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The Program for Automatic Control of Beam Transfer Lines

S. Ananian, R. Manoukian*

*on leave of absence from
Yerevan Physics Institute

ABSTRACT

A set of programs for the control of beam transfer lines is presented. The sequential maximal likelihood algorithms are used for the estimation of beam initial parameters (X_0,X'_0,Z_0,Z'_0,$\Delta P/P$) and for the calculation of optimal control parameters.The efficiency of the algorithms is demonstrated by the results of the simulations on the electron beam transfer line from $PETRA$ to $HERA$.The investigations show that the use of sequential algorithms reduces the number of calculations and can be efficient for the optimal control of beam transfer lines.

1. INTRODUCTION

The traditional manual tuning of the beam transfer lines becomes inefficient with the increasing of both the beamlines length and the number of the magnets. That's why computer controlled beamlines are widely used now /1/.

The set of programs is presented in this paper,which can be used for beamline automatic control.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

Let A_i be the transfer matrix between the monitors M_i and M_{i+1} and B_i be the influence of the correctors on the beam,which are located between these monitors.If X_i is the beam state vector on the i monitor,which is consisted of r and z coordinates and of their derivatives,then one will have

$$X_{i+1} = A_i X_i + B_i U_i, \qquad i = 0, 1, ..., n, \tag{1}$$

where U_i is the vector which includes the strength of the correctors and n is the number of the monitors.

Usually we get the coordinates r and z with some errors from the monitors.The observation vector on the i monitor will be

$$Z_i = H_i X_i + V_i, \qquad i = 0, 1, ..., n, \tag{2}$$

where H_i is the matrix for separation of the coordinates from the state vector,V_i is the measurement errors.

Let's discuss some versions of the mathematical formulation of the beamline optimal control versus the sequence of observation and control.

1. It is assumed that after sequential measurements of beam coordinates $Z_i, i = 0, 1, ...n$ we need to calculate the control vector U_k.If to take the result of minimizing

$$J = \frac{1}{2} \| X_n \|_S^2 + \frac{1}{2} \sum_{k=0}^{n-1} (\| X_k \|_Q^2 + \| U_k \|_R^2) \tag{3}$$

with the constraintes (1),(2),as a criterion of optimal U_k,we shall have a well known problem of optimal regulator /2/,where S,Q and R are weighting matrices.Here $\| U_k \|_R^2 = U_k^T R U_k$.Optimal control will be

$$U_k = -L_k \tilde{X}_k, \tag{4}$$

where the matrix L_k is a result of Ricatti equation solved backward in time

$$L_k = R^{-1} B_k^T (P_{k+1}^{-1} + B_k R^{-1} B_k^T)^{-1} A_k, \tag{5}$$

$$P_k = Q + A_k^T (P_{k+1}^{-1} + B_k R^{-1} B_k^T)^{-1} A_k, \tag{6}$$

where $P_n = S$ and k is changed from $n-1$ to $0.\tilde{X}_k$ is the estimation of the state vector formed by the Kalman filter /2/

$$\tilde{X}_{k+1}^{j+1} = \tilde{X}_k^j + P_{j+1}A_j^T H_{j+1}^T R^{-1}(Z_{j+1} - H_{j+1}A_j\tilde{X}_k^j - H_{j+1}B_jU_j), \quad (7)$$
$$P_{j+1} = P_j - P_jA_j^T H_{j+1}^T (H_{j+1}A_jP_jA_j^T H_{j+1}^T + R)^{-1}H_{j+1}A_jP_j, \quad (8)$$

where $\tilde{X}_k^0$ is the initial guess of X_k state vector and if more information is available it can be the mean value of X_k, R is the noise covariance matrix,P_0 is the error covariance matrix of $\tilde{X}$,it showes the uncertainty of the initial guess and during the iteration procedure its diagonal components are decreasing,because the increase of available information reduces the paprameter uncertainty.In every j iteration step we take a measurement $\underline{Z_{j+1}}$ from the monitor $\underline{k+j}$ and improve the $\tilde{X}_k^j$ estimation.Here $\underline{A_j}$ is the transfer matrix between monitors $\underline{k}$ and $\underline{k+j}$, $\underline{U_j}$ are the correctors between the monitors $\underline{k}$ and $\underline{k+j}$, $\underline{H_{j+1}}$ is the separation matrix on $\underline{k+j}$ monitor.We can also use the information from the monitors which situated are before $\underline{k}$ monitor and in that case $\underline{A_j}$ will be the inverse of the transfer matrix between monitors $\underline{k-j}$ and $\underline{k}$. In this algorithm if the aprior information is not sufficient to get the Kalman estimation i.e. the matrices P_0,R and the mean value of X_k not known,then one can use the sequential least square estimation or optimal estimation of maximal likelihood estimator ($\tilde{X}_k^0 = 0$ and $P_0 \rightarrow \infty$). It is important that the calculations structure is the same for all cases.These algorithms are attractive, because they are efficient in the case that there are unknown parameters in matrix A_j.

2. From the measurements of Z_i on the all monitors one can estimate state vector X_0 and after that calculate the optimal control U_i.

The estimation of $\tilde{X}_0$ one can do with the Kalman filter and the control U_k can be calculated by the (4),where the sequence of

$$X_{i+1} = A_iX_i + B_iU_i, \quad X_0 = \tilde{X}_0 \quad (9)$$

can be used as X_k.

3. It is easy to get

$$X = AX_0 + BU, \quad (10)$$
$$HX = HAX_0 + HBU \quad (11)$$

from (1),(2),where

$$X = (X_1, X_2, ..., X_n)^T,$$
$$U = (U_1, U_2, ..., U_n)^T,$$
$$Z = (Z_1, Z_2, ..., Z_n)^T,$$
$$H = (H_1, H_2, ..., H_n)^T,$$
$$V = (V_1, V_2, ..., V_n)^T.$$

The aim of optimal control is to have $\| HX \|^2 \rightarrow 0$. If $\| HX \|^2 \cong 0$,then

$$HAX_0 = -HBU, \qquad Z = HAX_0 + V,$$

$$Z = -HBU + V. \qquad (12)$$

If we consider (12) as a linear model of observation of the Z_i measurements,the optimal estimation of U can be realized with the sequential Kalman estimator.

We consider, that the least approach of optimal control formation must be most efficient practically in spite of its heuristical form, because it is simple in calculation and the aprior information about B matrix can be easily improved experimentally by the simple calculations.

3. THE STRUCTURE OF THE PROGRAM

A set of programs is written for the investigation of the estimation and control algorithms.The programs consist of several modules,each of them can be operated separately in real time scale.

The *'TRANSPORT'* /3/ formalism is used here. The structure of the beam-line is the input file of the program.It can include bending magnets,quadrupoles, synchrotron magnets,correctors.The tilts of the elementes are given by the rotation matrix.The corrector is assumed to be an element which gives a kick in the middle.

It is possible to simulate the disturbances of the elements by vertical and horizontal kicks in the middle of each element.The program is consisted of the following modules:

-the module of beam trajectory simulation;

-estimation of the beam parameters X,X',Z,Z',ΔP/P;

-the control module.

This modules can operate separately using the structure file of the beam-line.The control module is consisted of subroutines of optimization based on the least square method (ordinary and sequential) and the subroutines for making local corrections.

4. THE RESULTS OF SIMULATIONS

The efficiency of the algorithms are investigated on the electron beam transfer line from *PETRA* to *HERA*.The beam line is about 219 meter and is consisted of 19 bending magnets and 19 quadrupoles.The control system includes 20 monitores and 22 correctors: 12 vertical and 10 horizontal. Almost all the monitores and the correctors are attached to the quadrupoles.The tiltes of bending magnets and the quadrupoles cause the coupling of horizontal and vertical motion /4/.

The process of measurement from the monitor is simulated with the help of the beam trajectory simulation module and random number generation program.The normal distributed noise with 0 mean value and variance of 1mm (the accuracy of the monitors is of 0.5mm) is used.

