
Higgs mass predictions of public NMSSM spectrum generators

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Abstract The publicly available spectrum generators for the NMSSM often lead to different predictions for the mass of the standard model-like Higgs boson even if using the same renormalization scheme and two-loop accuracy. Depending on the parameter point, the differences can exceed 5 GeV, and even reach 8 GeV for moderate superparticle masses of up to 2 TeV. It is shown here that these differences can be traced back to the calculation of the running standard model parameters entering all calculations, to the approximations used in the two-loop corrections included in the different codes, and to different choices for the renormalization conditions and scales. In particular, the importance of the calculation of the top Yukawa coupling is pointed out.

1 Introduction

Now that run II of the Large Hadron Collider (LHC) has started, either the first clear hints for new physics beyond the standard model (BSM) will finally show up or many well-motivated theories for BSM physics will come under even greater pressure. While in run I a scalar was discovered [1,2] which has all properties of the long-expected Higgs boson of the standard model (SM), at the same time stringent limits were placed on the simplest scenarios for new physics. In particular, the minimal realization of supersymmetry – the minimal supersymmetric standard model (MSSM) – has lost at least some of its appeal: no hints for supersymmetric (SUSY) particles were found, and the observed value of the SM-like Higgs mass requires either multi-TeV stop masses or special values of the stop-mixing parameter. However, if the stops are heavy and the MSSM is assumed to be a valid description of nature far above the electroweak scale, then a large fine-tuning is required to stabilize the vacuum expectation value (VEV) of the Higgs field. On the other hand, the large stop mixing needed to generate a Higgs mass of 125 GeV for stop masses around one TeV turns out to be dangerous because of charge and color breaking minima [3,4,5,6,7]. This leads to the question of whether the MSSM is really a natural completion of the SM and has attracted more interest in other SUSY scenarios. This is especially the case for SUSY models that predict a larger Higgs mass already at tree level. The best-studied examples are models with singlet extensions, where the interaction between the Higgs fields and the gauge singlet can raise the Higgs mass [8,9] and significantly reduce the fine-tuning [10,11,12,13,14,15,16,17,18,19]. A $\mathbb{Z}_3$ symmetry is often introduced to forbid all dimensionful parameters in the superpotential and to solve the μ problem of the MSSM at the same time [8]. This realization of a singlet extension of the MSSM is the next-to-minimal supersymmetric standard model (NMSSM). The NMSSM comes with a rich collider phenomenology [20,21,22], and also provides possibilities to hide SUSY at the LHC [23].

To confront the NMSSM with the measured Higgs mass, a precise calculation is necessary. The first calculations of radiative corrections to the Higgs masses in the NMSSM assumed an effective field theory below the SUSY scale (the scale of the stop/sbottom masses), and integrated the renormalization group equations (RGEs)

for the corresponding couplings from the SUSY scale to the weak scale [24, 25, 26]. One-loop corrections¹ of $\mathcal{O}(\alpha_t + \alpha_b)$ in the effective potential approximation have been computed in Refs. [27, 28, 29, 30]. Two-loop corrections of $\mathcal{O}(\alpha_s \alpha_t, \alpha_t^2)$ have been included in the NMSSM in the leading-logarithmic approximation (LLA) in Refs. [31, 32], and one-loop corrections to the SM-like Higgs mass including NMSSM-specific and electroweak couplings in the LLA in Ref. [33]. Full one-loop calculations including the momentum dependence have been performed in the $\overline{\text{DR}}$ renormalization scheme² [35, 36], or in a mixed on-shell (OS)- $\overline{\text{DR}}$ scheme [37, 38]; at the two-loop level, the dominant $\mathcal{O}(\alpha_s(\alpha_b + \alpha_t))$ corrections in the $\overline{\text{DR}}$ scheme have become available a few years ago [35]. The two-loop corrections involving only superpotential couplings such as Yukawa and singlet interactions [39], and a two-loop calculation of the $\mathcal{O}(\alpha_s \alpha_t)$ corrections with the top/stop sector renormalized either in the OS scheme or in the $\overline{\text{DR}}$ scheme [40], have also been provided recently.

At the moment, five public spectrum generators for the NMSSM are available which make use of these results to compute the Higgs-boson masses and mixing matrices in the NMSSM for a given parameter point: the stand-alone codes **NMSSMTools** [41, 42, 43], **SOFTSUSY** [44, 45, 46] and **NMSSMCALC** [47], as well as NMSSM versions of **FlexibleSUSY** [48] and **SPheno** [49, 50], which are based on **SARAH** [51, 52, 53, 54, 55]. The various tools, however, often make predictions that differ by several GeV for the mass of the SM-like Higgs boson even if using the same renormalization scheme. The aim of this work is to identify the origin of these deviations for the $\overline{\text{DR}}$ calculation, and to determine the numerical significance of each difference between the codes.

The paper is organized as follows. In sec. 2 we introduce the model and the conventions used. In sec. 3 the different codes and their features are presented, and the results obtained in six test points when using the codes out-of-the box are shown. The differences in the Higgs-mass calculation between these codes are discussed in sec. 4. Here, we concentrate on a SUSY scale input, while we address additional effects in a GUT scenario in sec. 5. We conclude in sec. 6.

2 The NMSSM and its conventions

We consider in the following a model with the particle content of the MSSM extended by a gauge singlet superfield $\hat{S}$. The general, renormalizable and R -parity conserving superpotential for this model in terms of the (hatted) doublet $(\hat{H}_u, \hat{H}_d)$ and singlet superfields reads

$$\mathcal{W}_S = \mathcal{W}_{\text{Yukawa}} + \frac{1}{3}\kappa\hat{S}^3 + (\mu + \lambda\hat{S})\hat{H}_u\hat{H}_d + \frac{1}{2}\mu_s\hat{S}^2 + t_s\hat{S}, \quad (1)$$

with

$$\mathcal{W}_{\text{Yukawa}} = Y_u\hat{Q}\hat{U}\hat{H}_u + Y_d\hat{Q}\hat{D}\hat{H}_d + Y_e\hat{L}\hat{E}\hat{H}_d, \quad (2)$$

where we have suppressed color and isospin indices in every term. In general, the couplings in eq. (1) are complex parameters, and those in eq. (2) are complex 3×3 matrices. The most general soft SUSY-breaking terms for this theory are given by

$$\begin{aligned} V_{\text{soft}} = & m_s^2|S|^2 + m_{H_u}^2|H_u|^2 + m_{H_d}^2|H_d|^2 + \tilde{f}^\dagger m_{\tilde{f}}^2 \tilde{f} \\ & + M_1\lambda_B^2 + M_2\lambda_W^2 + M_3\lambda_g^2 + T_d\tilde{q}\tilde{d}H_d + T_u\tilde{q}\tilde{u}H_u + T_e\tilde{e}\tilde{l}H_d \\ & + \left(B_\mu H_u H_d + \frac{1}{2}B_s S S + \chi_s S + T_\lambda S H_u H_d + \frac{1}{3}T_\kappa S^3 + h.c. \right), \end{aligned} \quad (3)$$

with the left-handed squark and slepton doublets $\tilde{f} = \tilde{q}$ and $\tilde{l}$ and the right-handed squarks and sleptons $\tilde{f} = \tilde{u}_R, \tilde{d}_R, \tilde{e}_R$. All generation indices are suppressed. The soft SUSY-breaking masses m_s^2 , $m_{H_d}^2$ and $m_{H_u}^2$ are always real, and the sfermion masses $m_{\tilde{f}}^2$ are hermitian 3×3 matrices, while all other soft terms can be complex. The soft SUSY-breaking trilinears T_λ and T_κ are often expressed in an alternative way, as the product of the corresponding superpotential coupling times a free parameter with the dimensions of a mass

$$T_\lambda \equiv \lambda A_\lambda, \quad T_\kappa \equiv \kappa A_\kappa. \quad (4)$$

¹We define $\alpha_{t,b,\tau} = Y_{t,b,\tau}^2/(4\pi)$, where $Y_{t,b,\tau}$ are the third-family Yukawa couplings. We follow the standard convention of denoting, e.g., by $\mathcal{O}(\alpha_t)$ the one-loop corrections to the Higgs masses that are in fact proportional to $\alpha_t m_t^2$, i.e. proportional to Y_t^4 . Similar abuses of notation affect the other one- and two-loop corrections.

²Strictly speaking, all calculations have been done in the $\overline{\text{DR}}'$ -scheme where the ϵ scalars are decoupled [34].

Similarly, the trilinear soft SUSY-breaking interactions involving sfermions can be expressed as

$$T_i \equiv Y_i A_i, \quad i = u, d, e. \quad (5)$$

It is common to introduce a $\mathbb{Z}_3$ symmetry to forbid all dimensionful parameters (i.e., μ , μ_s and t_s) in eq. (1). This solves the μ problem of the MSSM [56] – at the price of introducing a domain wall problem [57] – and leads to the NMSSM, where the surviving terms in the superpotential are

$$\mathcal{W}_{\text{NMSSM}} = \mathcal{W}_{\text{Yukawa}} + \frac{1}{3}\kappa\hat{S}^3 + \lambda\hat{S}\hat{H}_u\hat{H}_d, \quad (6)$$

and the soft SUSY-breaking terms B_μ , B_s and χ_s in eq. (3) also vanish.

For suitable values of the remaining s -dependent SUSY-breaking terms m_s and T_κ , a VEV v_s for the scalar singlet S of the order of the SUSY-breaking scale is generated. This dynamically generates the μ term which gives masses to the higgsinos,

$$\mu_{\text{eff}} = \lambda\langle S \rangle. \quad (7)$$

Note, there are two common conventions: $\langle S \rangle = v_s$ or $\langle S \rangle = v_s/\sqrt{2}$. Finally, after electroweak symmetry breaking (EWSB) where also the neutral components of H_d and H_u receive VEVs v_d and v_u , the singlet mixes with the other doublets.³ In the CP-conserving case, this leads to three CP-even physical scalars and two CP-odd ones together with the neutral would-be Goldstone boson. It is common to define the basis for the CP-even states as (ϕ_d, ϕ_u, ϕ_s) where ϕ_i are the real parts of the neutral components of H_d , H_u and S . This leads to the tree-level mass matrix $(M_h^2)^{\text{tree}}$, derived from the tree-level scalar potential

$$(M_h^2)_{ij}^{\text{tree}} = \frac{\partial^2 V^{\text{tree}}}{\partial \phi_i \partial \phi_j}. \quad (8)$$

The pole masses m_{h_i} ($i = 1, 2, 3$), identified with the masses of the three CP-even physical scalars, are the eigenvalues of the momentum dependent, loop-corrected mass matrix

$$M_h^2(p^2) = (M_h^2)^{\text{tree}} + \sum_n^\infty \Pi^{(n)}(p^2), \quad (9)$$

each evaluated at $p^2 = m_{h_i}^2$. The $\Pi^{(n)}(p^2)$ denote 3×3 matrices of n -loop self-energies calculated for an external momentum p^2 . As mentioned in the introduction, only the one-loop contribution, $\Pi^{(1)}(p^2)$, is completely known so far. For $\Pi^{(2)}$ only partial results in the limit $p^2 = 0$ exist in the NMSSM. Different choices of the renormalization scheme lead to different results for the physical scalar masses if the perturbative series in eq. (9) is truncated at a given order n . The differences between the results are of $\mathcal{O}(\Pi^{(n+1)})$, and provide an estimate of the theoretical uncertainty of the calculation due to uncomputed higher-order corrections. The matrix that diagonalizes $M_h^2(p^2)$ is called Z^H in the following,

$$m_h^{\text{diag}} = Z_H M_h^2(p^2) Z_H^T. \quad (10)$$

In general, Z_H is also a p^2 -dependent quantity. The NMSSM spectrum generators adopt different definitions for the loop-corrected rotation matrix Z_H that they give as output: the external momentum is either set equal to zero, or set equal to one of the mass eigenvalues.

