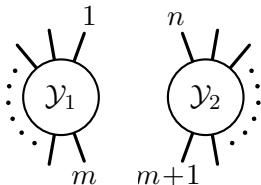
$$u_1 = u_3, \quad u_2 = u_4, \quad z = u_1 - u_2.$$



Deformations in ABJM: Gluing

Construct bigger deformed diagrams from four-vertex

Products: Invariance trivial, no constraints



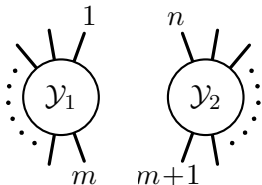
$$\mathcal{Y}(1 \dots n) = \mathcal{Y}_1(1 \dots m) \mathcal{Y}_2(m+1 \dots n)$$

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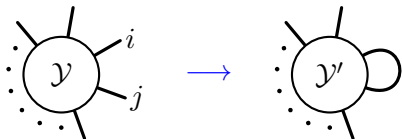
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Fusing:
$$\mathcal{Y}'(\dots) = \int d^{2|3} \Lambda_i d^{2|3} \Lambda_j \delta^{2|3}(\Lambda_i - i \Lambda_j) \mathcal{Y}(\dots, i, j, \dots)$$

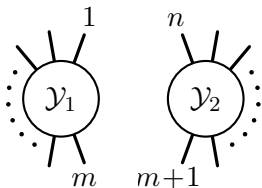


Constraint:

$$u_i = u_j$$

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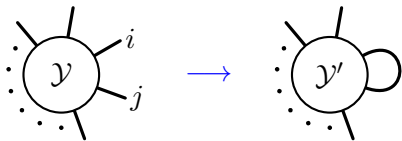
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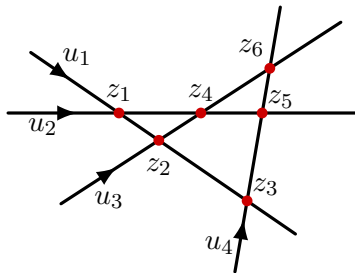
$$\textcolor{red}{u}_i = \textcolor{red}{u}_j$$

Can construct **all deformed diagrams** by iterated product and fusion

Deformations in ABJM: General Diagrams

Fundamental vertex is four-valent

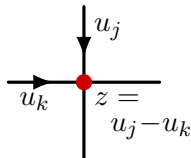
$\Rightarrow$ General $2k$ -point diagram can be drawn with k straight lines



Characterized by order-two permutation σ , $\sigma^2 = 1$

One evaluation parameter on each line, $u_j = u_{\sigma(j)}$

Vertex parameters z_i completely fixed by constraint



Deformations in ABJM: R-Matrix Formalism (I)

Reformulate invariants in terms of R-matrix construction.

$$R_{jk}(z) \circ \text{ (diagram of vertex } f \text{ with legs } j, k, \cdot, \cdot) = \text{ (diagram of vertex } f \text{ with legs } j, k, \cdot, \cdot \text{ and internal lines } \Lambda', \Lambda'' \text{ meeting at } z \text{)}$$

$$(R_{jk}(z) \circ f)(\Lambda_j, \Lambda_k) \equiv \int d\Lambda' d\Lambda'' \mathcal{A}_4(z)(\Lambda_j, \Lambda_k, i\Lambda', i\Lambda'') f(\Lambda'', \Lambda')$$

R-matrix kernel is four-point amplitude

Just a reformulation of gluing $\Rightarrow R_{jk}(z)$ trivially preserves invariance

R-operator intertwines two single-particle representations

Permutes evaluation parameters:

$$\hat{\mathfrak{J}}^a(\dots, u_j, u_k, \dots) R_{jk}(z) = R_{jk}(z) \hat{\mathfrak{J}}^a(\dots, u_k, u_j, \dots)$$

(on space of invariants)

Deformations in ABJM: R-Matrix Formalism (II)

R-operator permutes evaluation parameters:

$$\hat{\mathfrak{J}}^a(\dots, u_j, u_k, \dots) R_{jk}(z) = R_{jk}(z) \hat{\mathfrak{J}}^a(\dots, u_k, u_j, \dots)$$

Invariants are chains of R-matrices acting on vacuum Ω_{2k} :

$$\mathcal{Y}_{2k} = R_{i_\ell, j_\ell}(z_\ell) \dots R_{i_1, j_1}(z_1) \Omega_{2k}, \quad \Omega_{2k} = \prod_{j=1}^k \delta^{2|3}(\Lambda_{2j-1} + i\Lambda_{2j})$$

Vacuum permutation: $\sigma = [1, 2][3, 4] \dots [2k-1, 2k]$

Action of R-matrices conjugate the permutation:

$$\mathcal{Y}_{2k} \rightarrow R_{ij} \mathcal{Y}_{2k} \implies \sigma \rightarrow [i, j] \cdot \sigma \cdot [i, j]$$