The X_0,X_0',Z_0,Z_0',ΔP/P estimations quality and efficiency of control algorithms were investigated.

1. The beam parameters estimation.

The sequential maximal likelihood filter is used.The general results of the simulation are:

- the estimations convergence is not sensitive to the initial elements of covariance matrix P_0 if their are chosen to be 10^5 - 10^6 order of magnitude of the X_0 and the initial values of X_0 are 0;

- after 10 iterations estimated values are in the range of tolerable accuracy.

The typical process of parameters $X = 2mm$,$X' = 0.1mrad$, $Z = 1mm$, $Z' = 0.1mrad$,$\Delta P/P = 0.002$ estimation versus to the number of iteration are in fig.1-5.

2.The optimal control estimation.

The distorted trajectory is the result of not 0 initial X_0 values and/or disturbances of the beamline elementes.The estimation is done by the algorithm 3 without the X_0 estimation using sequential and not sequential least square algorethm (maximal likelihood).

The results of the simulation show that after 5 iterations the corrected trajectory is in the range of tolerable accuracy.

The typical results of the control algorithm are shown in fig.6-13.The corrected trajectories of the beam are shown in fig.10-13, the dash line showes the corrected trajectory when the ordinary least square method is applied and the full line is the one when the sequential least square method is applied.Here the only 22 correctors are used for the correction.

The trajectories before and after correction are shown in fig.6-9. Here 11 bending magnets which have one power supply,4 septumes (2 in $PETRA$ and 2 in $HERA$) and 22 correctores are used as control parameters.

5. CONCLUSION

The obtained results show that these algorithms can be used efficiently for the automatic control of beam transfer lines.

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REFERENCES

1. L.Dahl, F Ehrich ' Computer codes for automatic tuning of the beam transport line at the Unilac ', in ' Computing in accelerator design and operation ',Berlin (Germany) 1983.
   H.Pines, F.Selph ' Automatic tuning of the LBL Bvalac transfer line ',LBL-11580,IEEE NS-28 1983.
   M.Lee,S.Clearwate,E.Theil,V.Paxon ' Modern Approaches to Accelerator Simulation and On-line Control ' SLAC-PUB-4213 February 1987.
2. A.P.Sage ' Optimum System control ' 1968 by Prentice-Hall Inc. Englewood Cliffs,New Jersey 07632.
3. K.L.Brown,F.Rothacher,D.C.Carey,Ch.Iselin ' TRANSPORT ', SLAC-91 (1977).
4. G.Jacobs (DESY) private communication.

FIGURE CAPTIONS

Fig. 1-5 The estimation of beam initial parameters versus the number of iterations.

Fig. 6-9 The distorted (dash line) and corrected (full line) trajectories on the monitors after 5 iterations.

Fig. 10-13 The trajectories corrected by ordinary least square method (dash line) and the one corrected by sequential maximal likelihood algorithm (full line).