We conclude this section by presenting the set of input parameters that enter the calculation of the Higgs-boson masses and mixing matrices in the CP-conserving NMSSM. First, there are the three tadpole conditions that ensure the minimum of the Higgs potential at non-vanishing VEVs v_u, v_d, v_s ,

$$\left. \frac{\partial V^{\text{eff}}}{\partial \phi_i} \right|_{\text{min}} = 0, \quad i = d, u, s, \quad (11)$$

where V^{eff} represents the effective scalar potential computed at the same perturbative order as the mass matrix in eq. (9). These conditions fix three parameters. The simplest choice is to solve them for m_s^2 , $m_{H_d}^2$ and

³While in the CP violating NMSSM the VEVs can be taken complex, it turns out that, after exploiting the minimum conditions of the scalar potential, the NMSSM Higgs sector at tree level can be described by one complex phase, which is given by $\varphi_y = \varphi_\kappa - \varphi_\lambda + 2\varphi_s - \varphi_u$, where φ_u and φ_s are the complex phases of the VEVs of H_u and S , respectively, and φ_κ and φ_λ the complex phases of κ and λ .

$m_{H_u}^2$, and this is done in all the comparisons for every code we use. Since one combination of VEVs (namely, $v^2 = v_u^2 + v_d^2$) is constrained by experimentally measured observables, this leaves six free parameters in the Higgs sector of the model:

$$\lambda, \kappa, A_\lambda, A_\kappa, \tan\beta, \mu_{\text{eff}}, \quad (12)$$

where $\tan\beta \equiv v_u/v_d$, and μ_{eff} stands in for v_s . Under the assumption that the soft SUSY-breaking terms are flavor-diagonal and that only third-generation soft interactions are non-negligible, this reduces the additional input parameters required to fully define the model to the set

$$A_t, A_b, A_\tau, m_{\tilde{t}_L}, m_{\tilde{t}_R}, M_1, M_2, M_3, M_{\tilde{f}}, \quad (13)$$

with $m_{\tilde{t}_L}^2 \equiv m_{\tilde{q},33}^2$, $m_{\tilde{t}_R}^2 \equiv m_{\tilde{u},33}^2$ ⁴, $A_\tau = A_{e,33}$, $A_b = A_{d,33}$, $A_t = A_{u,33}$, and where we assumed for simplicity that all other diagonal entries of $m_{\tilde{f}}^2$ are degenerate and equal to $M_{\tilde{f}}^2$.

One can further reduce the number of input parameters by assuming a specific mechanism to transmit SUSY breaking from a hidden sector to our visible sector. In models inspired by minimal supergravity the following relations among the soft terms exist at the gauge-coupling unification scale:

$$m_{\tilde{f}}^2 = m_0^2 \mathbf{1}, \quad M_i = M_{1/2} \quad (i = 1, 2, 3), \quad A_j = A_0 \quad (j = e, d, u), \quad (14)$$

where $\mathbf{1}$ is the identity matrix in flavor space. In contrast to the fully constrained NMSSM, the Higgs and singlet soft masses are obtained again by the tadpole equations, and also the NMSSM-specific trilinear soft couplings are treated as free parameters. Thus, necessary input parameters are

$$m_0, M_{1/2}, \tan\beta, A_0, \lambda, \kappa, A_\kappa, A_\lambda, \mu_{\text{eff}}. \quad (15)$$

3 Spectrum generators for the NMSSM and their features

3.1 Features

The main features of the publicly available spectrum generators for the NMSSM are summarized in table 1. Here, two different kinds of spectrum generators are distinguished: dedicated tools which contain all physically relevant information about the NMSSM (vertices, mass matrices, renormalization group equations (RGEs), loop corrections, etc.) in a hardcoded way, and multi-purpose generators which use the **Mathematica** package **SARAH** to derive this information for a given model. The advantage of the first approach, which is adopted by **NMSSMTools**, **NMSSMCALC** and **SOFTSUSY**, is that the tools can be used directly after compilation without further effort. In contrast, to use **FlexibleSUSY** and **SPheno**, one needs to first install **SARAH**⁵, which is then used to generate the necessary source code to get an NMSSM spectrum generator. These additional steps allow these tools to be used for a much broader range of models, i.e. not only other singlet extensions beyond the NMSSM are supported, but also many other extensions of the MSSM. Also, the features of the NMSSM version (like boundary conditions at all relevant scales) can be adjusted in the input files for **SARAH** or **FlexibleSUSY** without the need to modify the final code⁶. **SARAH** is available at [hepforge](http://sarah.hepforge.org/).

<http://sarah.hepforge.org/>

More details on the different tools are given in the following, in the order in which the codes became publicly available.

⁴For further reference, we define also $m_{\tilde{u}_R}^2 \equiv m_{\tilde{u}_{11}}^2$, $m_{\tilde{d}_R}^2 \equiv m_{\tilde{d}_{11}}^2$, $m_{\tilde{u}_L}^2 \equiv m_{\tilde{q}_{11}}^2$.

⁵**FlexibleSUSY** also offers pre-generated code for selected models, with specific boundary conditions and EWSB outputs, allowing one to use these specific **FlexibleSUSY** spectrum generators without first installing **Mathematica** and **SARAH**.

⁶For easier comparison with other codes, we have created input files to allow for purely SLHA1 [58] input for **SPheno** and **FlexibleSUSY**, while by default a block form for all matrices is used as in SLHA2 [59]. Interested readers can obtain the files from the authors.

	FlexibleSUSY	NMSSMCALC	NMSSMTools	SOFTSUSY	SPheno
Code					
type	using SARAH	stand alone	stand alone	stand alone	using SARAH
language	C++	Fortran 77 and 90	Fortran 77	C++	Fortran 90
Models					
No $\mathbb{Z}_3$	✓	✗	✓	✓	✓
GUT model	✓	✗	✓	✓	✓
CPV	(✓)	✓	✗	✗	(✓)
Thresholds					
scale(s)	M_Z	M_t, M_{SUSY}	M_t, M_{SUSY}	M_Z	M_Z
EW parameters	full one-loop	OS definitions	full one-loop	full one-loop	full one-loop
Yukawas	full one-loop; two-loop QCD	one-loop (S)QCD; two-loop QCD	one-loop (S)QCD+Yukawa; two-loop QCD	full one-loop; two-loop QCD; optionally two-loop SQCD	full one-loop two-loop QCD
strong gauge	one-loop top+SUSY	—	—	one-loop top+SUSY	one-loop top+SUSY
Higgs mass calculation					
scheme	$\overline{\text{DR}}$	OS, $\overline{\text{DR}}$	$\overline{\text{DR}}$	$\overline{\text{DR}}$	$\overline{\text{DR}}$
one-loop	full	full	full	full	full
two-loop	$\alpha_s(\alpha_b + \alpha_t) + \text{MSSM}$	$\alpha_s \alpha_t$	$\alpha_s(\alpha_b + \alpha_t) + \text{MSSM}$	$\alpha_s(\alpha_b + \alpha_t) + \text{MSSM}$	$\alpha_s(\alpha_b + \alpha_t) + (\alpha_t + \alpha_b)^2 + \alpha_\tau(\alpha_\tau + \alpha_b) + (\alpha_\kappa + \alpha_\lambda)^2 + \alpha_\lambda(\alpha_t + \alpha_b)$
SUSY masses					
one-loop	✓	✗	✓	✓	✓
momentum effects	✓	✗	✓ ($\alpha_s, \alpha_t, \alpha_b$ only)	✓	✓
Other observables					
decays	✗	✓	via NMHDECAY	via NMHDECAY	✓
flavour, $g-2$	✗	✗	✓	✗	✓

Table 1 Public spectrum generators for the NMSSM and their features. The first two rows specify if the code is stand alone or gets necessary information from the `Mathematica` package `SARAH`, and in what language the code is written. The next three rows list the models that are supported: “GUT model” means that all parameters at the SUSY scale can be related by a constrained model at the GUT scale, and “CPV” shows if a Higgs sector with CP-even and -odd mixing is supported. Here, (✓) is used if this is only possible at the one-loop level so far. The rows under the sub-heading “Thresholds” give details of what SUSY and non-SUSY threshold corrections are included to calculate the running parameters entering the Higgs-mass calculation. The first row contains the scale (or scales) at which the threshold corrections are introduced, and the following three rows refer to different categories of parameters. Under “Higgs mass calculation” the main aspects of the calculation of the Higgs mass are given: the renormalization scheme (“scheme”), and what corrections at one- and two-loop are included. “full” at one-loop refers to a diagrammatic calculation including the entire p^2 dependence and all corrections up to those suppressed by first and second generation Yukawa couplings (which are also included in some codes). At two-loop the included corrections are shown. Here, we define $\alpha_X = X^2/(4\pi)$ for $X = (\lambda, \kappa)$. “+MSSM” refers to the approximation that the known MSSM results for the pure Yukawa interactions at two-loop are included. Under “SUSY masses” it is shown if the code calculates also one-loop corrections to SUSY states (neutralinos, charginos, sfermions, gluino) and if the effects from the external momenta are included in these calculations. Finally, it is summarized in “Other observables” if the tool gives also a prediction for flavor observables and/or other precision observables like $g-2$ and if it calculates the decays of at least the Higgs bosons.

3.1.1 NMSSMTools

The first public codes to be made available for the NMSSM were **NMHDECAY** [41,42], **NMSPEC** [43] and **NMGMSB**, which are today collected in the package **NMSSMTools**. **NMHDECAY** was based on a suitably modified version of **HDECAY** [60,61,62]. This package is available at

<http://www.th.u-psud.fr/NMHDECAY/nmssmtools.html>

So far **NMSSMTools** is restricted to the real NMSSM with and without $\mathbb{Z}_3$ symmetry, but a version to support also CP violation is under construction [63]. **NMSSMTools** allows to calculate the Higgs masses for a parameter point with three different options:

1. The most precise calculation makes use of the NMSSM corrections of ref. [35]. This provides a fully diagrammatic calculation of the Higgs masses at the one-loop level, including all contributions and the momentum dependence, plus the two-loop corrections of $\mathcal{O}(\alpha_s(\alpha_b + \alpha_t))$ at zero external momentum. In addition, the two-loop corrections known from the MSSM at $\mathcal{O}((\alpha_t + \alpha_b)^2 + \alpha_\tau(\alpha_\tau + \alpha_b))$ [64,65,66,67] are included.
2. The second option implements the same corrections as the first one, but neglects the momentum dependence of the one-loop self-energies. (This makes the code faster, which is useful for large scans.)
3. The third option is the original implementation of the Higgs mass calculation in **NMSSMTools**, which – as described in the appendix C of ref. [8] – implements only a partial computation of the one-loop contributions, and restricts the two-loop contributions to the leading-log approximation.

