

TBCI AND URMEI - NEW COMPUTER CODES FOR WAKE FIELD AND CAVITY MODE CALCULATIONS

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Summary

Wake force computation are most important for any study of instabilities in high current accelerators and storage rings. These forces are generated by intense bunches of charged particles passing cylindrically symmetric structures on or off axis. The adequate method for computing such forces is the time domain approach. The computer Code TBCI computes for relativistic as well as for nonrelativistic bunches of arbitrary shape longitudinal and transverse wake forces up to the octupole component. TBCI is not limited to cavity-like objects and thus applicable to bellows, beam pipes with varying cross sections and any other nonresonant structures. For the accelerating cavities one also needs to know the resonant modes and frequencies for the study of instabilities and mode couplers. The complementary code named URMEI computes these fields for any azimuthal dependence of the fields in ascending order. The mathematical procedure being used is very safe and does not miss modes. Both codes together represent a unique tool for accelerator design and are easy to use.

Introduction

When a bunch of charged particles traverses an accelerating cavity - or any other structure of varying shape - electromagnetic fields are excited. These wake fields cause serious problems in linear accelerators and storage rings: beam break up, emittance growth, head tail turbulence, bunch lengthening, higher order mode losses and beam loading.

The adequate method for computing such wake fields is solving Maxwell's equations in time domain including the presence of free moving charges. As a result one obtains all forces as a function of time and space coordinates for a given geometry and charge distribution. For many instability considerations one uses - so far - only forces that are averaged over a certain period. These forces - with all fast cycling terms averaged out - can be integrated in a comoving bunch frame. This integration yields so called wake potentials which depend now only on the relative position in the comoving bunch frame. The explicit time dependence has disappeared.

Forces that interact within one bunch length - so called short range forces - depend basically only on the shape of the environment. These forces seem to establish fundamental limitations to the performance of various accelerators in terms of charge per bunch. Since these forces do not depend on the material properties or other quality factors there is no simple way to overcome this limitation. A typical wake potential of a Gaussian bunch is shown in figure 1.

There is a rather important accelerator component that is usually treated in frequency domain: the rf accelerating cavities. For a given - complete - set of resonant modes the averaged wake potentials can also be written as a sum over the mode contributions¹. Unfortunately many modes are needed to describe accurately the forces inside the bunch since the sum over the modes converges very slowly. Furthermore there exist not many modes in a cavity with open side tubes, typically only a few tens. The only structure which is open and has an infinite set

of modes is an infinitely repeating chain of identical cavities. For the SLAC linac an infinitely repeating chain is a very good model and the computation of averaged forces for pill-box like structures can be found in the literature^{2,3,4} as well as perturbation theory approach for bellow-like objects⁵.

We are still left with the problem of forces that are carried by - those few - modes in cavities. Although a computation in time domain always yields the full answer a wake long behind the exciting bunch is dominated by resonant modes with a high quality factor and thus better described by the shunt impedance, quality factor and resonant frequencies.

These long range forces may cause multi-turn and/or multi-bunch instabilities in storage rings and linacs. Especially for superconducting cavities the forces are important since modes may not be damped within one turn even in very large rings. A typical long range wake potential is shown in figure 1. The resonant characteristic can be seen but only far behind the driving bunch whereas the short range wake shows no resonant structure. Although long range forces can always be avoided by means of damping antennas the mode patterns are of interest in order to place the antennas properly.

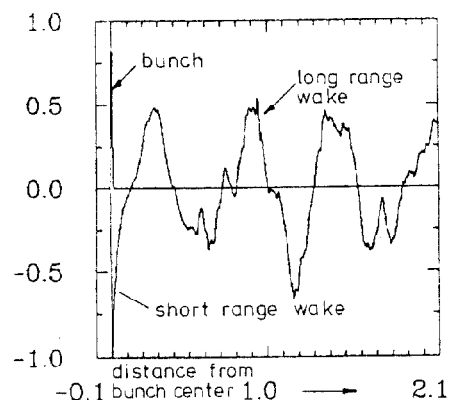


Figure 1:
Typical (normalized) decelerating wake potential behind a Gaussian bunch ($\sigma = 4$ mm) due to the PETRA cavities. Note that the resonant character shows up only far behind the bunch.

