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## Abstract

The next generation of electron positron colliding beam facilities (LEP, SLC) are likely to be severely limited in their performance by collective wake force effects. There are two methods for achieving quantitative answers for these wake forces: one using the frequency domain approach and the other integrating Maxwell's equations directly in the time domain. It is important - with respect to the design of future machines - to establish that the two approaches give the same results. In this paper, these methods are briefly reviewed and their results are compared. Without any adjustable constants good agreement is found for both the decelerating and deflecting forces down to very short bunches.

## Introduction

To solve the interaction between electron (or positron) bunches and the cavities through which they traverse exactly requires the self-consistent solution of Maxwell's Equations and the equation of motion. Fortunately the betatron and synchrotron wave length in high energy electron machines is large compared to the typical interaction length due to a cavity and we can split the problem into two parts. The first part, which we discuss in this paper, involves solving the fields due to rigid bunches traversing the cavities. The second part, of which we will not speak, takes these solutions as driving terms in the equation of motion. In discussing the beam-cavity interaction we restrict our considerations to the case of cylindrically symmetric structures, however allowing non-symmetric field distributions. The source is a particle (or a distribution of particles) moving at the speed of light parallel to the axis at some offset $r$ and azimuthal angle $\varphi$. No externally applied fields are present.

When a charged particle #1 passes off axis at a distance $r_1$ and at azimuthal position $\varphi_1$ through a beam pipe with a sudden change in geometry, changes in the electromagnetic fields are excited. Due to causality these changes can act only on a test particle #2 travelling at a distance (s) behind particle #1, see fig. 1. Similarly the force on a particle #2 inside a bunch can come only from that portion of the bunch that travels ahead of it. In either case the force experienced by particle #2 depends on both particles coordinates and time.

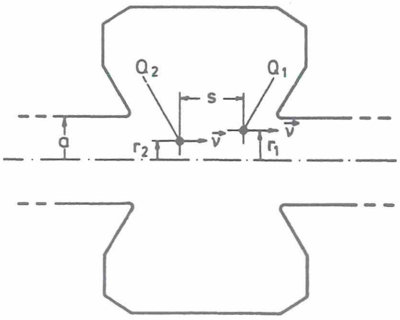



Figure 1:
Two particles traversing an accelerating cavity (of cylindrical symmetry) at the speed of light

Figure 2 shows the transient decelerating "wake forces" experienced by particles inside a Gaussian bunch passing through a PETRA an accelerating cavity with a long exiting side tube. It is found from TBCI results that the time structure of the forces depends strongly on the longitudinal and radial position of the test particle and that the time interval over which these forces are nonzero extends significantly over the cavity passage time.

However the wavelength of particle motion in linacs and storage rings is normally large compared to the interaction range of these self fields. Thus all we need for an accurate description of a particle's dynamics is the net effect of these fields over its passage. In order to obtain the net change in momentum one integrates the wake forces over the passage in a comoving coordinate system:

$$\int_{-\infty}^{+\infty}\vec F(r_1,\varphi_1,r_2,\varphi_2,s,t,z=ct-s)\,c\,dt = \vec W(r_1,\varphi_1,r_2,\varphi_2,s) \quad (1)$$

The time argument has disappeared; The "wake potential/1/2/" $\vec W$ depends only on the particle positions relative to each other. For an infinitely repeating structure the wake potential is defined as the average momentum change per period.

The wake potential due to a point charge $\vec W_\delta$, also called the "delta function wake potential", can serve as a Green's function and is related to the wake potential due to a smooth distribution $\rho(r,\varphi,s)$:

$$\vec W_\rho(r,\varphi,s) = \iiint \vec W_\delta(r',\varphi',s-s')\,\rho(r',\varphi',s')\,dV \quad (2)$$

Due to causality the integration over s' extends only over the region s'-s>0.