NMSSMTools also calculates the SUSY spectrum at the one-loop level, and the Higgs and sparticle decay branching fractions (the latter in **NMSDECAY** [68] using a suitable modification of **SDECAY** [69]). B -physics observables and the muon anomalous magnetic moment are computed following refs. [70,71], and compared to experimental constraints. Bounds on the Higgs sector from LEP [72] and on the couplings of the SM-like Higgs boson (from ref. [73]) are implemented. A link to **MicrOmegas** is included which allows for an easy calculation of the dark matter relic density, and direct and indirect dark matter detection cross sections.

It is possible to define a set of input parameters at the SUSY scale, or to define specific SUSY-breaking scenarios such as minimal supergravity using **NMSPEC**, or scenarios of gauge-mediated SUSY breaking (following ref. [74]) using **NMGMSB**. In all cases, the fine-tuning of the EWSB scale with respect to the input parameters can be estimated.

3.1.2 SPheno and SARAH

The main features of the **SPheno** version generated with **SARAH** for the NMSSM are a precise mass-spectrum calculation using the full two-loop RGEs of the NMSSM, with all flavor and CP effects, for the running between M_Z , M_{SUSY} and M_{GUT} . The RGEs are calculated using generic expressions given in Refs. [75,76,77,78,79,80,81,82]. **SPheno** calculates also the full one-loop corrections to all masses, including the entire momentum dependence in the $\overline{\text{DR}}$ scheme. At two-loop all corrections in the gaugeless limit with vanishing external momenta are included [83,84], by making use of generic expressions given in Refs. [85,86,87]. This approach makes it possible to include all two-loop corrections without making use of any MSSM approximations as done by other codes, i.e. also the NMSSM-specific corrections of $\mathcal{O}(\alpha_\lambda(\alpha_\lambda + \alpha_\kappa + \alpha_t))$ are fully included. However, it is optionally also possible to make use of results from the literature for the two-loop corrections in the MSSM and NMSSM. These routines are usually much faster but have to be used carefully for models beyond the MSSM. Higgs sectors with CP violation are so far supported only at the one-loop level.

Other features of the **SPheno** NMSSM version are the calculation of all important quark-flavor violating observables (B and K decays, $\Delta M_{B_{d,s}}/\Delta M_K$, ϵ_K) as well as lepton-flavor violating observables using the **FlavorKit** functionality [88]. Also, $(g-2)_l$ and electromagnetic dipole moments are predicted. Moreover, all two- and three-body decays of SUSY particles, and two-body decays of the Higgs scalars are calculated. The sparticle decays are purely tree-level, while for the Higgs bosons the NLO-QCD corrections to decays in two quarks, photons and gluons are included. Also decays in virtual vector bosons are taken into account. Finally, **SPheno** can make a prediction for the electroweak fine-tuning according to measures proposed in Refs. [89,90] and writes all necessary input files to test points with **HiggsBounds** [91,92,93] and **HiggsSignals** [94].

The basic **SPheno** version that is used together with **SARAH** can be downloaded from

<http://spheno.hepforge.org/>

A handy possibility to interface **SPheno** versions created by **SARAH** with Monte-Carlo tools like **MadGraph** [95,96] and **WHIZARD** [97,98] as well as **MicrOmegas** is available via the **BSM Toolbox** [99].

3.1.3 NMSSMCALC

NMSSMCALC allows for the computation of the Higgs masses both in the CP-conserving and CP-violating NMSSM, both at one-loop and at two-loop level. In contrast to the other codes, NMSSMCALC makes use of mixed $\overline{\text{DR}}$ -OS renormalization conditions for the computation of the Higgs masses. In the (s)top sector the user can choose between OS and $\overline{\text{DR}}$ renormalization. In addition, if the charged Higgs boson mass $M_{H^\pm}$ is given as an input instead of A_λ , $M_{H^\pm}$ enters the Higgs mass calculation as an OS parameter. For comparison between the codes, however, the option of $\overline{\text{DR}}$ renormalization of the (s)top sector and the option with the input A_λ are chosen. The Higgs mass calculation at one-loop level is performed including the full momentum dependence and all possible contributions [37,38]. At the two-loop level the $\mathcal{O}(\alpha_s\alpha_t)$ corrections are included [40]. They include the $\mathcal{O}(\alpha_s\alpha_t)$ part relating the vacuum expectation value to physical observables, which is missing so far in the other spectrum generators.

NMSSMCALC can be downloaded from

<http://www.itp.kit.edu/~maggie/NMSSMCALC/>

and comes together with an NMSSM extension of HDECAY [60,61,62] for the Higgs boson decays. The decays are calculated in the CP-conserving and CP-violating NMSSM. The decay widths include the dominant higher-order QCD corrections. Furthermore, the decays of the neutral Higgs bosons into a bottom pair include the higher order SUSY-QCD and the approximate SUSY-electroweak (EW) corrections up to one-loop order. The decays into a strange quark pair include the dominant resummed SUSY-QCD and the ones into τ pairs the dominant resummed SUSY-EW corrections. Accordingly the charged Higgs boson decays into fermion pairs include the higher order SUSY corrections. All two-body decays into SUSY particles have been implemented. For the CP-conserving case the decays into stop and sbottom pairs come with the SUSY-QCD corrections. Finally, all relevant off-shell decays into two massive gauge boson final states, into gauge and Higgs boson final states, into Higgs pairs as well as into heavy quark pairs are included. For more details and a complete list of references see [47].

3.1.4 SOFTSUSY

SOFTSUSY is a widely used spectrum generator for the MSSM with and without R -parity violation which has recently been extended to the NMSSM. The NMSSM spectrum generator is implemented to solve scenarios with and without the $\mathbb{Z}_3$ symmetry. The implementation of EWSB conditions allows one to choose as output for the $\mathbb{Z}_3$ -conserving case either the set $\{\kappa, v_s, m_s^2\}$ or the set $\{m_{H_u}^2, m_{H_d}^2, m_s^2\}$, and in the $\mathbb{Z}_3$ -violating case there is an additional option of giving out $\{\mu, B_\mu, \chi_s\}$. For the high-scale boundary conditions there are a number of pre-defined options including the mSUGRA-inspired semi-constrained NMSSM and a general high-scale boundary condition which allows one to set all soft SUSY-breaking parameters independently. The users may also easily create their own boundary conditions.

For the NMSSM, SOFTSUSY extends the full three-family one- and two-loop RGEs of the MSSM [75,100] using general two-loop expressions from the literature [75,76], with the NMSSM specialization given in Appendix D of the NMSSM manual [45]. These expressions were also cross-checked against two-loop NMSSM RGEs in the literature that use the 3rd-family approximation [8], and tested numerically against the RGEs from SARAH using an early development code from FlexibleSUSY. Finally, the RGEs for the vacuum expectation values are implemented using Refs. [81,82]. The one-loop self-energies and tadpole corrections for the Higgs bosons were extended to the NMSSM using the expressions in Refs. [35,36]. The NMSSM extension also includes the two-loop corrections at order $\mathcal{O}(\alpha_s(\alpha_t + \alpha_b))$ from Ref. [35] and in addition uses the MSSM results from Refs. [64,65,66,67].

Higgs and sparticle decays may be obtained by interfacing SoftSUSY with NMHDECAY [41,42] and NMSDECAY [68], which is based upon Ref. [69]. These are both distributed as part of the NMSSMTools package, and SOFTSUSY provides a script to do the interface with this package automatically.

The homepage of SOFTSUSY is

<http://softsusy.hepforge.org/>

3.1.5 FlexibleSUSY

FlexibleSUSY is a Mathematica and C++ package, which creates spectrum generators for user-defined (SUSY or non-SUSY) models. It makes use of SARAH to obtain model-dependent details, i.e. couplings, RGEs, self-

energies and tadpole corrections. The boundary conditions on the model parameters at different scales, as well as spectrum-generator specific configuration details are specified in a **FlexibleSUSY** model file. **FlexibleSUSY** can therefore generate spectrum generators for the different MSSM singlet extensions provided by **SARAH**, for example the NMSSM with and without a $\mathbb{Z}_3$ symmetry. The generated spectrum generators consist of C++ classes, which can be easily adapted allowing one to use the routines in a custom-made program for solving non-standard problems. For example one may build a tower of effective field theories, which could be useful for NMSSM scenarios with very heavy superparticles. Currently, **FlexibleSUSY** fully supports only real parameters. Complex parameters have been developed very recently in version 1.1.0, but they are still undocumented and undergoing testing. The automatic implementation of alternative boundary-value-problem solvers and automatic generation of towers of effective field theories are currently under development. Additionally, an extension to calculate decays, **FlexibleDecay**, for both Higgs and superparticles is currently being developed.

Due to the use of **SARAH**, **FlexibleSUSY** employs the full two-loop RGEs and one-loop corrections to the pole mass spectrum. At the two-loop level it makes use of Higgs mass corrections available in the literature. In the case of the NMSSM **FlexibleSUSY** uses the $\alpha_s(\alpha_t + \alpha_b)$ corrections given in Ref. [35]. In addition, the MSSM two-loop corrections based on Refs. [64,65,66,67] are used, though the user may choose to switch any of these two-loop corrections off using the SLHA file.

The homepage of **FlexibleSUSY** is

<http://flexiblesusy.hepforge.org/>

3.2 Used codes and options

For the comparison in the following, these versions of the different codes were used:

- **NMSSMTools** 4.7.0
- **SPheno** 3.3.6 together with **SARAH** 4.5.7
- **NMSSMCalc** 1.0.3
- **SOFTSUSY** 3.6.0
- **FlexibleSUSY** 1.1.0 together with **SARAH** 4.5.0

We always make use of the most precise calculation available in each code, i.e. all available two-loop corrections turned on and in **NMSSMTools** we only compare Higgs masses calculated using option 1, described in section 3.1.1, and do not consider any of the approximations from alternative options. We also concentrate on calculations done in the $\overline{\text{DR}}$ scheme. A discussion of the effects using an OS scheme is beyond the scope of this comparison and will be given elsewhere including also results from the upcoming NMSSM version of **FeynHiggs** [101,102,103,104,105].