Summarizing so far there are two groups of electromagnetic forces that generate the beam-environment interaction: short range and long range forces. Short range forces within one and the same bunch are adequately computed in time domain whereas long range forces are better described by resonant modes in frequency domain.

For cylindrically symmetric structures two complementary computer codes have been developed for the calculation of decelerating and deflecting forces excited by bunches on or off axis. Both codes TBCI⁶ and URMEI⁷ can process arbitrarily shaped geometries. They will be described briefly in the following.

Common features of TBCI and URMEI

Both codes are based on the FIT method in a rectangular grid⁸. Each cell may be further subdivided into two triangular sub cells. This subdivision improves significantly the approximation of odd shaped boundaries.

In frequency domain often only half a cavity cell is necessary for the computation due to some symmetries.

In time domain the full cell has to be used and represented in a mesh. On both sides a quasi open boundary condition eases the computation and makes long tapers unnecessary⁹. This special boundary condition also enables the treatment of structures with little variation in shape - such as bellows - which are a hard problem in frequency domain.

A beam traversing at the speed of light a cavity off axis at a distance a produces a current density that can be written as

$$(1) \quad \vec{J}(r, \phi, z, t) = \frac{e_z \beta c}{2\pi a} \sum_{m=-\infty}^{+\infty} \cos m\phi \delta(r-a) \lambda(z-\beta ct)$$

Each term of the sum represents a charged ring varying as $\cos(m\phi)$ in azimuthal direction and carrying a line charge density $\lambda(s=z-\beta ct)$. TBCI computes the fields of each ring separately. After the calculations are done the contributions can be summed but it is known³ that only a few azimuthal m -terms are necessary for beams that pass structures near the axis. For storage ring applications monopole ($m=0$) and dipole wakes seem to suffice whereas the quadrupole ($m=2$) term is important too in linacs¹⁰. In frequency domain resonant modes are computed for each given m starting from the lowest frequency up to the cutoff frequency of the beam pipes.

TBCI-time domain

The TBCI program is an extension of BC11,9,12,13 to deflecting fields ($m > 0$). Details of the theory are published elsewhere⁶. This program has been used with meshes of over 200,000 nodes. TBCI can deal with arbitrarily shaped cavities and charge distributions. The speed of the particles may have any value between zero and the speed of light $0 < \beta < 1$. In Fig. 2 the fields generated by a Gaussian bunch are shown that passes three PETRA cavity cells. The lines shown represent the direction of the electric field ($r \cdot H\phi = \text{const.}$).

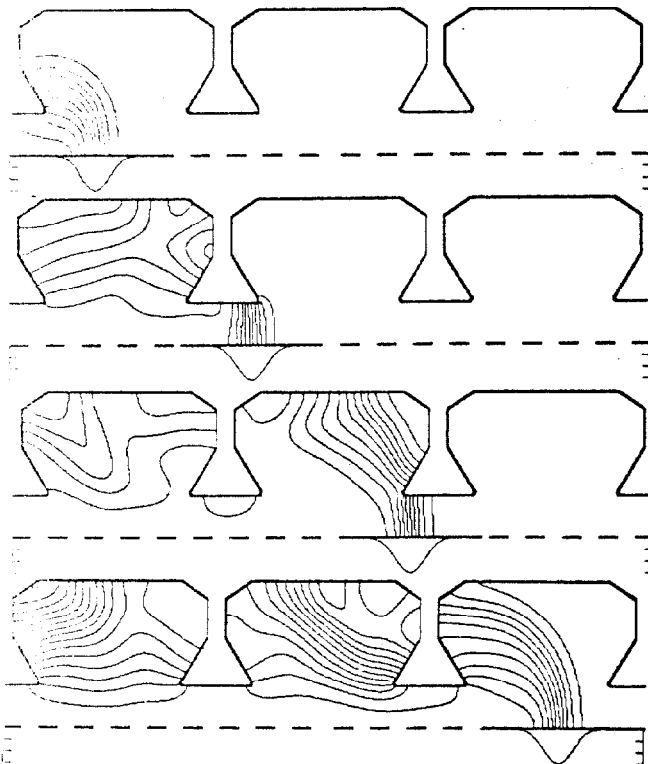


Figure 2: Fields excited by a Gaussian bunch ($\sigma = 2\text{cm}$) traversing a PETRA cavity.