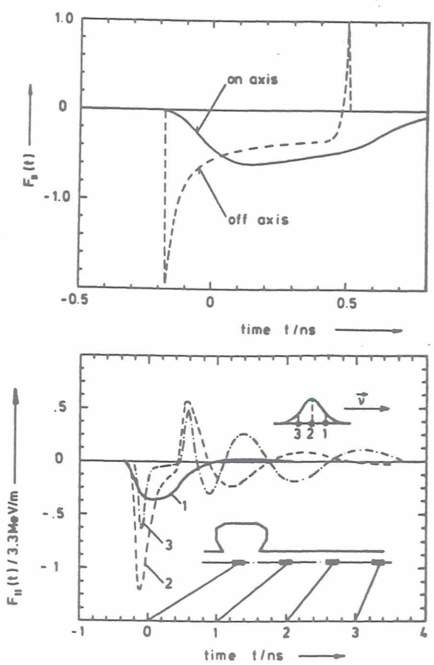



Figure 2: Decelerating monopole wake forces (m=0) seen by test particles travelling in a Gaussian bunch (charge 1µC, σ=3cm) along a PETRA cavity with adjacent beam pipe.
a) two particles on axis and near the beam pipe surface 2 σ ahead the center
b) three particles on axis at different longitudinal positions (2σ ahead of the middle, at the center and 2σ behind the middle)

Since we consider only structures of cylindrical symmetry it is useful to decompose the driving current density generated by particles #1 into a sum over it azimuthal Fourier components:

$$\vec{J} = \frac{cQ_1 \; \lambda(s)\, \delta(r-r_1)}{\pi \, r_1} \sum_{m=0}^{\infty} \frac{\cos m(\varphi-\varphi_1)}{1+\delta_{om}} \, \vec{e}_z \qquad (3)$$

Each term of the sum represents a charged ring the charge density of which varies as $\cos m(\varphi-\varphi_1)$ in azimuthal direction. For each ring we may calculate the electromagnetic field separately. The total wake potential is then just the sum over all contributions due to each ring.

$$\vec{W}(r_1,\varphi_1,r_2,\varphi_2,s) = \sum_{m=0}^{\infty} \vec{W}_m(r_1,\varphi_1,r_2,\varphi_2,s) \qquad (4)$$

For particles at the speed of light a very general and useful scaling property of the wake potentials holds /1/3/4/:
(For simplicity we choose $\varphi_1=0, \varphi=\varphi_2-\varphi_1$)

$$\vec{W}_m(r_1,r_2,\varphi,s) = m(r_1/a)^m(r_2/a)^{m-1} \; w_m(s) \cos m\varphi \; \vec{e}_r$$
$$- \; m(r_1/a)^m(r_2/a)^{m-1} \; w_m(s) \sin m \varphi \vec{e}_\varphi \qquad (5)$$
$$- \; (r_1/a)^m(r_2/a)^m \quad w_m'(s) \cos m \varphi \vec{e}_z$$

a is the beam hole radius and $w_m$ is a characteristic wake function. Note that the transverse and the longitudinal potentials are defined by only one function $w_m(s)$. Normally only the first few terms are needed (m=0,1,2) since $r_1$ and $r_2$ are small compared to the beam hole radius a.

The quantitative knowledge of wake potentials is of great importance for the design of any electron positron accelerator since it has been found that the performance of existing facilites - such as PETRA and PEP - is limited by wakefield effects. Two complementary computational methods have been developed in order to calculate the wake potential: time and frequency domain approaches. Both will be briefly discussed with their limitations and finally compared quantitatively using some test examples for which both methods apply.

## Time Domain Approach

The computer programs that have been developed for the computation of wake forces in the time domain are described in detail elsewhere /5/6/ (TBCI). We want to list here only a few characteristics:

1) Maxwell's equations are solved directly by the FIT /7/ discretization method in time domain.
2) The moving particles may have any smooth (but constant) distribution and any constant speed $0<\beta<1$.
3) The code handles very large grids (over 200.000 nodes) and can solve for arbitrarily shaped cylindrically symmetric geometries with open beam tubes.