3.3 Predictions for the CP-even and CP-odd Higgs masses

We discuss here the Higgs mass predictions of the various codes for six different test points (TP) with the following features:

- MSSM-like point (TP1)
- MSSM-like point with large stop splitting (TP2)
- Point with light singlet and λ close to the perturbativity limit (TP3)
- Point with heavy singlet and λ close to the perturbativity limit (TP4)
- Point with slightly lighter singlet. Additional matter for perturbativity needed; inspired by NMP9 [106] (TP5)
- Point with huge λ (TP6)

In table 2 we list the input parameters that are different for the six benchmark points. All values in this table apart from the one for $\tan\beta$ are defined at the scale Q . The shown value for $\tan\beta$ refers to the scale M_Z ⁷. The remaining soft SUSY-breaking parameters, common to all points, are

$$M_{\tilde{f}} = 1500 \text{ GeV} , A_{\tau} = 0 . \quad (16)$$

⁷Since **NMSSMCalc** and **NMSSMTools** expect $\tan\beta$ to be defined at Q as well, the input values have to be adjusted accordingly.

	Q	$\tan\beta$	λ	κ	A_λ	A_κ	μ_{eff}	M_1	M_2	M_3	A_t	A_b	$m_{\tilde{t}_L}$	$m_{\tilde{t}_R}$
TP1	1500	10	0.1	0.1	-10	-10	900	500	1000	3000	3000	0	1500	1500
TP2	1500	10	0.05	0.1	-200	-200	1500	1000	2000	2500	-2900	0	2500	500
TP3	1000	3	0.67	0.1	650	-10	200	200	400	2000	1000	1000	1000	1000
TP4	750	2	0.67	0.2	405	0	200	120	200	1500	1000	1000	750	750
TP5	1500	3	0.67	0.2	570	-25	200	135	200	1400	0	0	1500	1500
TP6	1500	3	1.6	1.61	375	-1605	614	200	400	2000	0	0	1500	1500

Table 2 Input parameters for the test points. All dimensionful parameters in units of GeV.

In addition, if possible via the input file, the codes were forced to calculate the masses directly at the input scale Q . This is possible for all tools except `NMSSMTools`. More details about the general handling of the scales in the different codes will be given in section 4.1.3. These points are chosen in a way that differences between the predicted Higgs masses are visible. However, they are not the result of an exhaustive scan to find points with the largest possible differences. Thus, we expect that these points are representative of a non-negligible fraction of the parameter space of the NMSSM.

TP1 and TP2 have small couplings between the Higgs doublets and the singlet. Thus, a mass of about 125 GeV for the SM-like state can only be obtained via large (s)top corrections as in the MSSM. In contrast, TP3-TP6 come with a large λ coupling and a small value for $\tan\beta$. This already increases the mass of the SM-like scalar at tree level. However, additional loop corrections beyond the MSSM are also expected to become more important. The difference between these four points is that TP3 and TP4 have values for λ and κ that give a consistent model without any Landau pole up to the GUT scale of 10^{16} GeV. For TP5 λ is not perturbative until the GUT scale unless additional matter is assumed to change the running. TP6 has a huge λ which always leads to a cut-off. Scenarios with these large λ couplings can be motivated by naturalness considerations [107].

	TP1	TP2	TP3	TP4	TP5	TP6
	h_1					
FlexibleSUSY	123.55	122.84	<i>91.11</i>	127.62	<i>120.86</i>	126.46
NMSSMCALC	120.34	118.57	<i>90.88</i>	126.37	<i>120.32</i>	123.45
NMSSMTOOLS	123.52	121.83	<i>90.78</i>	127.30	<i>119.31</i>	126.63
SOFTSUSY	123.84	123.08	<i>90.99</i>	127.52	<i>120.81</i>	126.67
SPHENO	124.84	124.74	<i>89.54</i>	126.62	<i>119.11</i>	131.29
	h_2					
FlexibleSUSY	<i>1797.46</i>	<i>5951.36</i>	126.58	<i>143.11</i>	125.08	<i>700.80</i>
NMSSMCALC	<i>1797.46</i>	<i>5951.36</i>	124.86	<i>142.59</i>	123.14	<i>701.02</i>
NMSSMTOOLS	<i>1797.46</i>	<i>5951.36</i>	127.28	<i>144.07</i>	126.95	<i>700.46</i>
SOFTSUSY	<i>1797.46</i>	<i>5951.36</i>	126.59	<i>143.02</i>	125.12	<i>701.01</i>
SPHENO	<i>1798.01</i>	<i>5951.35</i>	126.77	<i>144.04</i>	125.61	<i>689.30</i>
	h_3					
FlexibleSUSY	2758.96	6372.08	652.95	467.80	627.28	1369.53
NMSSMCALC	2756.37	6371.31	652.48	467.42	627.00	1368.68
NMSSMTOOLS	2758.51	6345.72	651.03	466.38	623.79	1368.90
SOFTSUSY	2758.41	6370.3	652.78	467.73	627.14	1369.19
SPHENO	2757.11	6366.88	651.21	467.5	624.02	1363.02

Table 3 Masses for the CP-even scalars (in GeV) for TP1–TP6 when using the spectrum generators “out-of-the-box”. The values correspond to the two-loop results obtained by the different tools. For `NMSSMCALC` the value using the $\overline{\text{DR}}$ scheme is shown and for `NMSSMTools` the value using the most precise calculation is given. The masses for the SM-like scalar are written in bold fonts, those for the singlet-like scalar in italics.

The scalar masses calculated for these six points by the five spectrum generators as they come “out-of-the-box” are listed in table 3, and the absolute values of the entries of the rotation matrix Z_H that correspond to the SM-like scalar are listed in table 4. For TP1, TP2 and TP6 the singlet-like scalar is very heavy and decouples, for TP4 the singlet-like scalar is just slightly heavier than the SM-like scalar, while for TP3 and TP5 it is the lightest scalar. We find that, even for the MSSM-like points, the differences in the mass of the SM-like scalar can exceed 5 GeV. For the points with very large λ the difference can be even larger. The masses for

	TP1			TP2			TP3		
	$ Z_{i1} $	$ Z_{i2} $	$ Z_{i3} $	$ Z_{i1} $	$ Z_{i2} $	$ Z_{i3} $	$ Z_{i1} $	$ Z_{i2} $	$ Z_{i3} $
FlexibleSUSY	0.1039	0.9946	0.0076	0.1034	0.9946	0.0004	0.2172	0.1888	0.9577
NMSSMCALC	0.1039	0.9946	0.0076	0.1034	0.9946	0.0004	0.2177	0.1923	0.9569
NMSSMTOOLS	0.1039	0.9946	0.0076	0.1038	0.9946	0.0004	0.2229	0.2115	0.9511
SOFTSUSY	0.1039	0.9946	0.0076	0.1034	0.9946	0.0004	0.2164	0.1883	0.958
SPHENO	0.104	0.9945	0.0074	0.1035	0.9946	0.0004	0.2269	0.2265	0.9472
FlexibleSUSY	0.0075	0.0069	0.9999	0.0096	0.0006	1.	0.2814	0.9274	0.2466
NMSSMCALC	0.0075	0.0068	0.9999	0.0095	0.0006	1.	0.2811	0.9265	0.2502
NMSSMTOOLS	0.0076	0.0069	0.9999	0.0101	0.0006	0.9999	0.2782	0.9216	0.2708
SOFTSUSY	0.0075	0.0068	0.9999	0.0096	0.0006	1.	0.282	0.9273	0.246
SPHENO	0.0075	0.0067	0.9999	0.0096	0.0006	1.	0.2725	0.919	0.2851
FlexibleSUSY	0.9946	0.1039	0.0068	0.9946	0.1034	0.0096	0.9347	0.3231	0.1483
NMSSMCALC	0.9946	0.104	0.0068	0.9946	0.1034	0.0095	0.9347	0.3234	0.1476
NMSSMTOOLS	0.9946	0.104	0.0068	0.9945	0.1038	0.0102	0.9338	0.3255	0.1484
SOFTSUSY	0.9946	0.104	0.0068	0.9946	0.1034	0.0096	0.9347	0.3234	0.1476
SPHENO	0.9945	0.104	0.0068	0.9946	0.1035	0.0097	0.935	0.3228	0.1468

	TP4			TP5			TP6		
	$ Z_{i1} $	$ Z_{i2} $	$ Z_{i3} $	$ Z_{i1} $	$ Z_{i2} $	$ Z_{i3} $	$ Z_{i1} $	$ Z_{i2} $	$ Z_{i3} $
FlexibleSUSY	0.4866	0.7908	0.3712	0.2867	0.4021	0.8695	0.2741	0.9417	0.195
NMSSMCALC	0.4869	0.7849	0.3832	0.2967	0.4433	0.8459	0.2746	0.9416	0.1947
NMSSMTOOLS	0.4855	0.7495	0.4501	0.3072	0.4711	0.8269	0.2743	0.9416	0.1953
SOFTSUSY	0.4867	0.7902	0.3725	0.2863	0.402	0.8697	0.2747	0.9417	0.1941
SPHENO	0.4835	0.7646	0.4262	0.3033	0.4703	0.8288	0.2712	0.9407	0.2039
FlexibleSUSY	0.0370	0.4059	0.9132	0.2189	0.8561	0.4681	0.2927	0.1115	0.9497
NMSSMCALC	0.0447	0.4158	0.9084	0.2055	0.8354	0.5099	0.2919	0.1113	0.9500
NMSSMTOOLS	0.0834	0.4727	0.8772	0.1974	0.8184	0.5396	0.2937	0.1114	0.9494
SOFTSUSY	0.0385	0.4066	0.9128	0.2195	0.8561	0.4679	0.2918	0.1107	0.9501
SPHENO	0.0682	0.4525	0.8892	0.1958	0.8204	0.5372	0.2945	0.1206	0.948
FlexibleSUSY	0.8728	0.4581	0.1683	0.9327	0.3245	0.1575	0.9161	0.3174	0.2451
NMSSMCALC	0.8723	0.4594	0.1674	0.9326	0.3251	0.1568	0.9162	0.3177	0.2443
NMSSMTOOLS	0.8703	0.4634	0.1670	0.9309	0.3291	0.1584	0.9157	0.3177	0.2460
SOFTSUSY	0.8727	0.4587	0.1675	0.9327	0.3249	0.1568	0.9162	0.3176	0.2444
SPHENO	0.8727	0.4589	0.1667	0.9326	0.3252	0.1567	0.9164	0.3172	0.2443

Table 4 Absolute values of the entries of the rotation matrix Z_H corresponding to the SM-like scalar for TP1–TP6 when using the spectrum generators “out-of-the-box”. The values correspond to the two-loop results obtained by the different tools. For NMSSMCALC the value using the $\overline{\text{DR}}$ scheme is shown and for NMSSMTOOLS the value using the most precise calculation is given.

the other two scalars usually agree quite well between the codes. Only **SPheno** gives masses of the singlet-like states, for TP5 and TP6, which differ by 1–2% compared to all other tools.