Integrating the longitudinal and transverse forces in a comoving bunch frame over the passage time yields the wake potentials. Fig. 3 shows for $m=0, 1$ and 2 the normalized wake potentials. The monopole terms is only decelerating whereas all higher contributions ($m > 0$) have decelerating and deflecting forces. We shall briefly summarize some applications of such TBCI calculations.

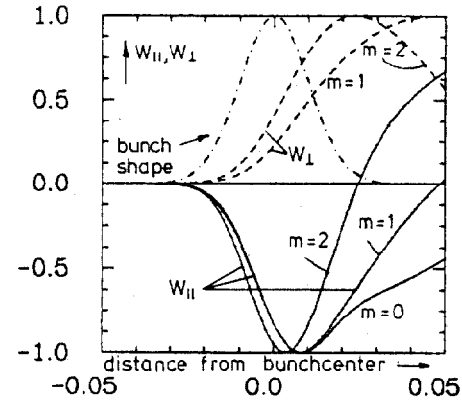


Figure 3: Normalized wake potentials due to PETRA cavities for monopole, dipole and quadrupole fields

First of all one can get the total energy lost by a beam due to the wake ($m=0$) potentials from:

$$(2) \quad U_{\text{tot}} = \int \lambda(s) w_{||}(s) ds.$$

Only a fraction of this total energy that the beam has lost remains in the cavities. Depending on the bunch length the fraction of the energy radiated into the side tubes can be significant¹⁴, see fig. 4.

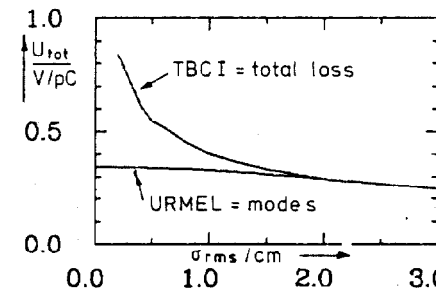


Figure 4: The total loss parameter as a function of bunch length for a single cell of the PETRA cavity computed with TBCI (all fields) and URMEL (modes only).

It is found that only 2/3 of the total energy lost by a beam of rms length 0,5 cm remains in the cavity. This part may be either computed via modes or with TBCI by integrating over the stored energy when the bunch has left the cavity for a while.

As a similar result one obtains the incoherent synchrotron tune and the head tail tune shift (all these integral quantities are computed by TBCI at the end of each run):

$$\frac{\Delta v_s}{v_{so}} \approx \frac{-1}{2V_{rf}} \int \lambda(s) \frac{\partial w_{||}(s)}{\partial s} ds; \quad \Delta v = \frac{-e\langle B \rangle_{rf}}{4\pi E_0} \int w_{\perp}(s) \lambda(s) ds$$

A much more important application of TBCI is found in particle tracking codes^{17,18,19}. The wake potentials generated by very short bunches can be used as Green's function for the particle-particle electromagnetic interaction^{18,20}.

URMEL - frequency domain

The most important type of structures that are usually investigated with a frequency domain approach are accelerating cavities. First of all the accelerating cavity has to be optimized with respect to the shunt impedance of the fundamental mode. Since accelerating structures makes normally use of the

axisymmetric fundamental mode it is sufficient for these purposes to have a computer code that solves for $m=0$ modes. Probably the best known code of this type is SUPERFISH21. A more complete list of similar programs may be found elsewhere7.

For other considerations but the fundamental mode the transverse deflecting modes are of great importance. In storage rings - especially when superconducting cavities are used - multi turn instabilities may limit severely the performance. For cooling reasons in superconducting cavities one also has to know what the field patterns look like and how much the modes couple to the beam.