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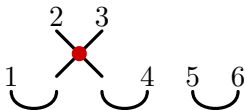
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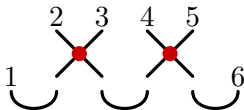
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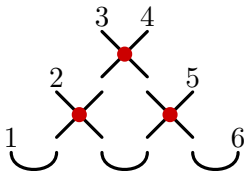
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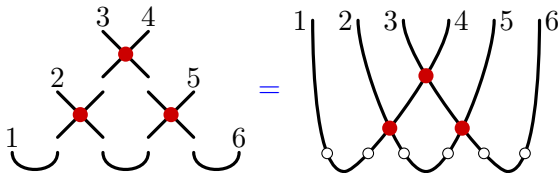
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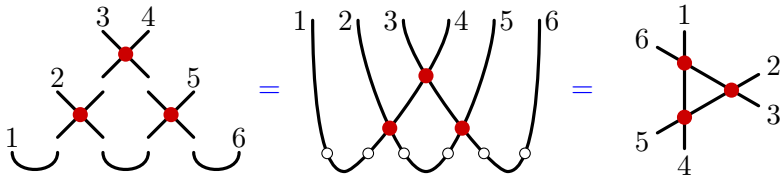
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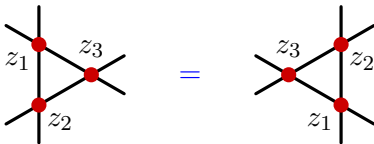
Yang–Baxter Equation

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Invariance requires that parameters z_i satisfy constraints.

R-operator satisfies Yang–Baxter equation (triangle equality):



$$R_{ij}(w-v)R_{j\ell}(w-u)R_{ij}(v-u) = R_{j\ell}(v-u)R_{ij}(w-u)R_{j\ell}(w-v)$$

Deformed Amplitudes?

Tree amplitudes are linear combinations of on-shell diagrams (BCFW)

$\mathcal{N} = 4$ SYM

Each n -point diagram:

n central charges, n evaluation parameters, n constraints

Sensible Yangian representation:

Requires same parameters on every diagram

Number of BCFW terms grows factorially

⇒ Constraints almost always outnumber parameters

[Beisert, Broedel]
Rosso, 2014]

Exception: n -point MHV and 6-point NMHV

Loops: Four-point integrand admits deformation → [Johannes' talk]

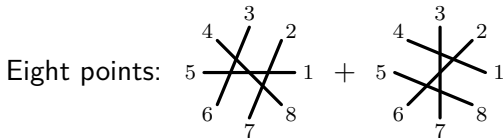
Integration non-trivial; result not easy to interpret

No admissible deformations for higher-point BCFW integrands.

Deformed Amplitudes?

ABJM

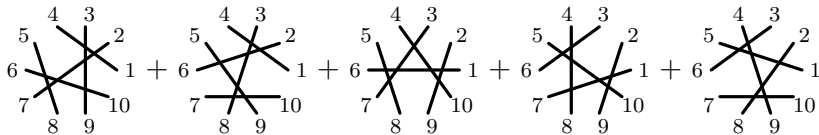
Four-point and six-point amplitudes are single diagrams. ✓



[Huang
Wen]

⇒ Allows for a one-parameter deformation: $u_1 - u_2$. ✓

Ten points:



⇒ No non-trivial deformation

[TB, Huang, Loebbert
Yamazaki, 2014]

General $(2p + 4)$ -point amplitude: $(2p)!/(p!(p + 1)!)$ diagrams.

[Huang
Wen]

⇒ No non-trivial deformation beyond eight points

Deformed Graßmannian Integral

Directly deforming BCFW decomposition fails in general

⇒ Need deformed “parent” object

Natural parent object: **Graßmannian integral**

Parametrized by **top cell** diagram ⇒ deformation straightforward

$\mathcal{N} = 4$ SYM:

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Yamazaki, 2014 Staudacher, 2014]

$$\mathcal{G}_{n,k}(\mathcal{W}_i, b_i) = \int \frac{d^{k \cdot n} C}{|\mathrm{GL}(k)|} \frac{1}{M_1 \dots M_n} \delta^{4k|4k}(C \cdot \mathcal{W})$$

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Parameters $u_j^\pm = u_j \pm c_j$ constrained by top-cell permutation:

$$u_j^+ = u_{\sigma(j)}^-, \quad \sigma(j) = j + k$$

Exponents b_j related to central charges:

$$\mathfrak{e}_j = -\mathcal{W}_j^c \frac{\partial}{\partial \mathcal{W}_j^c} \quad \Longrightarrow \quad b_i = \frac{1}{2}(u_i^- - u_{i-1}^-) = \frac{1}{2}(u_{i-k}^+ - u_{i-k-1}^+)$$

Deformed integral enjoys the **full Yangian symmetry**

Deformed Graßmannian Integral

ABJM:

$$\mathcal{G}_{2k}(\Lambda_i, b_i) = \int \frac{d^{k \times 2k} C}{|\mathrm{GL}(k)|} \frac{\delta^{k(k+1)/2}(C \cdot C^T) \delta^{2k|3k}(C \cdot \Lambda)}{M_1 \dots M_k}$$

Evaluation parameters fixed by permutation of top cell:

$$u_j = u_{j+k}$$

No central charges to fix $b_i \Rightarrow$ Act directly with generator:

$$\hat{\mathfrak{J}}(u_i) \mathcal{G}_{2k}(b_i) = 0 \quad \Rightarrow \quad b_j = u_j - u_{j-1} \quad (1 \leq j \leq k)$$

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$$u_j = u_{j+k}$$

No central charges to fix $b_i \Rightarrow$ Act directly with generator:

$$\hat{\mathfrak{J}}(u_i) \mathcal{G}_{2k}(b_i) = 0 \quad \Rightarrow \quad b_j = u_j - u_{j-1} \quad (1 \leq j \leq k)$$

What to Do With It?