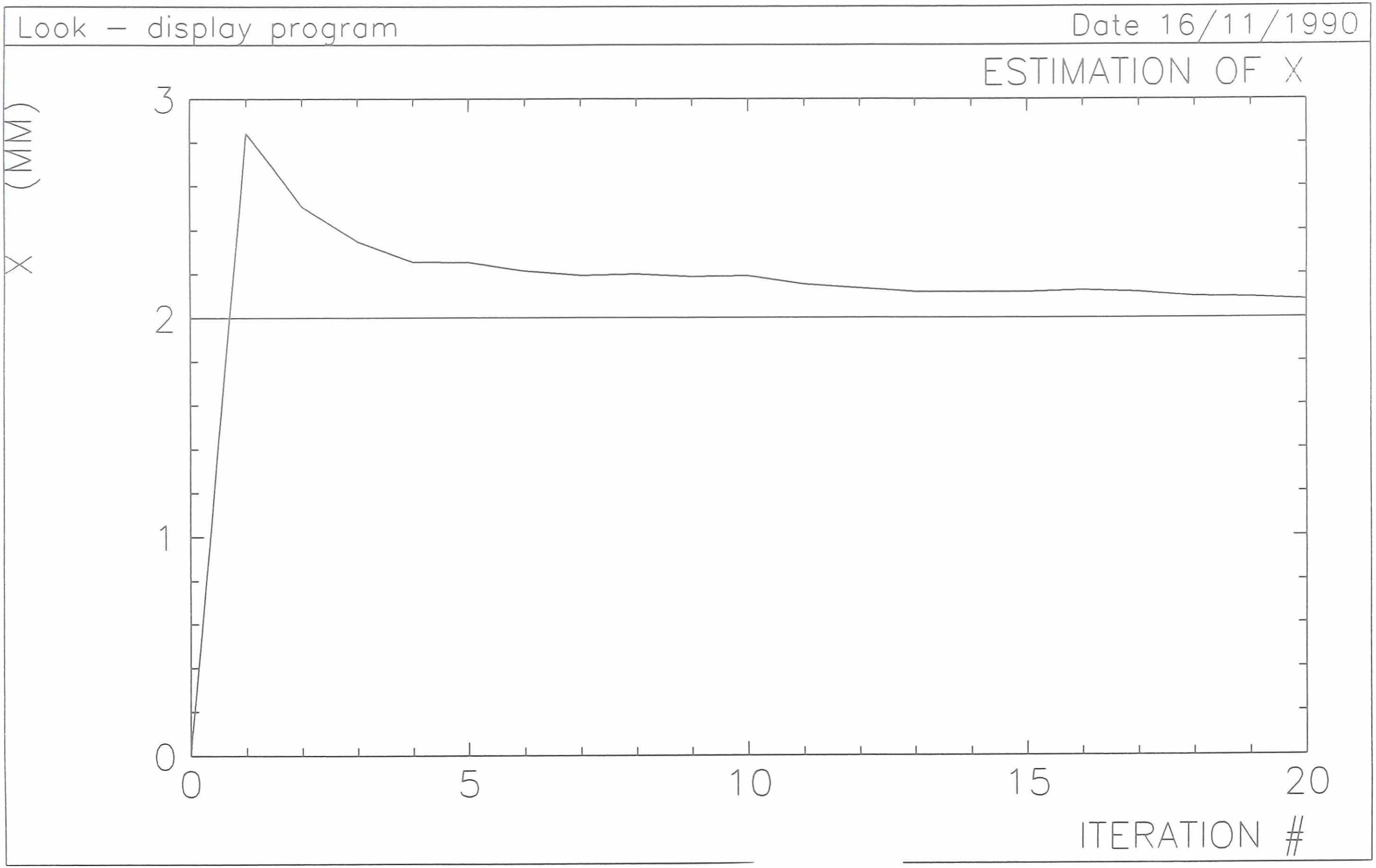
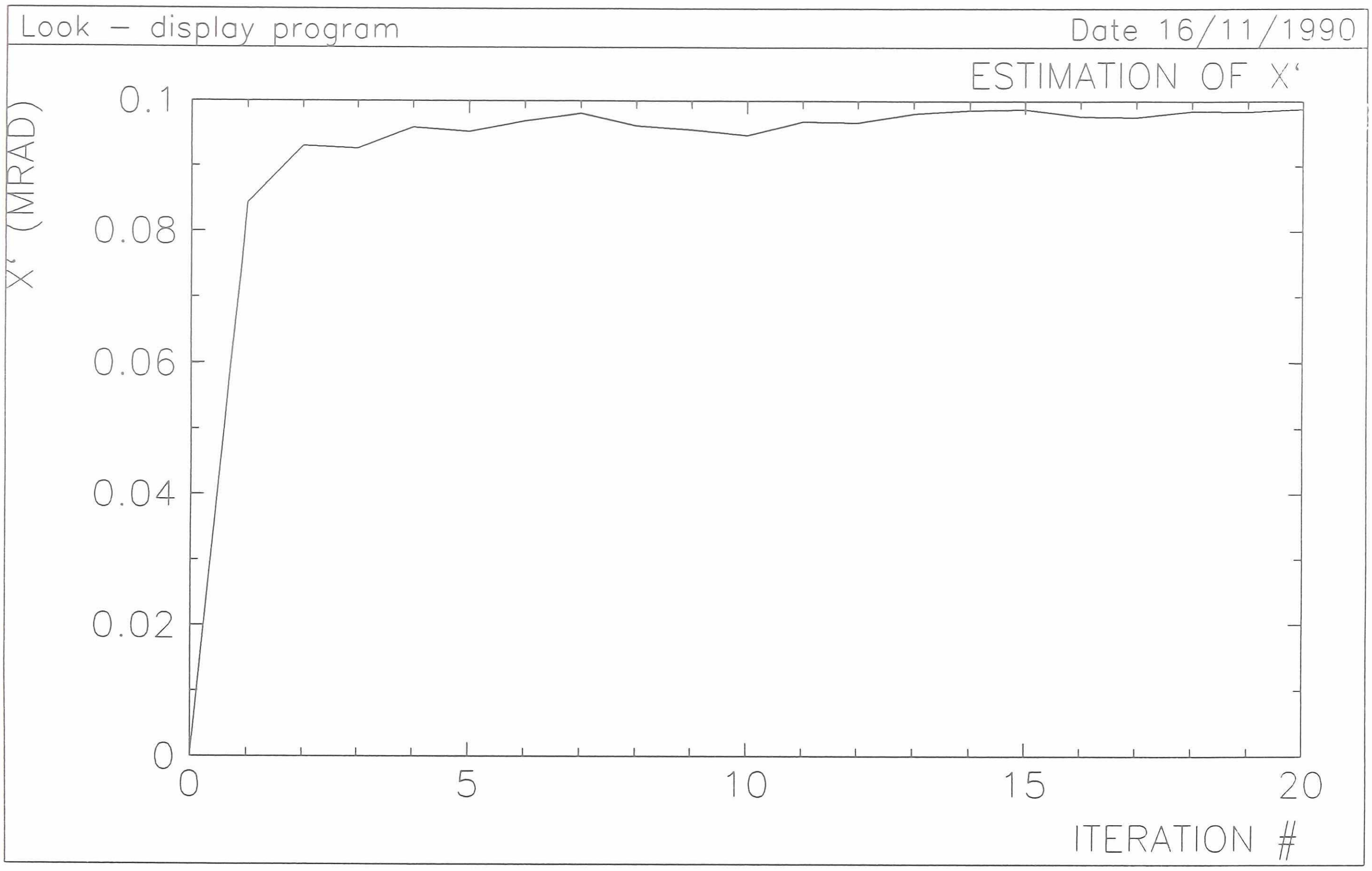
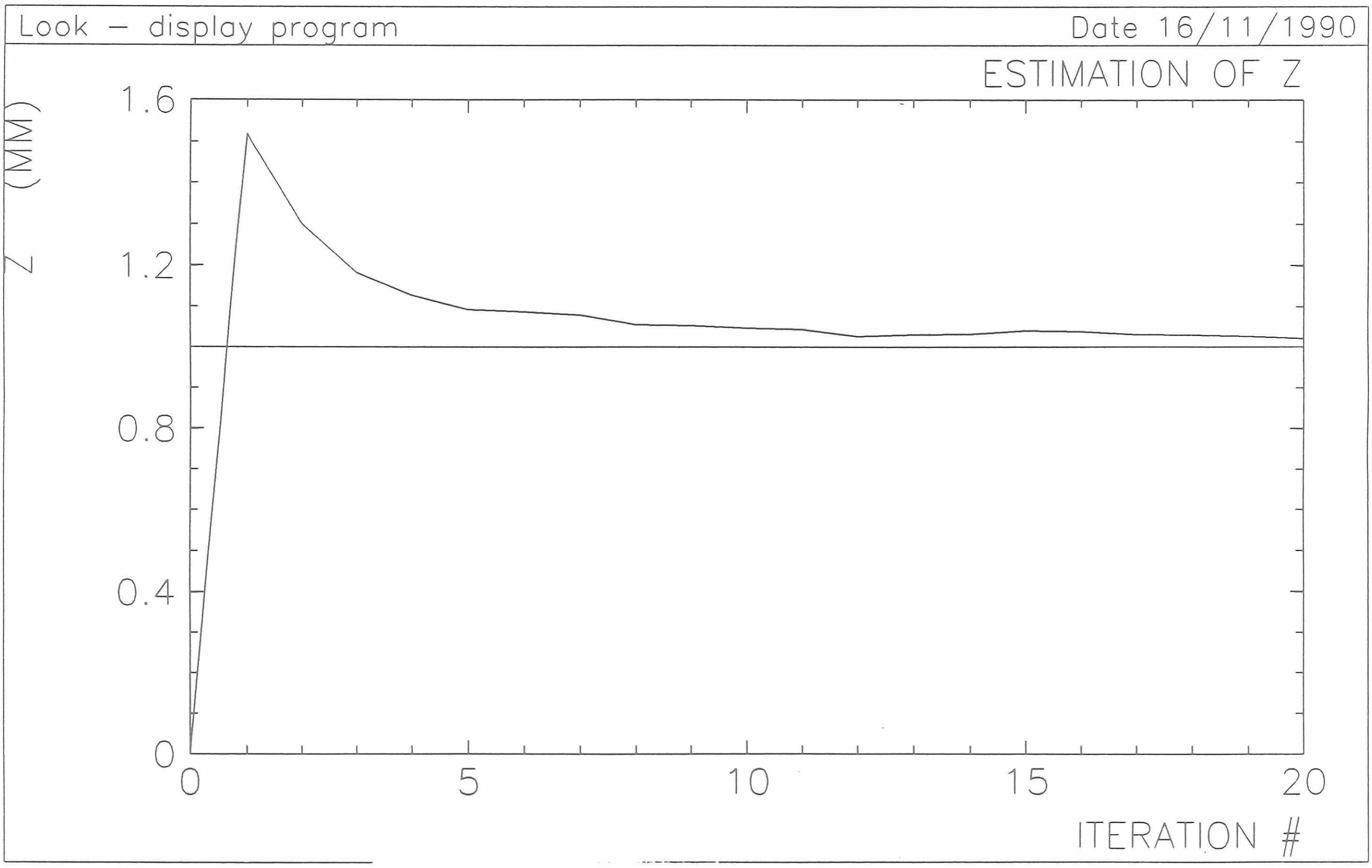

Look – display program
Date 16/11/1990
ESTIMATION OF X
X (MM)
3
2
1
0
0
5
10
15
20
ITERATION #

Fig. 1



Look – display program
Date 16/11/1990
ESTIMATION OF X'
X' (MRAD)
0.1
0.08
0.06
0.04
0.02
0
0
5
10
15
20
ITERATION #

Fig. 2



Look – display program
Date 16/11/1990
ESTIMATION OF Z
Z (MM)
1.6
1.2
0.8
0.4
0
0
5
10
15
20
ITERATION #

