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Theoretical Prediction
of Head Tail Tune Shift
in PETRA

by

R. Klatt, R.D. Kohaupt, T. Weiland

Abstract

The shift in frequency of the fundamental m = 0 head tail mode in a storage ring can be calculated from the optics and wake potentials. Such calculations have been performed for various beam pipe components in PETRA and are summarized here. Comparision with measurements shows good agreement for long bunches and that the theoretical description of the impedance is incomplete for short bunches (high frequencies).

List of symbols

q	charge
$w_{\delta\perp}(s)$	wake potential of a point charge at distance s behind it
$w_{\lambda\perp}(s)$	wake potential inside a bunch of longitudinal line charge density $\lambda(s)$
$\lambda(s)$	line charge density
β	beta function at the location of the impedance
$k_{\perp}$	transverse kick parameter
E_o	nominal particle energy
$\Delta p_{\perp}$	transverse momentum
c	speed of light
f_z	vertical betatron frequency
f_x	horizontal betatron frequency

1. Introduction

The shift in oscillation frequency of the head tail bunch modes in a storage ring results from wake field forces excited in many objects such as rf cavities, separator tanks and vacuum chamber joints. The wake field force, integrated over the transit time, is a wake potential that describes the distribution of momentum change inside a bunch of particles. If the beam offset from the ideal orbit is small the transverse wake potential gives the transverse kick imported by one particle to a second one at distance s:

$$c \cdot \Delta p_{\perp}^{1\to 2} = q_1 \cdot r_1 \quad w_{\delta\perp}(s) \qquad s > 0$$

$$c \cdot \Delta p_{\perp}^{1\to 2} = 0 \qquad\qquad s < 0$$

The kick experienced by test particle # 2 does not depend on its own radial position (i.e. offset). Taking r_1 as the bunch offsett and q_1 as the total charge in a bunch we obtain a kick distribution inside the bunch given by:

$$c \cdot \Delta p_{\perp}(s) = q_1 \cdot r_1 \cdot w_{\lambda\perp}(s)$$

A typical result is shown in figure 1.

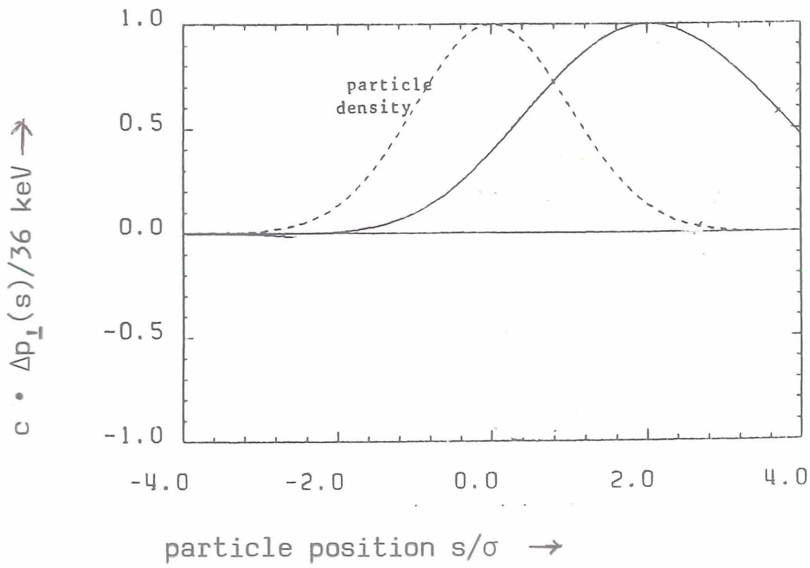

Figure 1: Transverse kick inside a Gaussian bunch after having passed a single cell PETRA accelerating cavity off axis (total charge = 1 μC, beam offset = 0.5 cm, σ = 2 cm)

The dipole moment of the total kick per unit offset r_1 and per unit charge q_1 inside a bunch with longitudinal density $\lambda(s)$ can be calculated as*)

$$k_\perp(\lambda) = \frac{\int_{-\infty}^{+\infty} \lambda(x)\, w_{\lambda\perp}(x)\, dx}{\left[\int_{-\infty}^{+\infty} \lambda(x)\, dx\right]^2}$$