4 Breaking down the differences

4.1 Main differences

The differences we have observed are larger than one usually finds for the MSSM when comparing the widely used tools that perform a $\overline{\text{DR}}$ calculation [67, 108]. We pinned down four sources which will be discussed in the following. Further differences, such as e.g. whether first and second generation fermions are included into the one-loop computation (NMSSMCALC, SPheno, FlexibleSUSY) or not (NMSSMTOOLS, SOFTSUSY), are numerically tiny.

4.1.1 Renormalization scheme

All codes but NMSSMCALC perform a pure $\overline{\text{DR}}$ renormalization. NMSSMCALC uses instead mixed $\overline{\text{DR}}$ -OS renormalization conditions at one-loop level. At two-loop level it offers an option to renormalize the (s)top sector not only OS but also $\overline{\text{DR}}$.

4.1.2 Calculation of the running parameters

A very crucial point is how the $\overline{\text{DR}}$ parameters for the gauge and Yukawa couplings at the SUSY scale are calculated, since these couplings affect all loop calculations.

Top Yukawa coupling The most important parameter entering the loop calculations in the Higgs sector is the top Yukawa coupling. To calculate the value of the $\overline{\text{DR}}$ -renormalized Yukawa coupling at the SUSY scale, there are two different approaches implemented in the codes:

1. **SUSY thresholds at M_Z :** **FlexibleSUSY**, **SOFTSUSY** and **SPheno** adapt to the NMSSM the approach presented in Ref. [109] to calculate the running parameters of the MSSM Lagrangian at the SUSY scale. In this setup all one-loop SUSY and non-SUSY thresholds are calculated at the weak scale, identified with the Z -boson mass M_Z , and also the known two-loop QCD corrections of [110, 111] are included. The evolution of the parameters from M_Z to the SUSY scale M_{SUSY} is done using two-loop SUSY RGEs. A detailed description of the calculation is given in Ref. [48] for **FlexibleSUSY**, Ref. [45] for **SOFTSUSY** and in Ref. [112] for **SPheno**. All three codes use in principle the same approach: the $\overline{\text{DR}}$ masses of the fermions are computed as

$$m_f^{\overline{\text{DR}}} = m_f^{\text{pole}} + \hat{m}_f \Sigma(\hat{m}_f^2), \quad (17)$$

where $\Sigma(p^2)$ is the (dimensionless) self-energy of the fermion for momentum p .⁸ The $\overline{\text{DR}}$ Yukawa couplings are then computed at M_Z by dividing the $\overline{\text{DR}}$ fermion masses by the appropriate VEVs. There is however a difference between the codes in the definition of the mass $\hat{m}_f$ entering the correction in eq. (17): **SOFTSUSY** and **FlexibleSUSY** use $\hat{m}_f = m_f^{\text{pole}}$, while **SPheno** uses $\hat{m}_f = m_f^{\overline{\text{DR}}}$. For top quarks, the effect of the different definition of $\hat{m}_f$ in the one-loop part of correction is partially compensated for by an appropriate shift in the two-loop QCD part of the correction.

2. **SUSY thresholds at M_{SUSY} :** In this scheme, the pole top mass is converted to the $\overline{\text{MS}}$ scheme at the scale m_t , and the RGEs of the SM are used for the running to M_{SUSY} . At M_{SUSY} the conversion of the $\overline{\text{MS}}$ top mass of the SM to the corresponding $\overline{\text{DR}}$ Yukawa coupling of the NMSSM takes place including the SUSY thresholds. This is the approach followed in **NMSSMTools** and **NMSSMCALC**. However, there remain some differences in the concrete realization. **NMSSMCALC** uses the NLO-QCD contributions to the top-mass RGE for the running up to M_{SUSY} , and includes only the SQCD threshold corrections at M_{SUSY} . **NMSSMTools** resums also large logarithms $\sim Y_t^2 \log(M_{\text{SUSY}}/m_t)$ and $\sim Y_t^2 \log(M_{\text{SUSY}}/M_A)$ (the latter in case the scale M_A of the heavy MSSM-like scalars differs from the scale M_{SUSY} of the stop/sbottom) and it includes also $\mathcal{O}(\alpha_t)$ threshold corrections from stop-Higgsino loops at M_{SUSY} .

Strong interaction coupling The strong interaction coupling at the scale M_{SUSY} , important for the two-loop corrections to the Higgs masses, is derived as follows:

- **FlexibleSUSY**, **SOFTSUSY**, **SPheno** calculate $\alpha_s^{\overline{\text{DR}}}(M_Z)$ via

$$\alpha_s^{\overline{\text{DR}}}(M_Z) = \frac{\alpha_s^{(5),\overline{\text{MS}}}(M_Z)}{1 - \Delta\alpha_s(M_Z)}, \quad (18)$$

$$\Delta\alpha_s(M_Z) = \frac{\alpha_s}{2\pi} \left(\frac{1}{2} - 2 \log \frac{m_{\tilde{g}}}{M_Z} - \frac{2}{3} \log \frac{m_t}{M_Z} - \frac{1}{6} \sum_{i=1}^6 \left(\log \frac{m_{\tilde{u}_i}}{M_Z} + \log \frac{m_{\tilde{d}_i}}{M_Z} \right) \right), \quad (19)$$

where $\alpha_s^{(5),\overline{\text{MS}}}$ denotes the strong coupling constant of the SM with $N_f = 5$ active flavors. Finally, $\alpha_s^{\overline{\text{DR}}}(M_Z)$ is run to M_{SUSY} using the two-loop RGEs of the NMSSM.

- **NMSSMCALC**: $\alpha_s^{\overline{\text{DR}}}(M_{\text{SUSY}})$ is computed by running $\alpha_s^{(5),\overline{\text{MS}}}(M_Z)$ up to the top pole mass m_t using the QCD part of the two-loop RGEs of the SM with $N_f = 5$ active flavors. Afterwards, the so-obtained $\alpha_s^{\overline{\text{MS}}}(m_t)$ is run to M_{SUSY} using the corresponding RGEs with $N_f = 6$ active flavors. With this procedure the one-loop

⁸More precisely, **SPheno** calculates the chiral and scalar self-energy contributions Σ_L , Σ_R , Σ_S to the 3×3 mass matrix, but this only leads to differences if flavor violation is included.

top threshold correction for $\alpha_s^{\overline{\text{MS}}}$ is taken into account implicitly. Finally, $\alpha_s^{\overline{\text{MS}}}(M_{SUSY})$ is converted to $\overline{\text{DR}}$ by

$$\alpha_s^{\overline{\text{DR}}}(M_{SUSY}) = \frac{\alpha_s^{\overline{\text{MS}}}(M_{SUSY})}{1 - \frac{\alpha_s^{\overline{\text{MS}}}(M_{SUSY})}{4\pi}}. \quad (20)$$

- **NMSSMTools**: $\alpha_s^{\overline{\text{MS}}}(M_{SUSY})$ is computed with a procedure analogous to the one of **NMSSMCALC**, but it is not converted to the $\overline{\text{DR}}$ scheme. This difference amounts to a three-loop (i.e., higher-order) effect in the calculation of the Higgs masses.

Electroweak interactions The EW gauge sector of the NMSSM is determined by three fundamental parameters, i.e. the gauge couplings⁹ g_1 and g_2 and the combination of Higgs VEVs $v = (v_u^2 + v_d^2)^{1/2}$. For a consistent determination of these three parameters, a choice of three physical inputs is required. The procedures adopted by the five codes differ in the choices of physical inputs, as well as, in the case of **NMSSMCALC**, the choice of renormalization scheme for the fundamental parameters.

1. **FlexibleSUSY**, **SOFTSUSY** and **SPheno** adapt again the approach of Ref. [109] to calculate g_1 and g_2 . The three physical inputs are chosen as the Z mass, the Fermi constant G_F and the electromagnetic coupling of the SM at the scale M_Z in the 5-flavor scheme, $\alpha_{em}^{(5),\overline{\text{MS}}}(M_Z)$. The threshold corrections to α_{em} from SUSY states, the charged Higgs and the top are calculated at the scale M_Z as

$$\alpha_{em}^{\overline{\text{DR}}}(M_Z) = \frac{\alpha_{em}^{(5),\overline{\text{MS}}}(M_Z)}{1 - \Delta\alpha(M_Z)}, \quad (21)$$

$$\Delta\alpha(M_Z) = \frac{\alpha}{2\pi} \left(\frac{1}{3} - \frac{16}{9} \log \frac{m_t}{M_Z} - \frac{4}{9} \sum_{i=1}^6 \log \frac{m_{\tilde{u}_i}}{M_Z} - \frac{1}{9} \sum_{i=1}^6 \log \frac{m_{\tilde{d}_i}}{M_Z} - \frac{4}{3} \sum_{i=1}^2 \log \frac{m_{\tilde{\chi}_i^+}}{M_Z} - \frac{1}{9} \log \frac{m_{H^+}}{M_Z} \right). \quad (22)$$

The Weinberg angle $\sin^{\overline{\text{DR}}} \Theta_W$ at the scale M_Z is obtained iteratively from the above-computed $\alpha_{em}^{\overline{\text{DR}}}(M_Z)$, plus G_F and M_Z , via the relation

$$\left(\sin^{\overline{\text{DR}}} \Theta_W \cos^{\overline{\text{DR}}} \Theta_W \right)^2 = \frac{\pi \alpha_{em}^{\overline{\text{DR}}}(M_Z)}{\sqrt{2} M_Z^2 G_F (1 - \delta_r)}, \quad (23)$$

where

$$\delta_r = \hat{\rho} \frac{\Pi_{WW}^T(0)}{M_W^2} - \frac{\Re \Pi_{ZZ}^T(M_Z^2)}{M_Z^2} + \delta_{\text{VB}} + \delta_r^{(2)}, \quad (24)$$

$$\hat{\rho} = \frac{1}{1 - \Delta\hat{\rho}}, \quad \Delta\hat{\rho} = \Re \left[\frac{\Pi_{ZZ}^T(M_Z^2)}{\hat{\rho} M_Z^2} - \frac{\Pi_{WW}^T(M_W^2)}{M_W^2} \right] + \Delta\hat{\rho}^{(2)}, \quad (25)$$

$\Pi_{VV}^T(p^2)$ ($V = Z, W$) are the $\overline{\text{DR}}$ -renormalized transverse parts of the self-energies of the vector bosons, computed at the renormalization scale $Q = M_Z$, and $\delta_r^{(2)}$ and $\Delta\hat{\rho}^{(2)}$ are two-loop corrections given in [113, 109]. The one-loop vertex and box corrections δ_{VB} implemented into **FlexibleSUSY** and **SOFTSUSY** are taken from [114, 115, 116], while the ones used in **SPheno** are automatically generated with **SARAH** for the NMSSM. We remark that, in this approach, the physical value of the W mass is an output of the calculation. At a generic scale Q , the relation between the pole mass M_V of a gauge boson V and its corresponding $\overline{\text{DR}}$ mass reads

$$M_V^{\overline{\text{DR}}}(Q) = \sqrt{M_V^2 + \Pi_{VV}^T(M_V^2)}, \quad (26)$$

where the self-energy is also computed at the scale Q .