Fig. 3

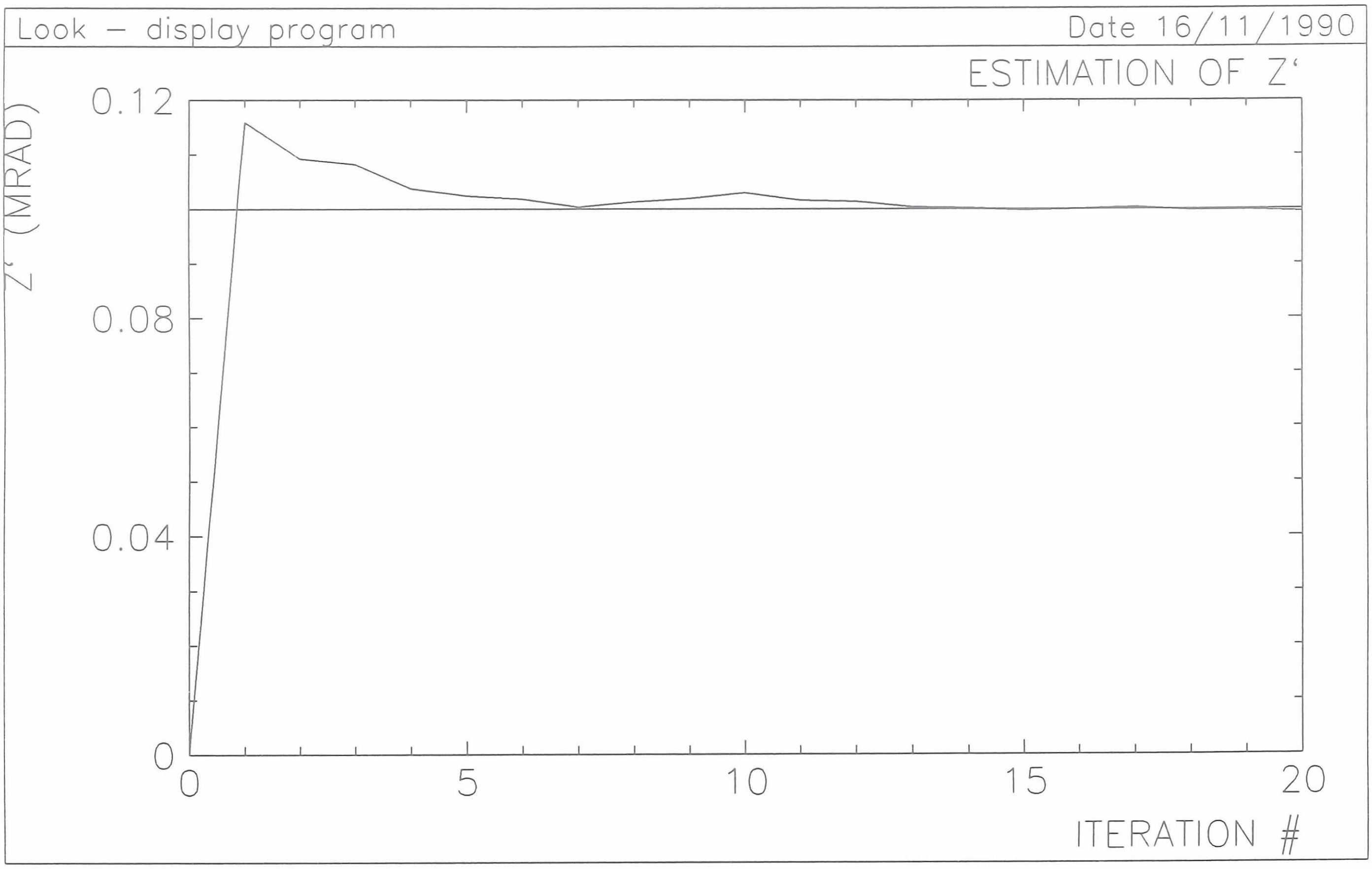

Look - display program
Date 16/11/1990
ESTIMATION OF Z'
Z' (MRAD)
0.12
0.08
0.04
0
0
5
10
15
20
ITERATION #