This "transverse kick parameter" has its equivalent in the case of higher order mode losses where one uses the loss parameter k.
The betatron tune shift of the m=0 mode due to a quadrupole error is given by[1]:

$$\Delta Q = \frac{-1}{4\pi} \oint_{\text{circum-ference}} \Delta k(s)\, \beta(s)\, ds$$

Since the wake force behaves like a quadrupole ($\vec{F} \propto \vec{r}$) we can obtain the tune shift from the wake potential as[2]:

$$\Delta Q = \frac{-e}{4\pi E_0} \langle\beta\rangle \int_{\text{bunch}} \lambda(x)\, w_{\lambda\perp}(x)\, dx$$

$$= \frac{-e}{4\pi E_0} \langle\beta\rangle \cdot k_\perp \cdot q$$

For all objects of cylindrical symmetry we can use TBCI[2] to obtain correct values[3] for the transverse kick parameters.

For nonsymmetric objects we need a fully threedimensional BCI program. Since this program is not yet in its final shape and not yet sufficiently tested we use cylindrical models of 3D objects.

Figure 2a shows the layout of all the cavity types in PETRA and figure 2b shows the cross section of PETRA vacuum chamber joint. The vacuum chamber joints are modeled by circular ones having the chamber height as radius.
Figure 2b also shows the quadrupole chamber joints and separator tanks in PETRA.
In table I, we list the transverse kick parameter $k_\perp$ for each of these components at various bunch lengths (assuming Gaussian shape).

