DESY M-83-25
August 1983

WAKE FORCE COMPUTATION IN THE TIME DOMAIN FOR LONG STRUCTURES

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Presented at 12th International Conference on High Energy Accelerators
Fermi National Accelerator Laboratory, Batavia, IL, August 11-16, 1983

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Abstract

One is often interested in calculating the wake potentials for short bunches in long structures using TBCI. For ultra-relativistic particles it is sufficient to solve for the fields only over a "window" containing the bunch and moving along with it. This technique reduces both the memory and the running time required by a factor that equals the ratio of the structure length to the window length. For example, for a bunch with σ_z of one picosecond traversing a single SLAC cell this improvement factor is 15. It is thus possible to solve for the wakefields in very long structures: for a given problem, increasing the structure length will not change the memory required while only adding linearly to the CPU time needed.

Theory

For a rigid bunch of high energy electons moving at constant velocity parallel to the axis of a cavity, the wakefield effects depend only very weakly on the energy γ: we can normally replace the bunch speed by the speed of light with no loss of accuracy in the solution of the wake potentials. Once this approximation is made, then due to causality it follows that: 1) no fields can ever precede the first particle of the bunch and 2) a particle at any set position within or behind the bunch will never be affected by anything that happens behind it. These two observations allow us to solve for the fields in a time-dependent code such as TBCI[1] only over a mesh window moving with the bunch without introducing any errors in the wake potentials. The front of this window is at the longitudinal coordinate of the first particle; the back is defined by the last position for which the wake potential is desired (see Fig. 1).

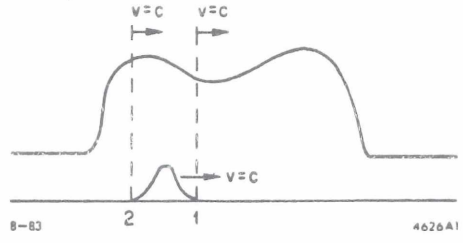

Fig.1. The moving mesh. If the first particle of the bunch is at point 1 and point 2 is the last position for which the wake potential is desired then the vertical dashed lines bound the moving mesh over which calculations are done.

This method of solving for the wake potentials can yield great savings in the memory and CPU time required for wake potentials whose range is shorter than the length of the accelerating structure. In cylindrically symmetric structures where N_r and N_z are respectively the number of mesh points in the radial and axial directions, the CPU time necessary for one time step is normally proportional to the number of mesh points N_p ($=N_rN_z$). Often we are only interested in the wake potentials over the bunch length (for example, 8σ of a Gaussian bunch) and if this length is small compared to the structure length the number of time steps needed for the bunch to pass the structure is approximately proportional to N_z. Then the total CPU time T needed for the bunch to pass is given by

$$T \approx C_1 N_r N_z^2 \qquad (1)$$

where C_1 is a constant of proportionality. But with the moving window method described above the same problem's space and time requirements become

$$N_p = N_r N_b \qquad \text{and} \qquad T \approx C_1 N_r N_z N_b \qquad (2)$$

where N_b is the number of mesh points over the bunch length. Thus, for example, for a truncated Gaussian bunch with $\sigma = \frac{1}{4}$ mm (and assuming a total bunch length of 8σ) traversing one SLAC cell (whose gap length is 2.9 cm) an improvement factor of 15 is found for both the number of mesh points and the CPU time needed.

From Eq.(2) we see that the moving window approach allows us to solve for the wake potentials of very long structures. Or conversely, given a structure, we can solve for the wake potentials of much shorter bunches than with the static mesh method. Thus the moving mesh method greatly increases the range of parameters over which a time domain code such as TBCI is useful.

Results

A modified version of TBCI was created incorporating a moving window. As an example of its use we describe results obtained for a Gaussian bunch with $\sigma = 1$ mm traversing one SLAC accelerator section. Each SLAC accelerator section is a constant gradient (disk loaded) structure, with 86 cells of equal length but otherwise varying in dimension.[2] The SLAC wake potentials calculated up to now were done using the cell of Fig. 2 repeated several times.[3] (Figure 2 represents SLAC cell #45, the 'average' SLAC cell, although in the real structure the irises are round.) The model used for the calculations here is shown schematically in Fig. 3. This represents every other cell of an accelerator section (a total of 44) with the round irises again squared off. The outer dimension for all cavities was taken to be a constant 2.5 cm. This lessens the number of mesh points needed without changing the results, since a 1 mm bunch never sees the outer walls of the SLAC cavities.

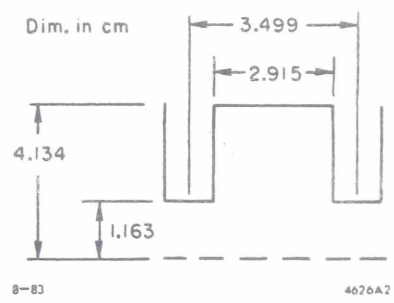

Fig.2. Model of SLAC cell, #45, the average SLAC cell, as used in Ref.3.

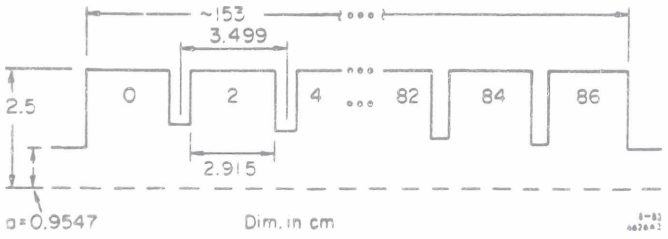

Fig.3. Model of SLAC accelerator section used here. Cavities #0 and #86 represent the coupling cavities. The other iris radii are given in Ref. 2.