Deformed Graßmannian integral

$$\mathcal{G}_{n,k}(\mathcal{W}_i, b_i) = \int \frac{d^{k \cdot n} C}{|\mathrm{GL}(k)|} \frac{1}{M_1^{1+b_1} \dots M_n^{1+b_n}} \delta^{4k|4k}(C \cdot \mathcal{W})$$

Interpret this directly as the deformed tree amplitude?

Properties

Can no longer localize $n(k-2) - k^2 + 4$ integrations on residues

Can set some b_i to zero and still localize on tree contour

⇒ Known deformed BCFW sums for MHV and $n = 6$ NMHV

⇒ In general requires all $b_i = 0$

⇒ No new deformations

Integrate this (not by residues) on some contour? → [Matthias' talk]

Deformed Momentum-Twistor Diagrams

For $\mathcal{N} = 4$ SYM:

Momentum-Twistor Diagrams

→ [Song's talk]

Mathematically the same as conventional diagrams, $\mathcal{W} \rightarrow \mathcal{Z}$

Can deform in exactly the same way

Yangian now based on **dual** superconformal symmetry, $\mathfrak{J}^A_{\mathcal{B}} = \mathcal{Z}^A \frac{\partial}{\partial \mathcal{Z}^B}$

BCFW Decomposition

Amplitudes $\mathcal{A}_{n,k} / \mathcal{A}_{n,k}^{\text{MHV, tree}}$: Sum of momentum-twistor diagrams

Deformation parameters v_i, c_i^{dual} again constrained by $v_j^+ = v_{\sigma(j)}^-$

Simplest non-trivial example: Six-point NMHV. Three terms.

Admits two-parameter deformation.

Again no deformations admitted at higher points.

Also no deformations at loop level

Momentum-Twistor Graßmannian Integral

Momentum-twistor Graßmannian integral

$$\tilde{\mathcal{G}}_{n,k}(\mathcal{Z}_i, \tilde{b}_i) = \int \frac{d^{k \cdot n} \tilde{C}}{|\mathrm{GL}(k)|} \frac{1}{\tilde{M}_1^{1+\tilde{b}_1} \dots \tilde{M}_n^{1+\tilde{b}_n}} \delta^{4k|4k}(C \cdot \mathcal{Z})$$

Relation to conventional twistor integral

Reduce twistor to momentum-twistor Graßmannian: [Arkani-H., Bourjaily, Goncharov
Cachazo, Postnikov, Trnka, 2012]

Reduction from C to $\tilde{C}$ reduces k to $k - 2$. Relation between minors:

$$M_i = \langle i, i+1 \rangle \dots \langle i+k-2, i+k-1 \rangle \tilde{M}_{i+1}$$

Induces relation between twistor and momentum-twistor invariants:

$$\mathcal{Y}(\mathcal{W}) \simeq \frac{\delta^4(P) \delta^8(Q)}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle} \tilde{\mathcal{Y}}(\mathcal{Z}) \quad (\text{undeformed})$$

For deformed Graßmannian integrals:

$$\mathcal{G}_{n,k}(\mathcal{W}_i, b_i) \simeq \frac{\delta^4(P) \delta^8(Q)}{\langle 12 \rangle^{\gamma_1} \dots \langle n1 \rangle^{\gamma_n}} \tilde{\mathcal{G}}_{n,k-2}(\mathcal{Z}_i, b_{i-1}), \quad \gamma_j = 1 + \frac{u_j^- - u_{j+1-k}^-}{2}$$

Summary

- ▶ Deformed on-shell graphs for ABJM
- ▶ R-matrix construction for $\mathfrak{osp}(6|4)$ Yangian invariants
- ▶ Deformed Grassmannian integral for $\mathcal{N} = 4$ SYM
- ▶ Deformed momentum-twistor Grassmannian integral
- ▶ Deformed Grassmannian integral for ABJM

Outlook/Questions

Can deformed BCFW expansion be rescued?

Different set of central charges for each diagram?

Integration contour for Graßmannian integral?

→ [Matthias' talk]

Lift to deformed amplituhedron?

Likely requires global coordinates

→ [Nima's talk]

Deformation at loop level?

Helpful for integrating the on-shell integrand?

Do the deformed amplitudes have any direct physical meaning?

What do the deformations mean for the geometric & differential structure of the Graßmannian?

Relation between Yangian & dual Yangian in deformed case?

Deformations compatible with exact symmetry?