In **FlexibleSUSY** $\sin^{\overline{\text{DR}}} \Theta_W$ can be alternatively calculated from the $\overline{\text{DR}}$ W and Z masses, via

$$\sin^{\overline{\text{DR}}} \Theta_W(M_Z) = \sqrt{1 - \left(\frac{M_W^{\overline{\text{DR}}}(M_Z)}{M_Z^{\overline{\text{DR}}}(M_Z)} \right)^2}. \quad (27)$$

⁹ For the $U(1)$ coupling g_1 we adopt the SM normalization, i.e. $g_1 = g'$.

In this case the three physical inputs used to calculate g_1 and g_2 are M_Z , M_W and $\alpha_{em}^{(5),\overline{\text{MS}}}(M_Z)$. This approach is the original one implemented in **FlexibleSUSY** 1.0.0 and when we refer to “out-of-the-box” results for **FlexibleSUSY** it will be using this option.

The so-obtained values for $\alpha_{em}^{\overline{\text{DR}}}$ and $\sin^{\overline{\text{DR}}} \Theta_W$ are used to get $g_1^{\overline{\text{DR}}}(M_Z)$ and $g_2^{\overline{\text{DR}}}(M_Z)$, which are then evolved to the SUSY scale using two-loop RGEs of the NMSSM.

2. **NMSSMTools** uses M_Z , M_W and G_F as physical inputs for the EW sector. It calculates the $\overline{\text{DR}}$ values of g_1 and g_2 at the scale M_{SUSY} by making use of the relations between the fundamental EW parameters and the $\overline{\text{DR}}$ masses of the gauge bosons, the latter computed at the scale $Q = M_{SUSY}$ as in eq. (26):

$$\sqrt{g_1^{\overline{\text{DR}}}(M_{SUSY})^2 + g_2^{\overline{\text{DR}}}(M_{SUSY})^2} = 2 M_Z^{\overline{\text{DR}}}(M_{SUSY}) / v^{\overline{\text{DR}}}(M_{SUSY}) , \quad (28)$$

$$g_2^{\overline{\text{DR}}}(M_{SUSY}) = 2 M_W^{\overline{\text{DR}}}(M_{SUSY}) / v^{\overline{\text{DR}}}(M_{SUSY}) , \quad (29)$$

with $v^{\overline{\text{DR}}}(M_{SUSY})$ calculated via eq. (34)¹⁰ below.

3. **NMSSMCalc** uses M_Z , M_W and the electromagnetic gauge coupling e as physical inputs for the EW sector. Moreover, it does not compute $\overline{\text{DR}}$ values for the EW gauge couplings, but adopts the OS conditions

$$g_1 = e \frac{M_Z}{M_W} , \quad g_2 = \frac{e}{\sqrt{1 - M_W^2/M_Z^2}} , \quad (30)$$

where M_W , M_Z and e are the measured values.

Electroweak VEV The last SM parameter that enters the calculation of the Higgs masses is v , which gets translated into the two Higgs VEVs v_d and v_u using the input value of $\tan \beta$. The parameter v at the scale M_{SUSY} is obtained as follows in the different codes:

1. **FlexibleSUSY** and **SOFTSUSY** calculate the $\overline{\text{DR}}$ value at M_Z as

$$v^{\overline{\text{DR}}}(M_Z) = \frac{2 M_Z^{\overline{\text{DR}}}(M_Z)}{\sqrt{g_1^{\overline{\text{DR}}}(M_Z)^2 + g_2^{\overline{\text{DR}}}(M_Z)^2}} . \quad (31)$$

$v^{\overline{\text{DR}}}(M_{SUSY})$ is then obtained using two-loop RGEs with the ξ -dependent terms calculated in the Feynman gauge [81,82].

2. **SPheno** calculates $v^{\overline{\text{DR}}}$ directly at M_{SUSY} as

$$v^{\overline{\text{DR}}}(M_{SUSY}) = \frac{2 M_Z^{\overline{\text{DR}}}(M_{SUSY})}{\sqrt{g_1^{\overline{\text{DR}}}(M_{SUSY})^2 + g_2^{\overline{\text{DR}}}(M_{SUSY})^2}} , \quad (32)$$

For completeness, we note that $v^{\overline{\text{DR}}}(M_Z)$, which is used in the threshold corrections to the gauge and Yukawa couplings, is calculated separately via the relation

$$v^{\overline{\text{DR}}}(M_Z) = \sqrt{M_Z^{\overline{\text{DR}}}(M_Z)^2 \frac{(1 - \sin^2 \Theta_W^{\overline{\text{DR}}}) \sin^2 \Theta_W^{\overline{\text{DR}}}}{\pi \alpha^{\overline{\text{DR}}}}} . \quad (33)$$

In this way, $v^{\overline{\text{DR}}}(M_Z)$ and $v^{\overline{\text{DR}}}(M_{SUSY})$ are not strictly correlated via an RGE evolution, but differences of a higher order are present.

3. **NMSSMTools** calculates $v^{\overline{\text{DR}}}(M_{SUSY})$ via

$$v^{\overline{\text{DR}}}(M_{SUSY})^{-2} = \sqrt{2} G_\mu \left(1 - \frac{\Pi_{WW}^T(0)}{M_W^2} - \delta_{\text{VB}} \right) , \quad (34)$$

where δ_{VB} denotes the one-loop correction to the muon decay amplitude. In the latter, **NMSSMTools** includes only the SM contribution.

4. **NMSSMCalc** uses also for v an OS condition in terms of the three physical inputs:

$$v = \frac{2 M_W}{e} \sqrt{1 - M_W^2/M_Z^2} . \quad (35)$$

¹⁰ All expressions assume $v \simeq 246$ GeV.

4.1.3 Scales

We have so far discussed differences in the threshold calculations, and how the codes perform those calculations at different scales. There are two other important scales involved in the calculation of the Higgs-boson masses: the scale at which the input parameters are taken by the codes, and the scale at which the pole masses are calculated. Thus, for a detailed comparison it is necessary that these scales be exactly the same in all codes. However, as mentioned in sec. 3.3, it is not possible to unify these two scales among the codes without modifications in **NMSSMTools**. In the following, we describe how these two scales are treated in general by the different codes, and how – and to what extent – they can be fixed to particular numerical values using the provided options in the input.

1. **SPheno**: If no scale is defined in the input, the input parameters are taken at $Q = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$ and the masses are calculated at the same scale. If a scale is set via the entries **EXTPAR**[0] or **MODSEL**[12], the input parameters are defined at this scale and the masses are calculated there as well.
2. **NMSSMCalc** uses a similar approach to **SPheno** and always calculates the masses at the same scale where the input parameters are taken. However, if no explicit scale is defined, the default choice is $Q = \sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}}$.
3. **SOFTSUSY** by default takes the input scale to be the gauge-coupling unification scale. However, one may instead choose the input scale by either explicitly specifying it in **EXTPAR**[0] or by setting **EXTPAR**[0] to -1 in order to set the input scale to $Q = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$. In contrast to **SPheno**, changing this entry does not affect the scale at which the pole masses are calculated. **SOFTSUSY** calculates the pole masses at $Q = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$ by default, like **SPheno**. To change this one may set an alternative scale¹¹ in the special **SOFTSUSY** block, using entry 4.
4. **FlexibleSUSY**: Both the parameter input scale and the scale at which the pole masses are calculated may be chosen by the user in the model file used to generate the code. In the NMSSM model file used in this section, both scales are set via the entry **EXTPAR**[0]. In the model file used in sec. 5 to study GUT scenarios the parameter input scale is set to be the gauge-coupling unification scale, and the scale where the pole masses are calculated is set to a generalization of the SUSY scale which reduces to $Q = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$ in the absence of family mixing.
5. **NMSSMTools** takes by default $Q = \sqrt{(2m_{\tilde{u}_L} + m_{\tilde{u}_R} + m_{\tilde{d}_R})/2}$ for the input parameters and $Q' = \sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}}$ to calculate the Higgs, stop and sbottom pole masses. It is possible to change Q to a fixed value via **EXTPAR**[0] or **MINPAR**[0], but fixing Q' is only possible by modifying the code (which is done for the present study). The running between the two scales is performed with two-loop RGEs for the NMSSM.

4.1.4 Included two-loop corrections

While all codes use complete calculations of the one-loop corrections to the Higgs masses [35,36,37,38], the partial two-loop corrections implemented in the codes cover different contributions. All the mentioned two-loop corrections are calculated for zero external momentum.

1. **NMSSMCalc** includes only the two-loop corrections of $\mathcal{O}(\alpha_s \alpha_t)$ [40], but accounts also for the two-loop corrections to the electroweak VEV, which are omitted in the other codes.
2. **SOFTSUSY**, **FlexibleSUSY** and **NMSSMTools** use the two-loop corrections of $\mathcal{O}(\alpha_s(\alpha_t + \alpha_b))$ which were calculated in the context of the NMSSM in Ref. [35]. In addition, these codes use the MSSM results for the two-loop corrections of $\mathcal{O}((\alpha_t + \alpha_b)^2)$ [66]. Moreover, **SOFTSUSY** and **FlexibleSUSY** include also routines to calculate the MSSM results for the tiny two-loop corrections involving α_τ [64,65,66,67].
3. **SPheno** performs a two-loop calculation in the gaugeless limit. Namely, any two-loop correction in the neutral Higgs sector that is independent of g_1 and g_2 is included. However, **SPheno** includes also the available routines for the NMSSM corrections of $\mathcal{O}(\alpha_s(\alpha_t + \alpha_b))$, as well as MSSM corrections involving only Yukawa couplings similar to **SOFTSUSY** and **FlexibleSUSY**. If wished by the user, these routines can be used instead of the full calculation.