*) $k_\perp$ is directly calculated by the TBCI program

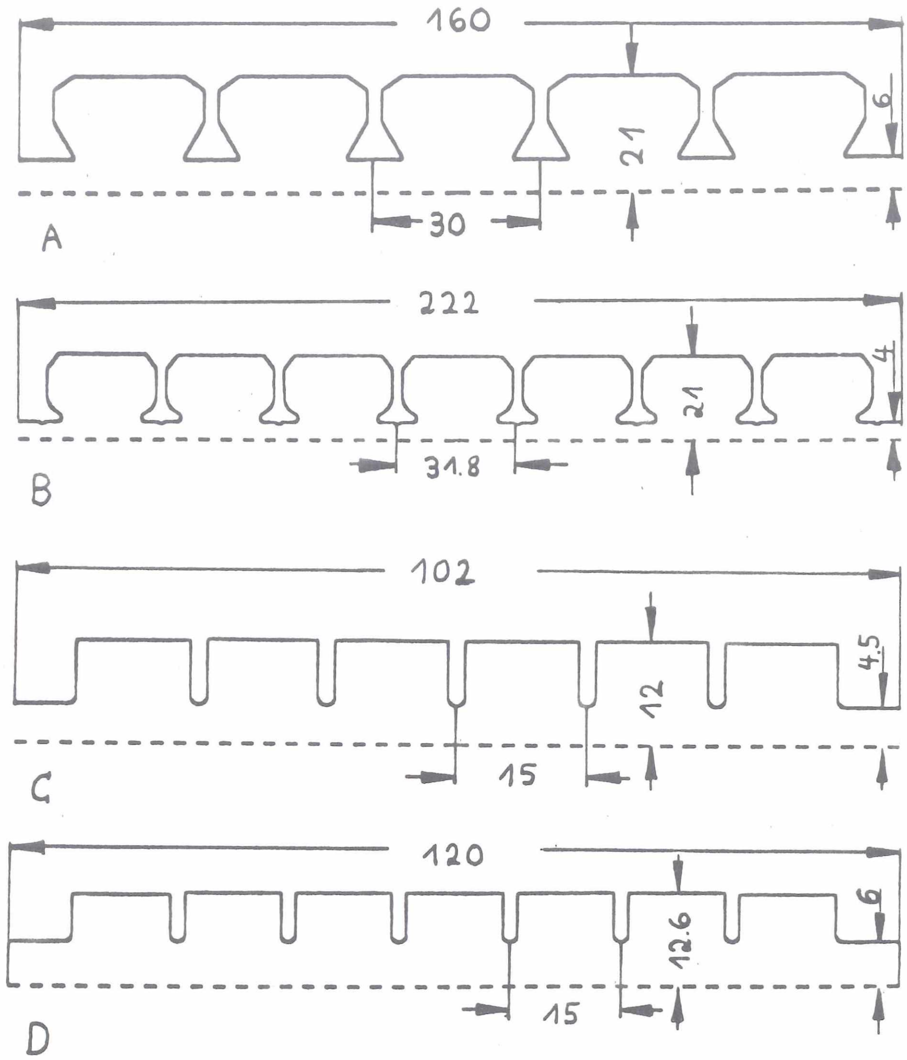

Figure 2a: Layout of PETRA cavities

A 500 MHz five cell

B 500 MHz seven cell

C 1 GHz six cell

D 1 GHz seven cell

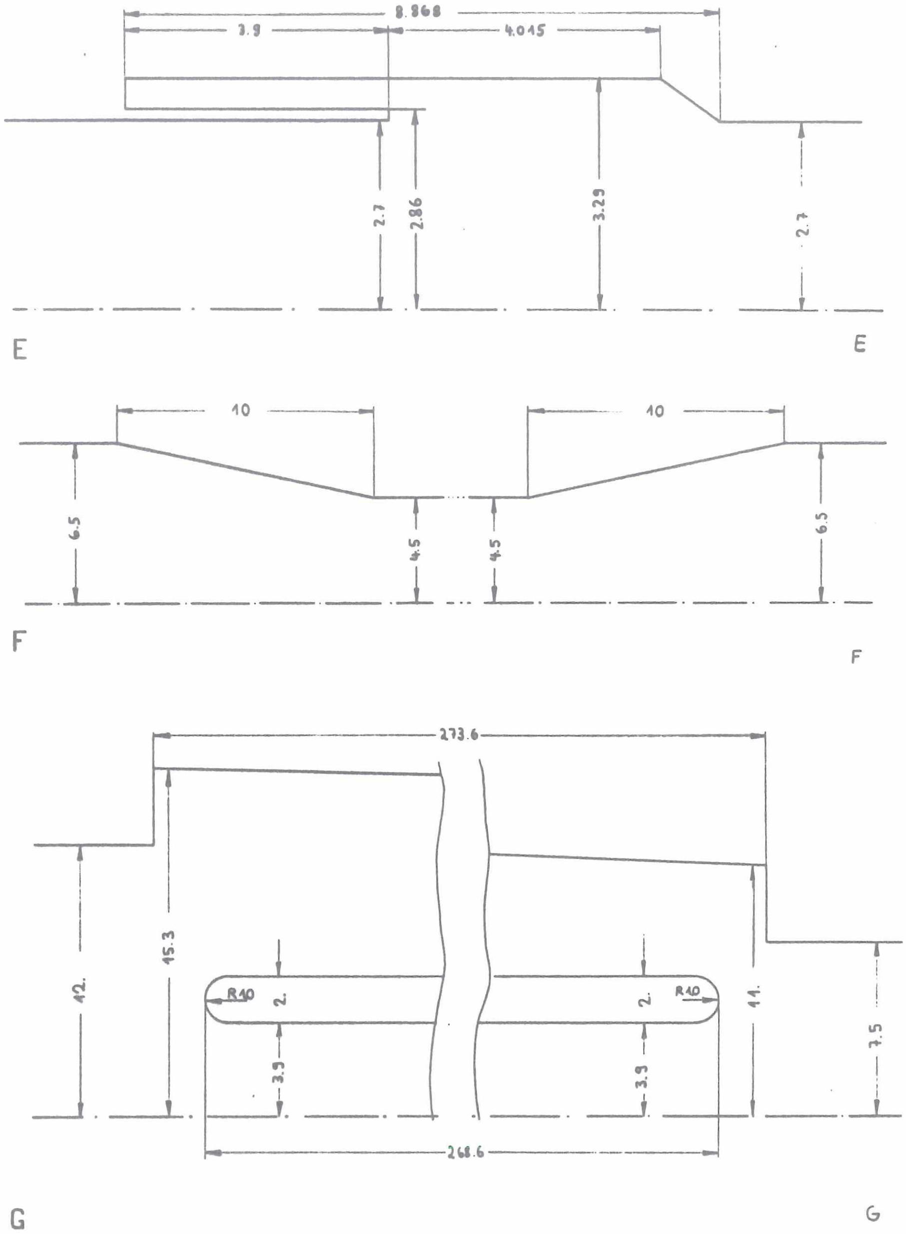

Figure 2b: Geometry of various components in PETRA

E vacuum chamber joint, bending magnet

F vacuum chamber joint, quadrupole magnet

G separator tank

In order to simplify the use of table I we give the formula for the tune shift in PETRA as:

$$\frac{\Delta f}{Hz} = - \frac{0.01135}{(E_o/7GeV)} \left(\frac{I_b}{mA}\right) \left(\frac{\langle\beta\rangle}{m}\right) \left(\frac{k_\perp}{V/pCm}\right)$$

Table I: Transverse kick parameter of various components in PETRA

A: 5 cell 500 MHz cavity

σ/cm	1	2	3	4	5
$k_\perp$/(V/pCm)	10	14	15	15	13

B: 7 cell 500 MHz cavity

σ/cm	1	2	3	4	5
$k_\perp$/(V/pCm)	32	39	37	33	29

C: 6 cell 1000 MHz cavity

σ/cm	1	2	3	4	5
$k_\perp$/(V/pCm)	19	21	20	17	14

D: 7 cell 1000 MHz cavity

σ/cm	1	2	3	4	5
$k_\perp$/(V/pCm)	10	12	12	11	9

E: Vacuum chamber joint in arc (vertical)

σ/cm	1	2	3	4	5
$k_\perp$/(V/pCm)	3.7	2.8	2.0	1.5	1.1

F: Vacuum chamber joint in straight section (vertical and horizontal)

σ/cm	1	2	3	4	5
$k_\perp$/(V/pCm)	0.86	0.73	0.52	0.36	0.26

G: Separator tank (vertical)

σ/cm	1	2	3	4	5
$k_\perp$/(V/pCm)	15	22	25	27	27

2. Examples

For PETRA at injection energy with a MI100 optics and

- 60 five cell cavities
- 232 vacuum chamber joints type E
- 48 vacuum chamber joints type F
- 8 separator tanks

we obtain

σ/cm	1	2	3	4	5
Δf_z/Hz	276	290	271	252	215
Δf_x/Hz	109	149	157	156	135