Figure 4 shows how the longitudinal, dipole and quadrupole (m = 0,1,2) wake potentials per cell differ as the number of cells taken in the model was changed. As usual, the wake potentials for a ring of charge with radius equal to the side tube radius a(= .9547 cm) and with a charge distribution which varies as cos mφ (where φ is the azimuthal angle) is given. The other components, or the results for other values of radius can be obtained from simple scaling equations (see Ref. 4). The transition to the 44 cell results is seen to be smooth. Figure 5 shows the 44 cell results compared with the earlier results[3] where cell #45 was taken to be representative of the entire structure. In each case the tapered model gives somewhat higher wake

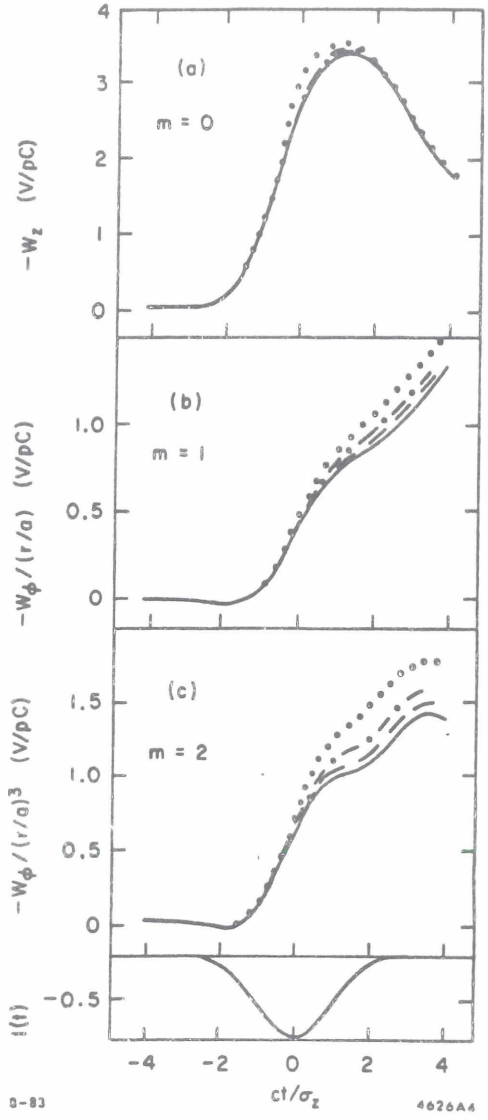

Fig.4. Wake potentials per cell when taking 6(dots), 12(dot dashes), 18 (dashes) and 44(solid line) evenly distributed cells in the model. For m = 0 the longitudinal wake potential W_z is given; for m = 1,2 the azimuthal component W_ϕ is given. The current distribution is also shown with the front of the bunch to the left. σ = 1 mm, a = .9547 cm.

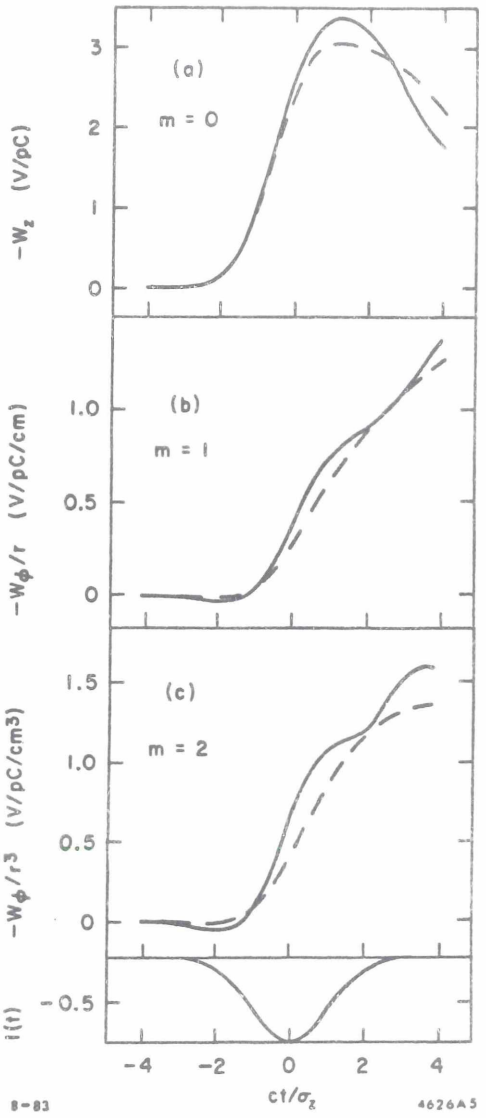

Fig.5. The m = 0,1,2 wake potentials per cell taking the 44 cells of Fig. 3 (solid line) compared with the results when taking only the cell of Fig. 2 repeated[3] (dashes). Again the front of the bunch is to the left. σ = 1 mm.

potentials. The CPU time for the 44 cell model running on an IBM 3081 was 10(20) minutes for m = 0 (m = 1,2); the number of mesh points used was N_p = 4141.

References

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2. G. A. Loew, et al., The Stanford Two-Mile Accelerator, (W. A. Benjamin, Inc., New York, 1968), p. 145.
3. K. Bane and T. Weiland, Paper submitted to this conference.
4. A. Chao, SLAC-PUB-2946, June 1982.