Fig. 4

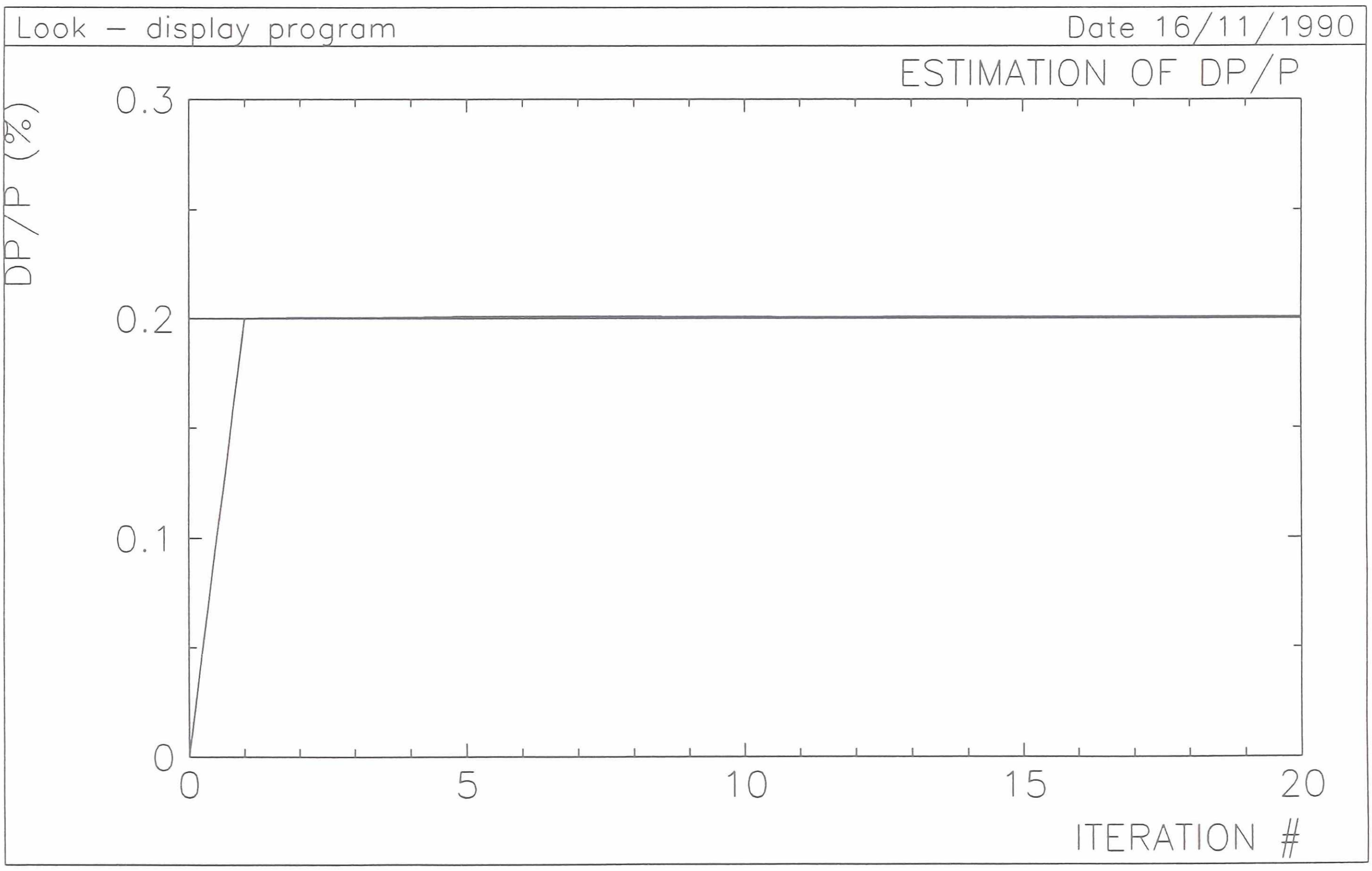
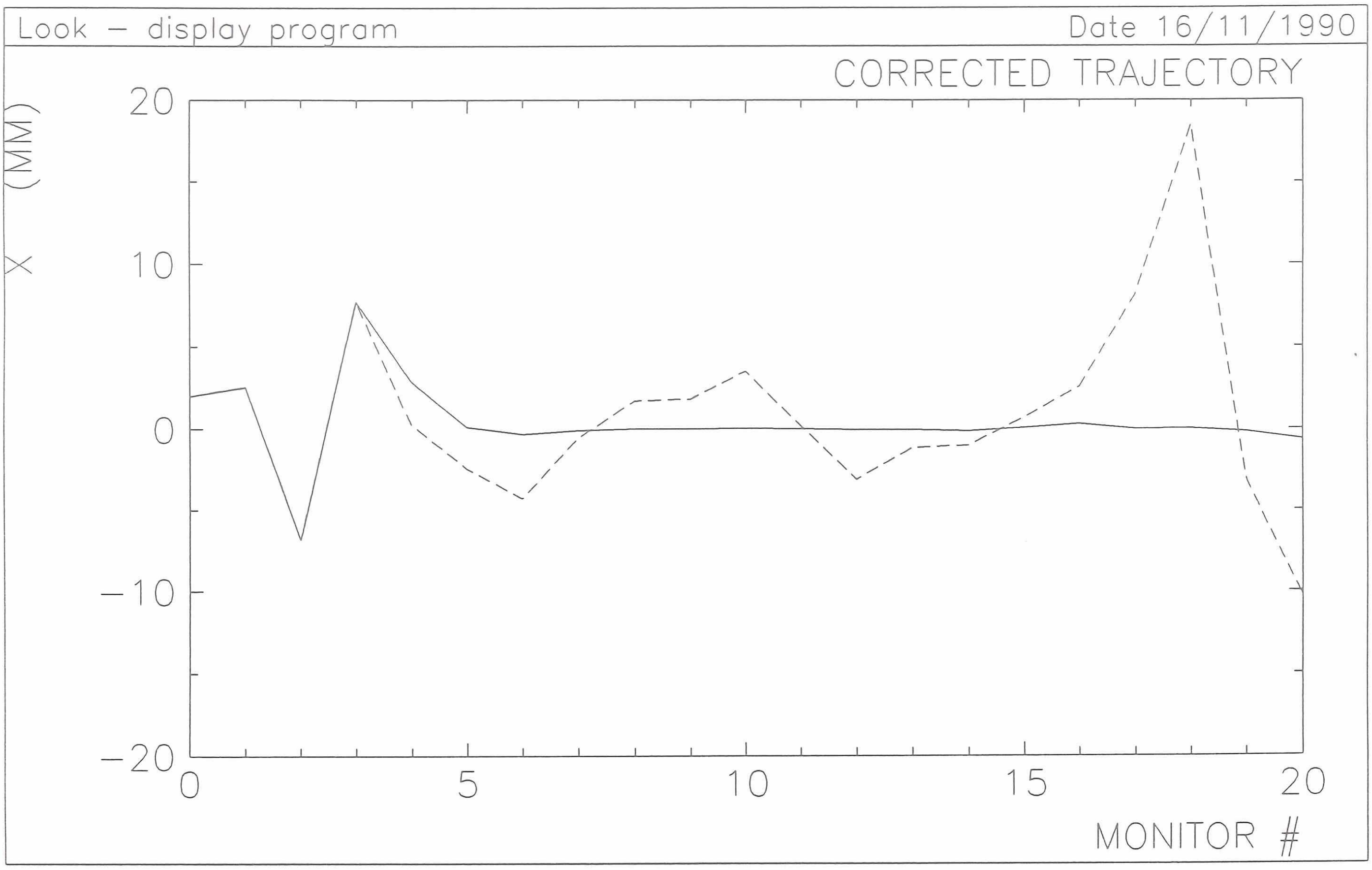
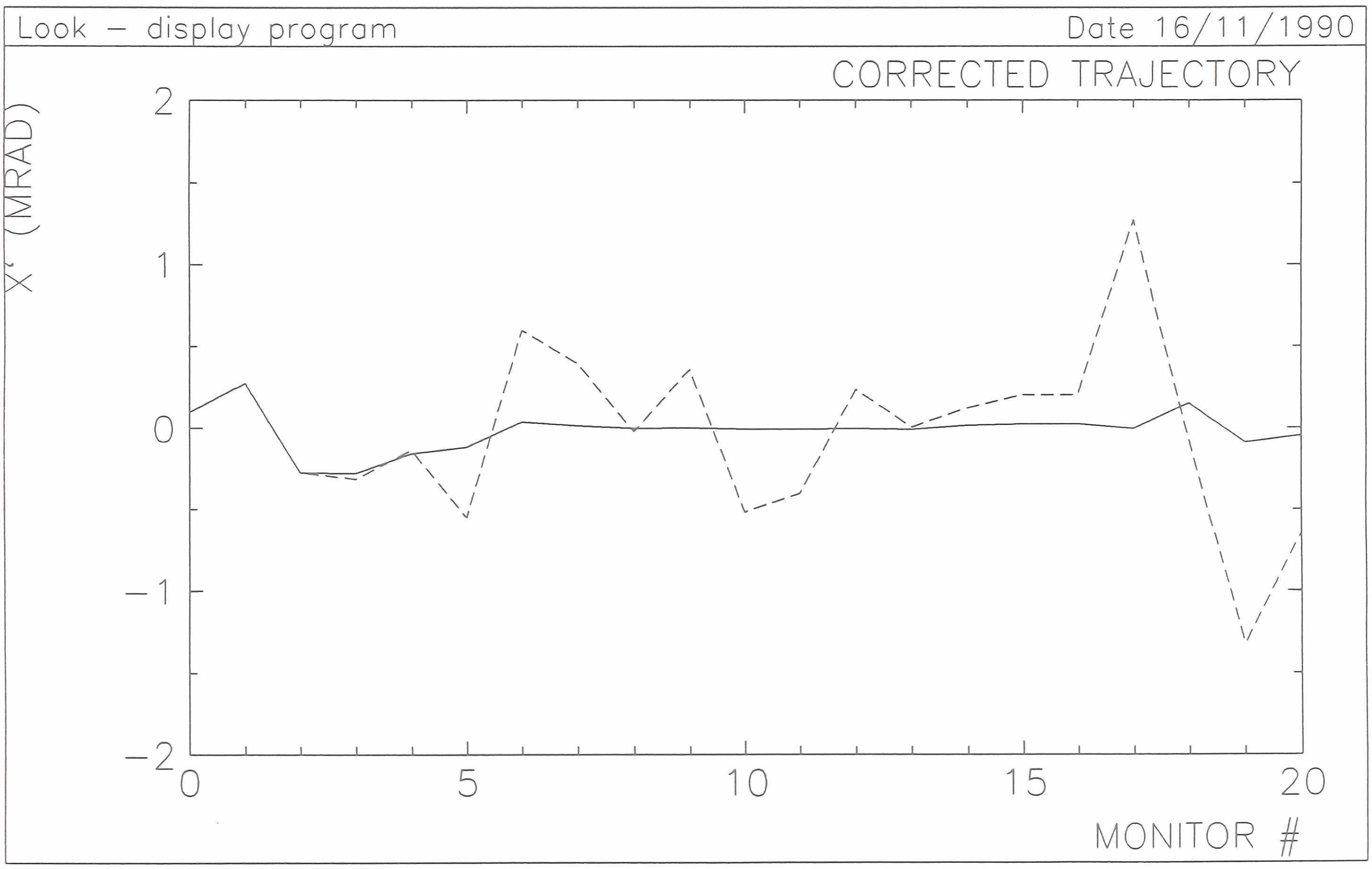

Look - display program
Date 16/11/1990
ESTIMATION OF DP/P
DP/P (%)
0.3
0.2
0.1
0
0
5
10
15
20
ITERATION #

Fig. 5



Look – display program
Date 16/11/1990
CORRECTED TRAJECTORY
X (MM)
20
10
0
–10
–20
0
5
10
15
20
MONITOR #

Fig. 6



Look – display program
Date 16/11/1990
CORRECTED TRAJECTORY
X' (MRAD)
2
1
0
–1
–2
0
5
10
15
20
MONITOR #