There is one more problem intrinsic to the time domain calculations as can be seen from the time structure of the forces in figure 2. Electromagnetic energy is radiated into the downstream beam pipe and this effect increases with shorter and shorter bunches. As a consequence one would have to use larger and larger grids in order to complete the electromagnetic interaction taking place downstream far outside the cavity. There are two solutions to the problem. One makes use of eq. 5 and integrates the wake force at the tube surface/4/. This yields the correct wake potentials but not the forces. (See figure 2.) The infinite range for the integration is reduced to the cavity opening since outside the gap the bunch moves along a tube wall and the tangential forces on a perfectly conducting surface are zero. A second way of solving this problem is by using an "infinitely large" mesh but only a comoving window for the actual computation as discussed elsewhere /8/. This method includes correctly all the transient details of the wake forces.

## Frequency Domain Approach

The wake functions corresponding to $\vec{W}_\delta$ can be written in terms of the eigenfrequencies $\omega_{mn}$ and loss parameters $k_{mn}$ (provided such modes exist) as

$$/1/2/: \quad w_m'(s) = 2Q \sum_{n=1}^{\infty} k_{mn} \cos(\omega_{mn} \, s/c) \qquad (6)$$

The components of $\vec{W}_\delta$ may then be obtained by eq. 5. The loss parameter $k_{mn}$ defined for each mode represents a quantity which is purely geometric and measures the coupling of a speed of light particle to the mode. (Basically this is the geometric impedance r/Q). Note that this approach a priori neglects the transient character of the wake forces.

A practical problem in applying this formalism is the fact that the sum over the modes n - for a given m - does not converge very well /1/9/. In fact, on the order of a thousand modes are necessary to calculate sufficiently accurate wake potentials inside short bunches. Besides a perturbation theory ansatz /10/, there are two computer codes that can handle that many modes: KN7C /11/ and TRANSVERS /12/.

However these codes solve for infinitely repeating periodic structures of rather special shape ("pill box" with side tubes, see figure 3). For realisticly shaped single cavities resonant modes can be obtained by mesh codes such as URMEL /13/ but only up to the cut off frequency of the beam port. Typically this gives only a few tens of modes for each m.

In principle the modal analysis yields via eq. 6 the wake potential behind a point charge, - which could not be obtained in time domain -. However the approximation is hidden in the fact that one never can calculate "all" modes and often a semiempirical analytic extension has to be added. Finally one gets an approximation for the point charge wake potential whereas time domain calculations approximate point charges by very short bunches.
Characteristics of the modal method are:

1) It is applicable only to resonant structures
2) It is applicable -so far- only to specific cavities
3) Large number of modes are necessary ($\sim$ 1000 for each m)

## Comparison

In order to compare the modal and time domain approach we had to choose a cavity which can be solved for by KN7C, TRANSVERS and TBCI as well. A pill-box like structure has been used as shown in figure3. This cavity is a good model for the SLAC linac cavity (#45) and several hundred modes have been computed for this structure previously /1/.

There are three possibilities of checking the results of the codes. One is to compare single modes in the frequency domain obtained from different codes, one is to compare integral quantities such as parasitic energy loss but the most important one is to compare the wake potentials inside the bunch directly for the shortest bunch that TBCI can handle.

The first method is -as mentioned before- rather limited since only very few modes can be compared between KN7C/TRANSVERS and URMEL. However a very good agreement was found for frequencies and loss parameters for the lower modes.

Another important quantity - though rather intensitive to errors - is the total parasitic loss $k_{tot}$ of a bunched beam into the cavity modes. In earlier comparisons /14/ agreement was found to be better than 5 % for realistic parameters, see fig.4.

Comparison of integral quantities cannot give information about details in the wake potentials. The ultimate comparison is done by running TBCI with bunches of various lengths down to the shortest one possible. The modal result for the Green's function is folded into the smooth bunches and can thus be compared with TBCI. To similate an inifinitely repeating structure each TBCI result here gives is the difference of the wake potential for four cells minus that for three cells.