4.2 Adjusting the codes

To check if the differences between the codes listed in the previous section can really explain the sizable discrepancies in the Higgs-mass predictions, we have modified the codes, choosing **SOFTSUSY** as reference. This

¹¹For this scale to be used directly it must be larger than M_Z . See documentation for full details.

	original		modified				
	one-loop	two-loop	one-loop	$\alpha_s \alpha_t$	$\alpha_s(\alpha_t + \alpha_b)$	$\alpha_s(\alpha_t + \alpha_b) + \text{MSSM}$	‘full’
TP1							
FlexibleSUSY	119.48	123.55	119.73	124.82	124.82	123.81	—
NMSSMCALC	115.49	120.34	119.75	124.85	—	—	—
NMSSMTOOLS	—	123.52	119.75	124.84	124.84	123.84	—
SOFTSUSY	119.75	123.84	119.75	124.84	124.84	123.84	—
SPHENO	120.69	124.84	119.75	124.84	124.84	123.84	123.84
TP2							
FlexibleSUSY	116.28	122.84	116.46	123.79	123.79	123.06	—
NMSSMCALC	111.80	118.57	116.49	123.82	—	—	—
NMSSMTOOLS	—	121.83	116.49	123.82	123.82	123.08	—
SOFTSUSY	116.49	123.08	116.49	123.82	123.82	123.08	—
SPHENO	118.00	124.74	116.49	123.81	123.81	123.08	123.05
TP3							
FlexibleSUSY	124.16	126.58	124.15	127.55	127.55	126.59	—
NMSSMCALC	121.76	124.86	124.15	127.56	—	—	—
NMSSMTOOLS	—	127.28	124.15	127.56	127.56	126.60	—
SOFTSUSY	124.15	126.59	124.15	127.56	127.56	126.59	—
SPHENO	124.79	126.77	124.15	127.55	127.55	126.59	126.10
TP4							
FlexibleSUSY	124.72	127.62	124.59	128.23	128.23	127.51	—
NMSSMCALC	122.85	126.37	124.59	128.23	—	—	—
NMSSMTOOLS	—	127.30	124.59	128.24	128.24	127.52	—
SOFTSUSY	124.59	127.52	124.59	128.24	128.24	127.52	—
SPHENO	124.89	126.62	124.59	128.23	128.23	127.52	126.33
TP5							
FlexibleSUSY	122.53	125.08	122.54	126.11	126.11	125.11	—
NMSSMCALC	121.95	123.14	122.54	126.12	—	—	—
NMSSMTOOLS	—	126.95	122.54	126.12	126.12	125.12	—
SOFTSUSY	122.54	125.12	122.54	126.12	126.12	125.12	—
SPHENO	122.90	125.61	122.53	126.12	126.12	125.12	124.85
TP6							
FlexibleSUSY	121.92	126.46	122.09	127.7	127.7	126.64	—
NMSSMCALC	118.55	123.45	122.12	127.73	—	—	—
NMSSMTOOLS	—	126.63	122.12	127.73	127.73	126.67	—
SOFTSUSY	122.12	126.67	122.12	127.73	127.73	126.67	—
SPHENO	123.37	131.29	122.09	127.71	127.71	126.65	129.91

Table 5 The mass in GeV of the SM-like scalar after applying the modifications listed in sec. 4.2 for TP1–TP6 at the different loop levels and including different two-loop corrections. All two-loop corrections are for zero external momentum, while at one-loop the entire p^2 dependence is taken into account. ‘full’ refers to a calculation which includes all corrections in the gaugeless limit where electroweak gauge couplings are neglected. Since the original version of `NMSSMTools` does not provide a flag to turn off the two-loop corrections, we don’t give any one-loop value here.

choice resulted from mere convenience: `FlexibleSUSY` is very close to `SOFTSUSY`, and the differences between the two codes were already fully understood. This made it easy to adapt `FlexibleSUSY` and left just three codes which required more modifications. In particular, we have changed the following:

- **Calculation of the $\overline{\text{DR}}$ parameters:** Changing the codes to mimic the calculation of the running couplings of `SOFTSUSY` would be, of course, a big task beyond the scope of this comparison¹² Therefore, the chosen approach was to read in directly the values of g_i , Y_i , v and $\tan\beta$ as calculated by `SOFTSUSY` and use them in the calculation of the Higgs masses. This was done for `NMSSMTools`, `NMSSMCALC` and `SPheno`. In `FlexibleSUSY` no actual modifications to the code are made, but we will show results for both options for calculating the Weinberg angle. The option which matches the approach in `SOFTSUSY` will be treated as the modified version, while the approach implemented in `FlexibleSUSY` 1.0.0 will be treated as the original version.

¹²We want to stress that it was double checked in the context of this comparison that the corresponding calculations performed by the codes are free of bugs.

- **Renormalization scale:** `NMSSMTools` was forced to calculate the Higgs mass at the same scale as all other codes.
- **Renormalization scheme:** The finite parts of the counterterms of the W boson mass δM_W , the Z boson mass δM_Z , and the electric charge δe were put to zero in `NMSSMCALC`. Note that at two-loop level it is more convenient to introduce a counterterm for v , since in the gaugeless limit only a combination of δM_W and δM_Z survives, which corresponds to δv . This counterterm also needs to be set to zero to adjust to the other codes.
- **Loop corrections:** Of course no additional loop corrections have been implemented in any of the codes in the context of this analysis. However, we added options to the different codes to turn off specific loop corrections, to get a clear picture of the masses at one-loop and at two-loop including (i) $\mathcal{O}(\alpha_s \alpha_t)$, (ii) $\mathcal{O}(\alpha_s(\alpha_t + \alpha_b))$ (iii) $\mathcal{O}(\alpha_s(\alpha_t + \alpha_b))$ plus MSSM approximations for the corrections involving only Yukawa couplings, (iv) the full NMSSM calculation in the gaugeless limit.

4.3 Masses after the adjustments

The results for the SM-like Higgs mass after applying all changes to the codes are listed in Tab. 5. Here, we disentangled also the different loop corrections at the one- and two-loop level. The entries $|Z_{i1}^H|$ and $|Z_{i3}^H|$ of the corresponding rotation matrix are listed in Tab. 6.

The big differences that were observed before are totally gone when comparing equivalent calculations. Thus, for TP1–TP5 all codes show an impressive agreement at $\mathcal{O}(\alpha_s \alpha_t)$. The corrections of $\mathcal{O}(\alpha_s \alpha_b)$ are completely negligible for all considered points (they could, however, give a sizable effect for very large $\tan \beta$). When including the purely Yukawa corrections at the two-loop level in the MSSM approximation, which is possible in all codes but `NMSSMCALC`, very good agreement is also found among the different tools. However, this approximation might not always be very good, and fails in particular for large λ . This is manifest in the comparison with the full NMSSM result, which is available only for `SPheno`.

4.4 Size of the different effects

We discuss now in turn the impact of the different changes we applied.

1. **FlexibleSUSY:** The entire difference between `FlexibleSUSY` and `SOFTSUSY` comes from the calculation of the Weinberg angle $\sin^{\overline{\text{DR}}} \Theta_W$ at the scale M_Z , as described in Section 4.1.2.
2. **SPheno:** In Tab. 7 the effects of adjusting successively the Yukawa couplings, the gauge couplings, and the electroweak VEV are shown. One can see that the biggest change comes from the Yukawa couplings, in particular the top Yukawa coupling.
3. **NMSSMCALC:** Tab. 8 shows the effects of applying successively the different adjustments. Here we show the effect of changing the Yukawa couplings, changing the renormalization of the EW sector, and neglecting $\mathcal{O}(\alpha_s \alpha_t)$ contributions due to the conversion of the electroweak VEV to the $\overline{\text{DR}}$ parameter, which were omitted in the other spectrum calculators. The effect of the latter on the Higgs masses is proportional to λ , and hence more visible for large λ .
4. **NMSSMTools:** Tab. 9 shows the effects of applying successively the different adjustments: the effects of adjusting successively the scale Q' for the Higgs-mass calculation, the running Yukawa and gauge couplings and the running electroweak VEV. The changes in the scale have only an effect for TP2, because for the other points Q was chosen as $\sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}}$, which coincides with the scale Q' used by `NMSSMTools`. The only sizable effects are due to the top Yukawa coupling and the changes in the VEV.

One can see from this comparison that the main difference between the predictions of the five codes is caused by the $\overline{\text{DR}}$ -renormalized top Yukawa coupling used in the calculation of the Higgs masses. In calculations that adopt the $\overline{\text{DR}}$ scheme, a change in the determination of $Y_t^{\overline{\text{DR}}}$ often has a larger impact than additional two-loop corrections to the Higgs mass matrix. Thus, a more-precise calculation of this coupling might be expected to result in a more-precise prediction for the Higgs masses. However, the precise calculation of the Higgs decays in `NMSSMCALC` includes only (S)QCD corrections. Therefore, to ensure an overall consistent calculation, the code includes only (S)QCD corrections in the calculation of $Y_t^{\overline{\text{DR}}}$ as well. This results in a prediction for the SM-like Higgs mass a few GeV below the ones of the other codes, which include additional corrections in the determination of $Y_t^{\overline{\text{DR}}}$.

	original		modified			
	one-loop	two-loop	one-loop	$\alpha_s \alpha_t$	$\alpha_s(\alpha_t + \alpha_b)$ + MSSM	‘full’
TP1						
FlexibleSUSY	0.1039,0.0076	0.1039,0.0076	0.1039,0.0076	0.1039,0.0076	0.1039,0.0076	—
NMSSMCALC	0.1039,0.0076	0.1039,0.0076	0.1039,0.0076	0.1039,0.0076	—	—
NMSSMTOOLS	—	0.1039,0.0076	0.1039,0.0076	0.1039,0.0076	0.1039,0.0076	—
SOFTSUSY	0.1039,0.0076	0.1039,0.0076	0.1039,0.0076	0.1039,0.0076	0.1039,0.0076	—
SPHENO	0.1040,0.0076	0.1040,0.0074	0.1039,0.0076	0.1039,0.0076	0.1039,0.0076	0.1039,0.0075
TP2						
FlexibleSUSY	0.1034,0.0004	0.1034,0.0004	0.1034,0.0004	0.1034,0.0004	0.1034,0.0004	—
NMSSMCALC	0.1034,0.0004	0.1034,0.0004	0.1034,0.0004	0.1034,0.0004	—	—
NMSSMTOOLS	—	0.1038,0.0004	0.1034,0.0004	0.1034,0.0004	0.1034,0.0004	—
SOFTSUSY	0.1034,0.0004	0.1034,0.0004	0.1034,0.0004	0.1034,0.0004	0.1034,0.0004	—
SPHENO	0.1035,0.0004	0.1035,0.0004	0.1034,0.0004	0.1034,0.0004	0.1034,0.0004	0.1034,0.0004
TP3						
FlexibleSUSY	0.2687,0.2975	0.2813,0.2470	0.2693,0.2970	0.2835,0.2400	0.2819,0.2465	—
NMSSMCALC	0.2664,0.3081	0.2811,0.2502	0.2705,0.2935	0.2842,0.2385	—	—
NMSSMTOOLS	—	0.2782,0.2708	0.2705,0.2935	0.2842,0.2384	0.2828,0.2442	—
SOFTSUSY	0.2693,0.2970	0.2820,0.2460	0.2693,0.2970	0.2836,0.2399	0.2820,0.2460	—
SPHENO	0.2701,0.2951	0.2725,0.2851	0.2693,0.2969	0.2836,0.2398	0.2820,0.2459	0.2719,0.2863
TP4						
FlexibleSUSY	0.4852,0.3689	0.4866,0.3712	0.4853,0.3698	0.4862,0.3938	0.4868,0.3722	—
NMSSMCALC	0.4858,0.3625	0.4869,0.3832	0.4845,0.4098	0.4836,0.4476	—	—
NMSSMTOOLS	—	0.4855,0.4501	0.4845,0.4098	0.4837,0.4472	0.4851,0.4235	—
SOFTSUSY	0.4853,0.3700	0.4867,0.3725	0.4853,0.3700	0.4862,0.3941	0.4867,0.3725	—
SPHENO	0.4852,0.3810	0.4835,0.4262	0.4853,0.3698	0.4862,0.3939	0.4867,0.3723	0.4840,0.4133
TP5						
FlexibleSUSY	0.0171,0.8787	0.2189,0.4681	0.0205,0.8754	0.2445,0.3881	0.2191,0.4685	—
NMSSMCALC	0.0154,0.9162	0.2055,0.5099	0.1259,0.7020	0.2621,0.3276	—	—
NMSSMTOOLS	—	0.1974,0.5396	0.1259,0.7020	0.2621,0.3276	0.2485,0.3758	—
SOFTSUSY	0.0208,0.8749	0.2195,0.4679	0.0208,0.8749	0.2447,0.3876	0.2195,0.4679	—
SPHENO	0.0605,0.8192	0.1958,0.5372	0.0206,0.8752	0.2449,0.387	0.2197,0.4672	0.1806,0.5766
TP6						
FlexibleSUSY	0.2741,0.1946	0.2741,0.1950	0.2746,0.1937	0.2747,0.1942	0.2747,0.1941	—
NMSSMCALC	0.2742,0.1947	0.2746,0.1947	0.2748,0.1935	0.2749,0.1939	—	—
NMSSMTOOLS	—	0.2743,0.1953	0.2748,0.1935	0.2749,0.1939	0.2749,0.1939	—
SOFTSUSY	0.2746,0.1937	0.2747,0.1941	0.2746,0.1937	0.2747,0.1941	0.2747,0.1941	—
SPHENO	0.2748,0.1938	0.2712,0.2039	0.2746,0.1937	0.2747,0.1942	0.2747,0.1941	0.2711,0.2037