Fig. 7

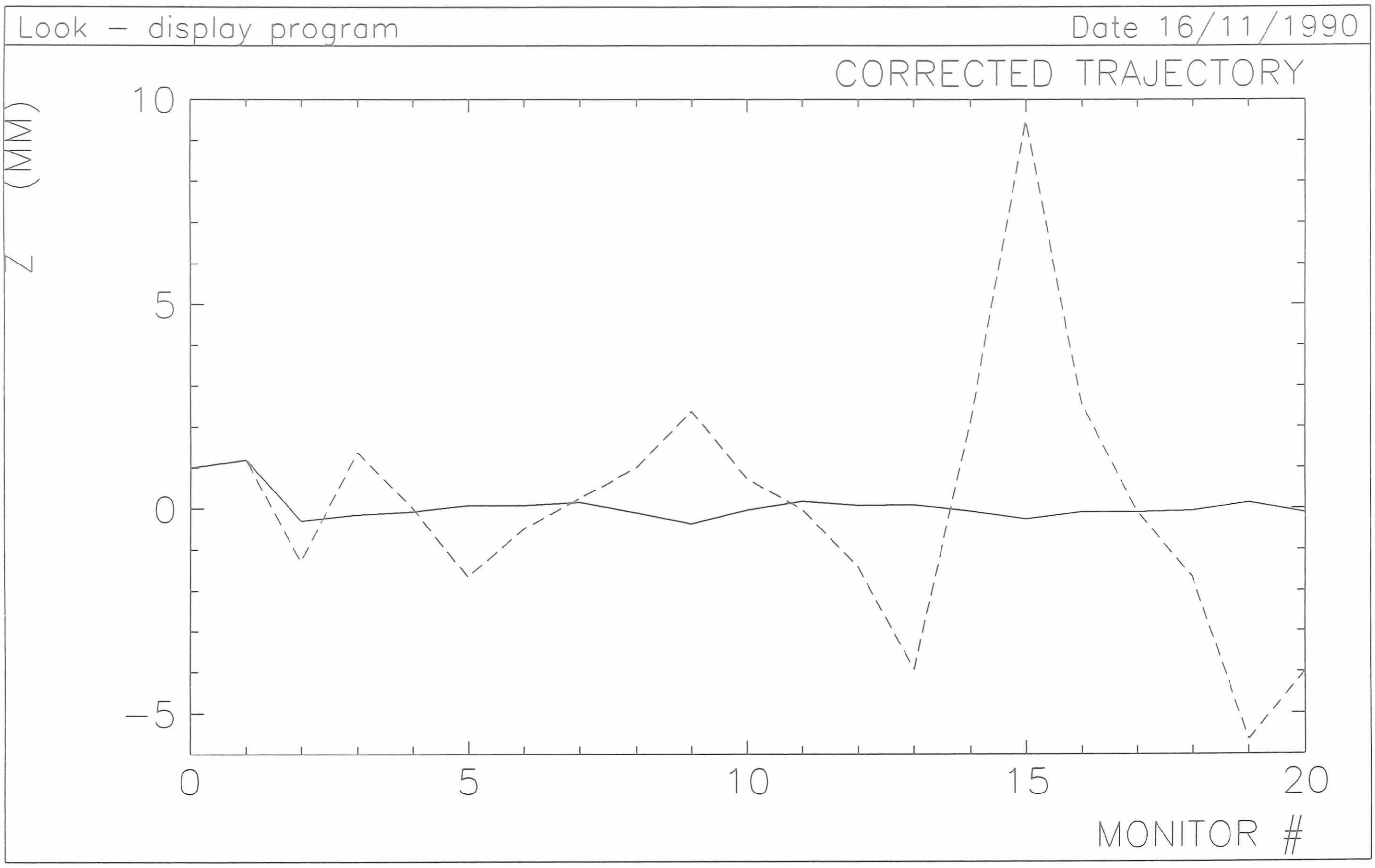

Look – display program
Date 16/11/1990
CORRECTED TRAJECTORY
Z (MM)
10
5
0
–5
0
5
10
15
20
MONITOR #

Fig. 8

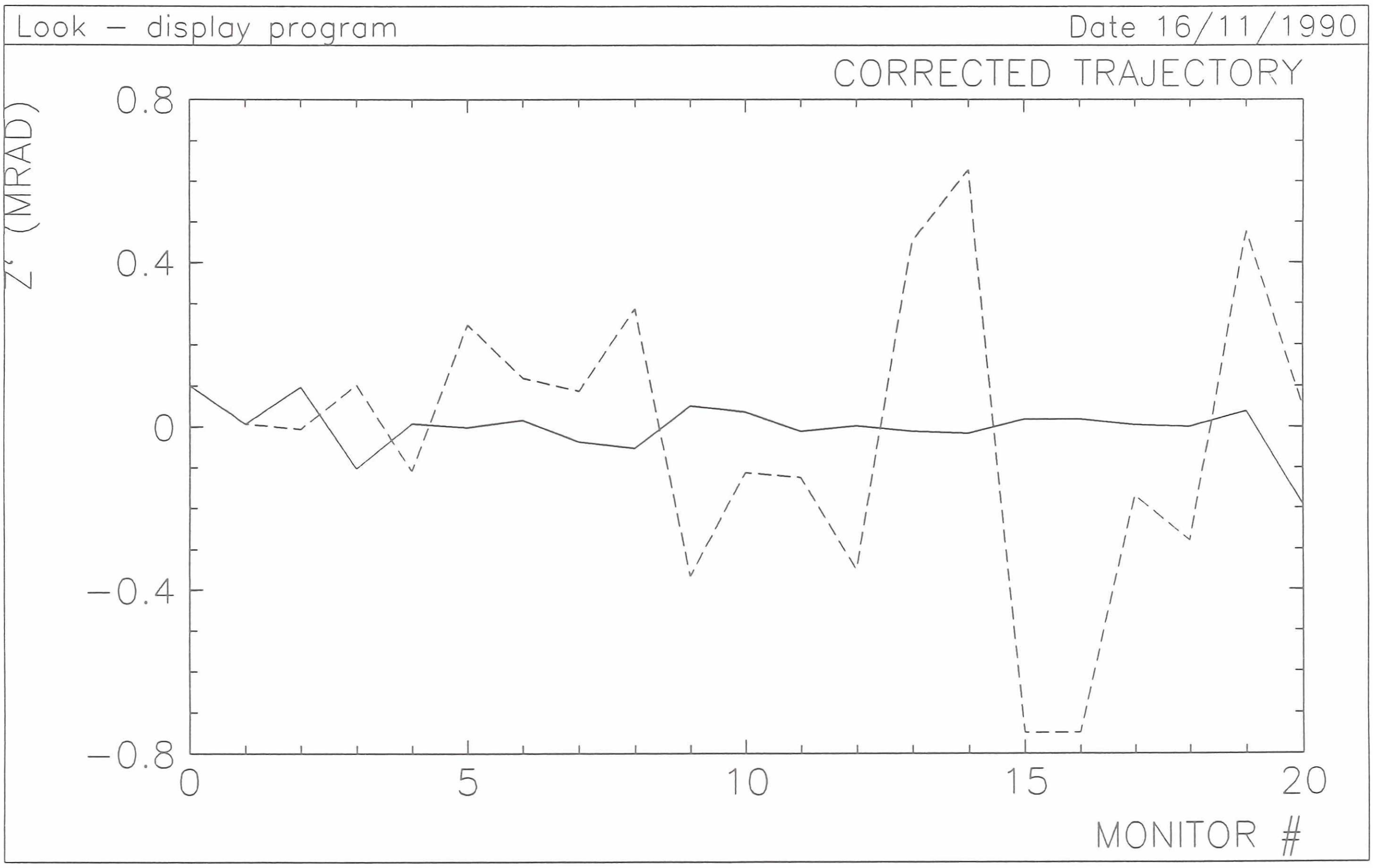

Look – display program
Date 16/11/1990
CORRECTED TRAJECTORY
Z' (MRAD)
0.8
0.4
0
–0.4
–0.8
0
5
10
15
20
MONITOR #