Good agreement was found for the SLAC linac cavity as test example down to 1 mm short bunches. Figure 5 shows results for monopole, dipole and quadrupole wake potentials (m=0,1,2). The agreement -as one expects- gets better with longer bunches. In all the comparisons made /15/ the discrepancy never exceeded 10 % inside the bunch (± 4σ).

## Conclusion

Collective wakefield effects in future high energy electron positron facilities can be studied a priori by means of particle simulation codes and analytical approaches. However, a key element in all these computations is a good quantitative knowledge of the wake forces. Now that two independent methods have been developed and compared resulting in good agreement confidence can be given to predictions about the performance of future facilities provided that particle simulation and the analytical methods can be corroborated by measurements in existing machines such as PETRA and PEP.

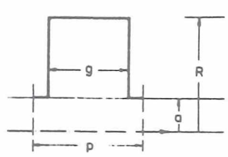



Figure 3: Infinitely repeating structure with periodicity p as used in the computer codes KN7C and TRANSVERS for modelling the SLAC linac cavity

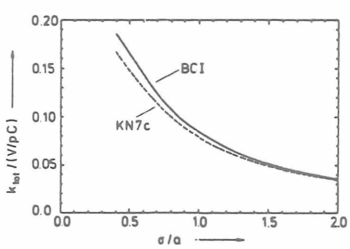
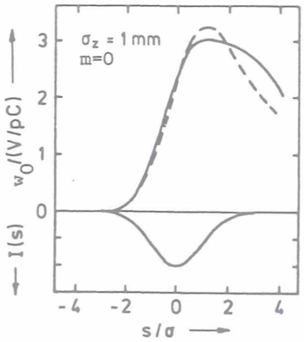
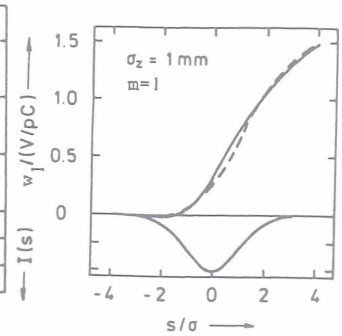



Figure 4:
Comparison of the total loss parameter computed with KN7C and TBCI for a cavity as shown in figure 3
(p = 3 m, g = 1 m, R = 1m, a = .125 m)



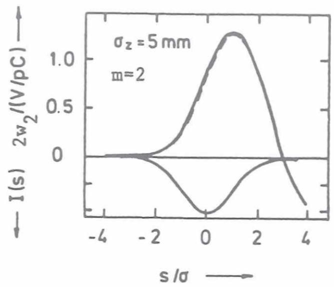
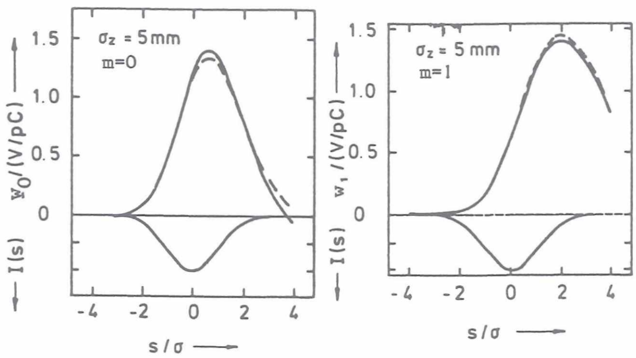
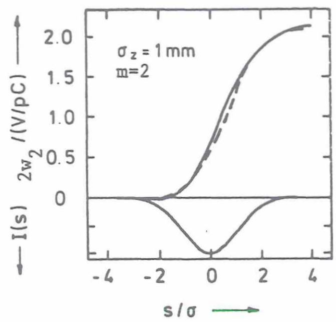




Figure 5:
Comparison of wake potentials for the SLAC linac model cavity obtained by TBCI (solid lines) and KN7C (for m=0)/TRANSVERS (for m>0) (dashed lines)
(p = 3.499cm, g = 3.15cm, R = 4.13cm, a = 1.163cm)

a) $\sigma_z$ = 1 mm




b) $\sigma_z$ = 5 mm

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