There are two extensions that became known based on SUPERFISH-like approaches22,23. These two codes use triangular meshes and suffer from their choice of variables: $E\phi$ and $H\phi$. The code that will be described here -URMEL- uses field components in $\phi = \text{const.}$ plane: E_r and E_z . This choice and the FIT-method finally yield a simple algebraic eigenvalue problem the eigenvalue of which is directly the frequency squared. More details are described elsewhere7.

An important feature of URMEL is that modes are found with increasing frequency automatically in the right order. There is no chance to miss modes as there is in other codes22,23. The version that is presently in use computes up to 20 transverse modes and 50 modes for $m=0$. Together with the symmetries one can easily find all modes up to the cutoff frequency. In a typical cavity - the LEP or PETRA accelerating cavity - this amounts to more than 100 modes.

Typical cpu time on an IBM 3081 is 3 min per transverse modes and about 300 sec for each mode with $m=0$. URMEL computes all usual quantities such as loss parameter, quality factor and shunt impedance. A detailed user guide is available24.

Fig. 5 shows some modes of the new seven-cell PETRA 500 MHz cavity and figure 6 some modes in the new superconducting PETRA cavity25. As an example for a typical URMEL application TABLE I gives a list of all modes ($m=0, 1$ and 2) found in the superconducting cavity and some measurements for comparison.

Table I: Computed and measured modes in the superconducting PETRA cavity

mode	computed			measured		
	f/MHz	R/Q/ Ω/m	Q	f/MHz	R/Q/ Ω/m	Q
TM010	988.5	762.	29153	991.8	763.	29200
"TE111"	1358.9	124.	37938	1359.9	-	-
"TM110"	1398.1	307.	32140	1404.2	-	-
"TE211"	1838.9	29.	43049	1837.6	-	-
"TM210"	1858.7	27.	37738	-	-	-
TM011	1965.5	195.	34659	1967.4	-	-
"TE122"	2010.9	84.	35258	2014.	-	-
TM020	2015.5	.2	42652	2026.	-	-

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$$f_{\text{res}} = 1557. \text{ MHz}$$

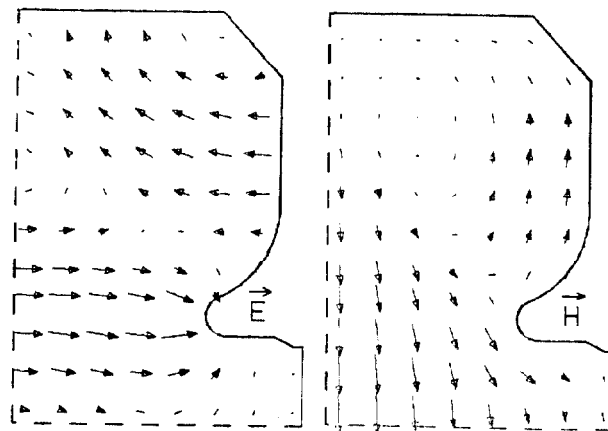


Figure 5: Deflecting dipole modes in the new PETRA cavity

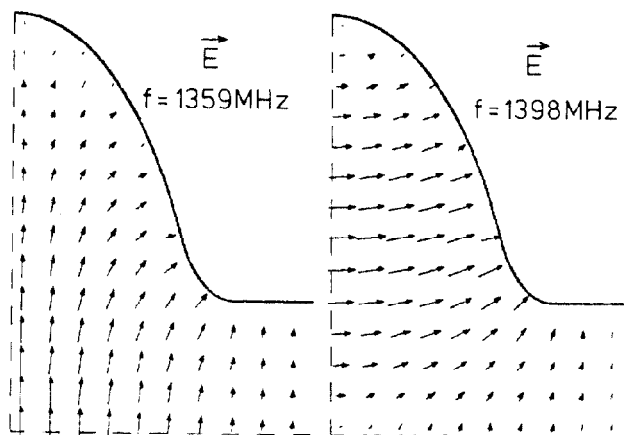


Figure 6: Deflecting dipole modes in the superconducting PETRA cavity

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