Fig. 9

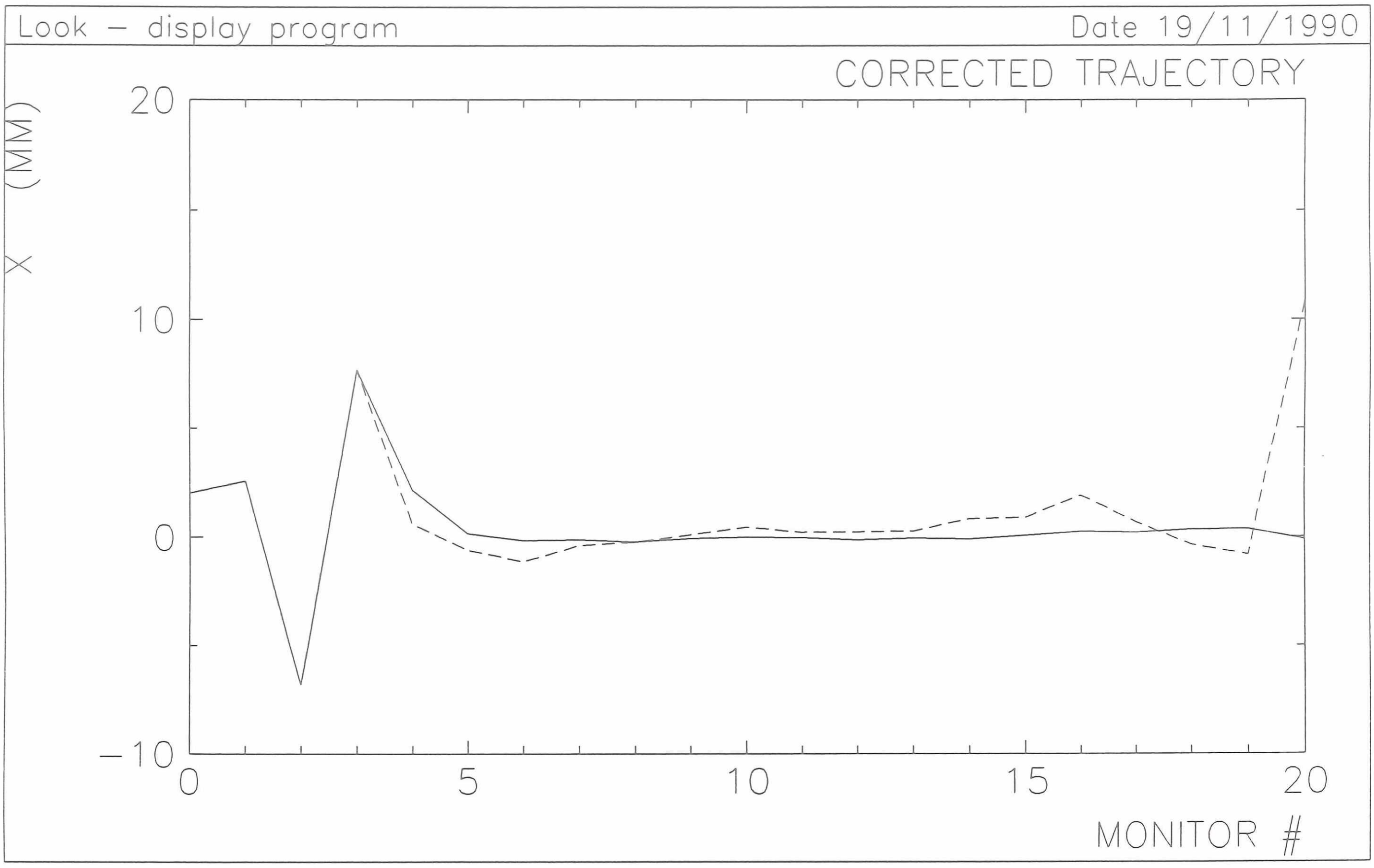

Look – display program
Date 19/11/1990
CORRECTED TRAJECTORY
X (MM)
20
10
0
–10
0
5
10
15
20
MONITOR #

Fig. 10

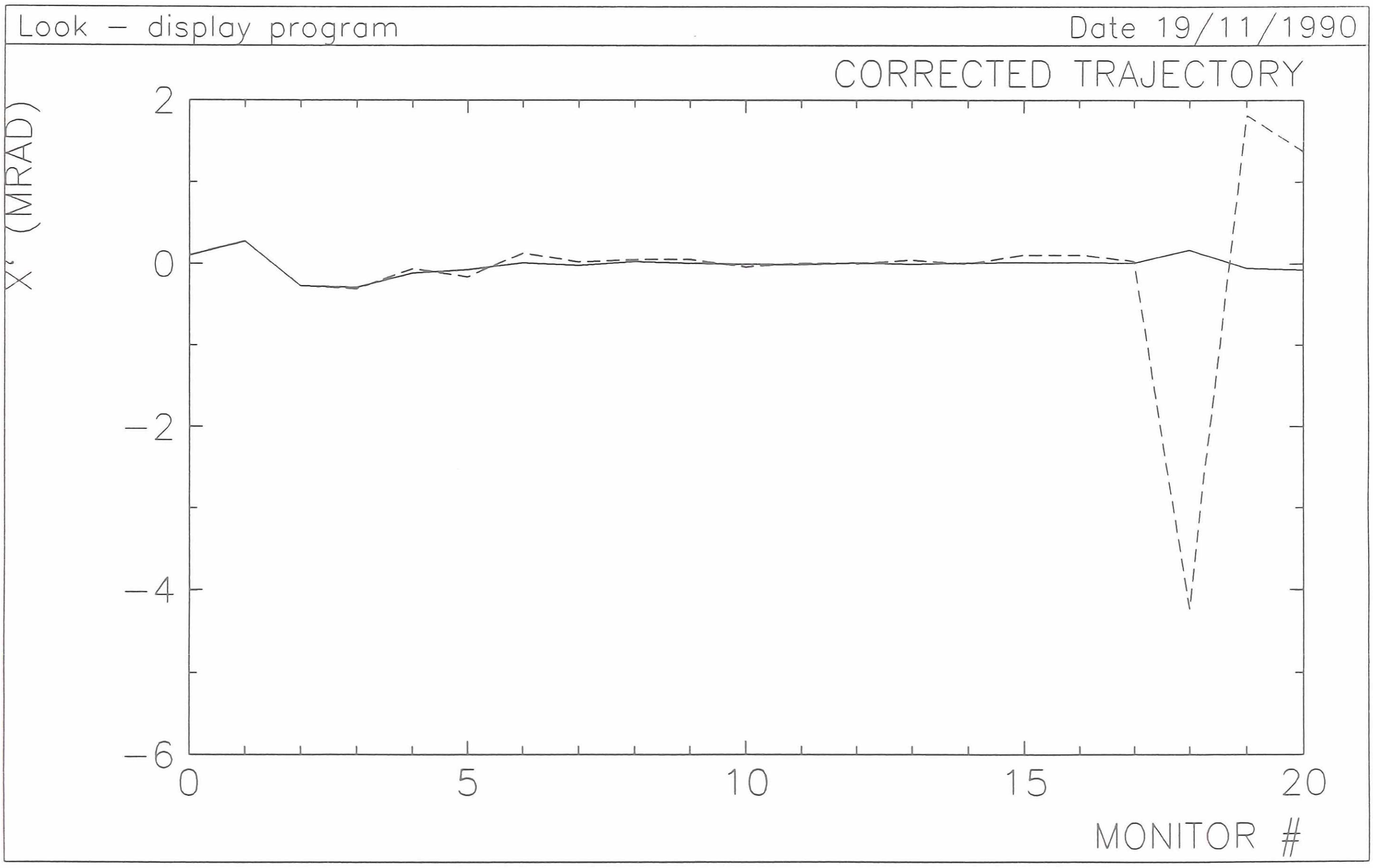

Look – display program
Date 19/11/1990
CORRECTED TRAJECTORY
X' (MRAD)
2
0
–2
–4
–6
0
5
10
15
20
MONITOR #