However measurements[4] show much higher vertical tune shift than predicted, namely:

σ/cm	Δf_z/Hz theory	Δf_z/Hz measured
1.05	277	580
2.10	273	445
5.00	215	270

More recently, using, for example, the optics MI100G and with

- 56 five cell cavities 500 MHz
- 56 seven cell cavities 500 MHz
- 16 six cell cavities 1000 MHz
- 8 seven cell cavities 1000 MHz
- 288 vacuum chamber joints type E
- 56 vacuum chamber joints type F
- 8 separator tanks

we obtain

σ/cm	1	2	3	4	5
Δf_z/Hz	676	753	704	633	544
Δf_x/Hz	474	586	572	522	453

In this case measurements show

σ/cm	Δf_z/Hz theory	Δf_z/Hz measured
1.04	679	960
4.85	463	490

3. Conclusions

We find a clear discrepancy between measured and predicted tune shifts especially for short bunches (σ = 1 cm). For long bunches however, we find good agreement and also no significant discrepancy between the tune shift in both planes, horizontal or vertical.

It follows that for long bunches the symmetric impedance is dominated by the cavities and that predictions are quite accurate.

For short bunches the impedance is dominated by vacuum chamber joints and other unindentified components outside the cavity region.

In order to complete our theoretical knowledge of this complex system we must do more accurate computations with the threedimensional BCI program[5] and investigate other sources of non-geometric impedance such as aluminium oxide in the vacuum system[6].

4. Literature

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/6/ A. Piwinski, private communication

5. Acknowledgements

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