Fig. 11

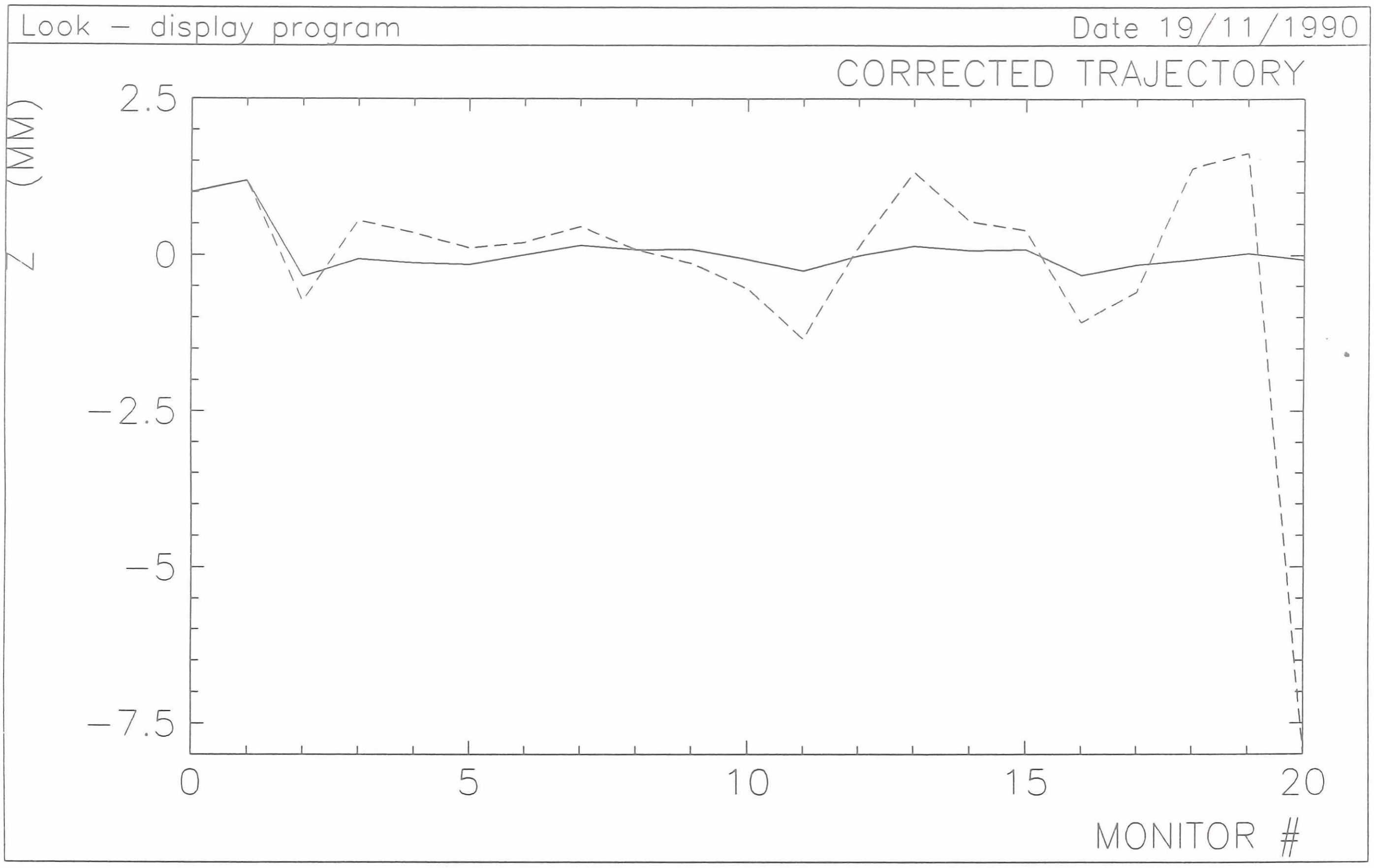

Look - display program
Date 19/11/1990
CORRECTED TRAJECTORY
Z (MM)
2.5
0
−2.5
−5
−7.5
0
5
10
15
20
MONITOR #

Fig. 12

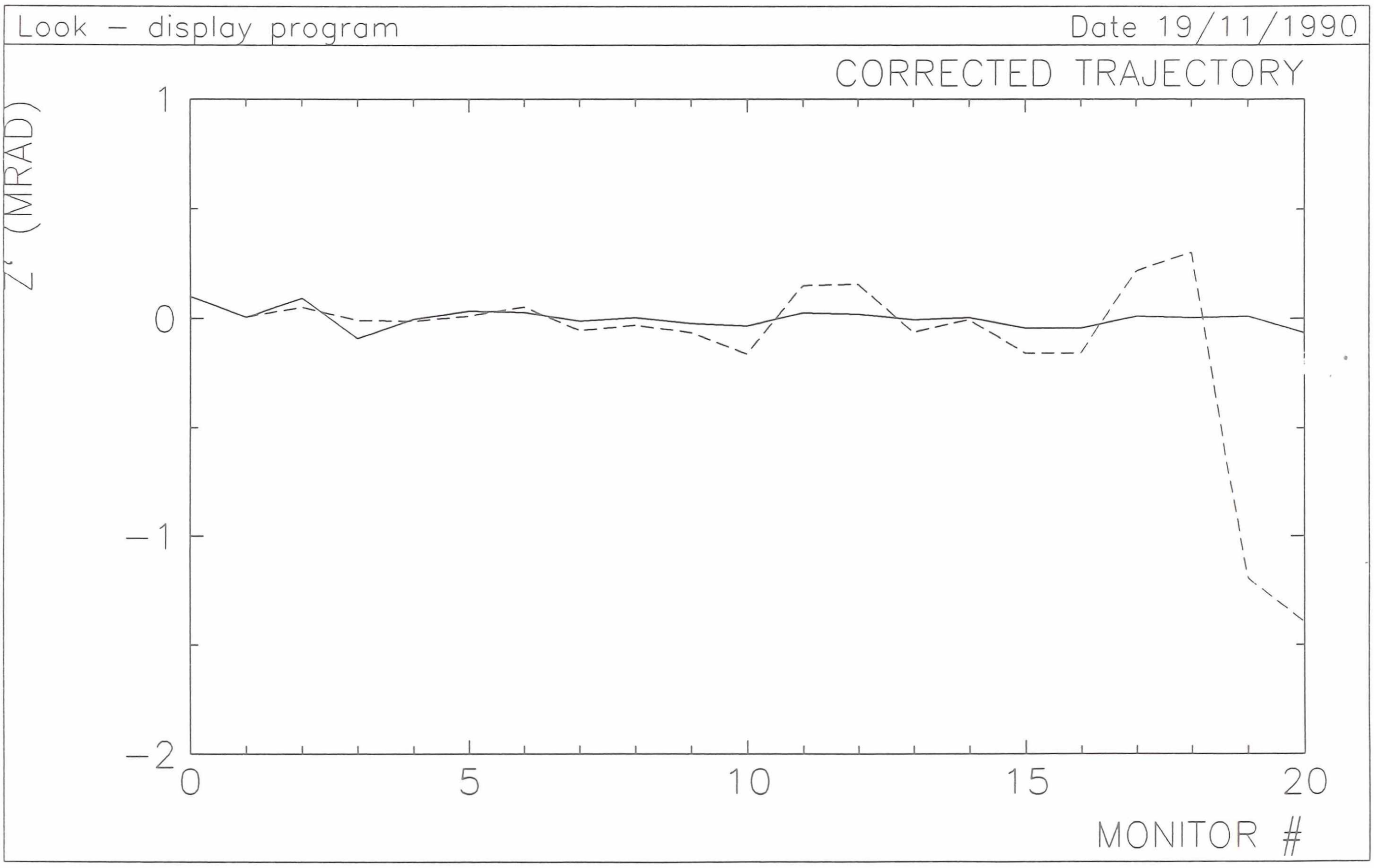

Look - display program
Date 19/11/1990
CORRECTED TRAJECTORY
Z' (MRAD)
1
0
−1
−2
0
5
10
15
20
MONITOR #

Fig. 13