Table 6 Absolute value of the H_d and singlet components ($|Z_{i1}^H|$, $|Z_{i3}^H|$) of the SM-like Higgs with index i after the modifications listed in sec. 4.2 for TP1–TP6. Because of the size of the table, we do not show explicitly the largest entry $|Z_{i2}^H|$, but it can easily be calculated as $|Z_{i2}^H|^2 = 1 - |Z_{i1}^H|^2 - |Z_{i3}^H|^2$. The remaining differences are caused by the external momentum p^2 used to calculate Z^H : NMSSMCALC and NMSSMTOOLS set $p^2 = 0$, while the other codes set $p^2 = m_{h_1}^2$. The conventions for the given corrections are the same as in Tab. 5, but we do not show the results for $\alpha_s(\alpha_b + \alpha_t)$ explicitly because they fully agree with the ones for $\alpha_s \alpha_t$ for the number of digits used here.

5 Expected effects for GUT scenarios

We have focused so far on the Higgs-mass calculation in which all SUSY parameters are fixed by the input at the SUSY scale. However, most codes offer also the possibility to study a GUT scenario where the soft SUSY-breaking parameters are fixed at the GUT scale. The differences among the codes discussed in the previous sections are going to play an important role here as well. Indeed, because of the additional RGE running, the discrepancies between the results of the different codes are usually of the same size as, and often even bigger than those found in the case of SUSY-scale inputs with comparable parameter choices. In particular, the differences in the determination of the $\overline{\text{DR}}$ couplings can play a crucial role, because the different values for the couplings at the SUSY scale affect the entire running between M_{SUSY} and M_{GUT} . This is especially important for points in which the input value of λ at M_{SUSY} is close to the perturbativity limit. Small changes in the threshold corrections to the weak-scale couplings can change the value of λ at the GUT scale considerably. This

Point	original	Y	g	v	modified
TP1	124.84	123.65	123.61	123.84	123.84
TP2	124.74	123.18	123.13	123.05	123.05
TP3	126.77	126.06	126.00	126.10	126.10
TP4	126.62	126.21	126.16	126.33	126.33
TP5	125.61	124.89	124.84	124.85	124.85
TP6	131.29	130.06	130.01	129.91	129.91

Table 7 The Higgs prediction for the SM-like Higgs mass by **SPheno** after applying successively the different adjustments for the Yukawas (Y), gauge couplings (g), and the electroweak VEV (v). Here, “original” refers to the results when using the code without any modification, while for “modified” all adjustments are turned on.

Point	original	Y	δ_1	δ_2	modified
TP1	120.34	124.41	124.85	124.85	124.85
TP2	118.57	123.31	123.82	123.82	123.82
TP3	124.86	127.55	127.50	127.56	127.56
TP4	126.37	128.32	128.18	128.23	128.23
TP5	123.14	126.21	126.03	126.12	126.12
TP6	123.45	127.26	127.55	127.73	127.73

Table 8 The prediction for the SM-like Higgs mass by **NMSSMCALC** after adjusting the $\overline{\text{DR}}$ Yukawa couplings (Y); performing a $\overline{\text{DR}}$ renormalization of the EW sector by changing the values of g_1 , g_2 and v and removing the finite parts of the one-loop counter-terms δM_W , δM_Z and δe (δ_1); removing also the finite counterterm for v at the two-loop level (δ_2). The same conventions as for Tab. 7 are used.

Point	original	$Q' = Q$	Y	g	v	modified
TP1	123.52	123.52	123.96	123.99	123.84	123.84
TP2	121.83	121.44	123.46	123.52	123.08	123.08
TP3	127.28	127.28	127.43	127.43	126.60	126.60
TP4	127.30	127.30	127.13	127.07	127.52	127.52
TP5	126.95	126.95	127.34	127.45	125.12	125.12
TP6	126.63	126.63	127.56	127.66	126.67	126.67

Table 9 Changes in the prediction by **NMSSMTools** for the SM-like Higgs mass after forcing the SUSY scale and the scale for the mass calculation to be identical ($Q' = Q$), changing the Yukawa couplings (Y), the gauge couplings (g), and the electroweak VEV (v). The same conventions as for Tab. 7 are used.

will then change the running of all soft terms from the GUT scale to the SUSY scale. While the modifications in the sfermion parameters are not necessarily large, A_λ and A_κ can be strongly affected. This will then have an impact on the singlet-like scalar.

To demonstrate this effect, we consider the two points given in Tab. 10. Both points provide an SM-like Higgs mass in the preferred range together with a light singlet, but the first (GTP1) is characterized by small values of λ and κ , while the second (GTP2) is characterized by larger values of those couplings, not far below the perturbativity limit. To show the large sensitivity of GTP2 compared to GTP1 to the parameters at the SUSY scale, we vary the top mass within the uncertainty: $m_t = 173.2 \pm 1.5$ GeV. The masses of the three CP-even Higgs bosons computed by the four codes that allow for GUT-scale boundary conditions, i.e. **FlexibleSUSY**, **NMSSMTools**, **SOFTSUSY** and **SPheno**, are shown in Tab. 11.

	m_0	$M_{1/2}$	A_0	$\tan \beta$	λ	κ	A_λ	A_κ	μ_{eff}
GTP1	1000	1000	-3000	10	0.05	0.05	-150	-300	100
GTP2	1000	1000	-3000	10	0.48	0.46	-150	-300	100

Table 10 Input values for GTP1 and GTP2. All dimensionful parameters in units of GeV. The parameters in the left section of the table are defined at the GUT scale, whereas those in the right section are defined at the scale M_{SUSY} , with the exception of $\tan \beta$ which is defined at M_Z .

	$m_t = 171.7$ GeV	$m_t = 173.2$ GeV	$m_t = 174.7$ GeV
FlexibleSUSY	100.1, 122.2, 951.9	100.0, 123.3, 967.0	100.0, 124.5, 982.0
NMSSMTools	100.0, 119.3, 982.1	100.0, 120.1, 974.4	99.9, 120.8, 1014.0
SOFTSUSY	100.1, 122.5, 958.6	100.0, 123.6, 974.4	100.0, 124.8, 990.2
SPheno	100.0, 123.7, 976.6	99.9, 124.9, 992.3	99.9, 126.0, 1008.0

	$m_t = 171.7$ GeV	$m_t = 173.2$ GeV	$m_t = 174.7$ GeV
FlexibleSUSY	102.8, 123.3, 961.9	97.9, 124.6, 974.1	92.5, 126.1, 986.1
NMSSMTools	96.6, 118.8, 987.7	91.4, 119.1, 1000.8	no output
SOFTSUSY	102.4, 123.7, 968.2	97.3, 125.2, 981.1	91.4, 126.7, 994.0
SPheno	95.1, 125.3, 981.9	89.3, 126.8, 994.6	82.5, 128.3, 1007.5

Table 11 The masses of the three CP-even Higgs bosons, $m_{h_{1,2,3}}$, for GTP1 (top) and GTP2 (bottom) for three different values of the top quark mass, m_t .

We see that, for GTP1, the SM-like Higgs mass varies by about 2 GeV for all codes when m_t is varied within one standard deviation, while the light-singlet mass is fairly independent of the top mass, and the predictions of all codes for the light-singlet mass agree within 0.1 GeV. For GTP2, the variation with m_t in the SM-like Higgs mass is about 3 GeV, but the singlet is very sensitive to the value of the top mass: its mass can change by about 10 GeV, and the different codes predict masses that differ from each other by several GeV. In addition, for $m_t = 174.7$ GeV **NMSSMTools** runs into the Landau pole and is not able to calculate the spectrum. The situation can be even more dramatic when taking for instance the benchmark points proposed in Ref. [117]: for each of the three points BP1–BP3, at least two codes do not produce an output. When they are forced to produce an output (for instance by ignoring tachyonic states which appear at tree-level), the differences between the codes can reach 20–30 GeV for the light singlet.

Points with very large λ at the SUSY scale are attractive since they seem to soften the SUSY fine-tuning problem [10, 11, 12, 13, 14, 15, 16, 17, 18, 19]. However, if λ is too close to the perturbativity limit, the output produced by the spectrum generators has to be taken with some care: these points can be very sensitive to small variations of the input parameters. Changing the SM input parameters within their respective uncertainties – or changing the details of the determination of the running couplings – can lead to a completely different spectrum, or even result in the spectrum generator classifying the parameter point as unphysical.

6 Conclusions

We have discussed the differences in the predictions for the scalar Higgs masses and mixings by public spectrum generators for the NMSSM. It has been shown that sizeable discrepancies can show up for various parameter points, but the origins of those discrepancies are fully understood. The main sources of discrepancies are in the approach used to calculate the running parameters that enter the Higgs-mass calculation. The most important parameter here is the top Yukawa coupling. Further discrepancies arise from the different accuracy of the two-loop corrections included in the codes.

This has important consequences for phenomenological studies: For MSSM-like points, **FlexibleSUSY**, **NMSSMTools**, **SOFTSUSY** and **SPheno** usually agree quite well with each other, while **NMSSMCALC** returns in general a smaller value for the SM-like Higgs boson mass. The main reasons for this discrepancy are the value of $Y_t^{\overline{\text{DR}}}$ at M_{SUSY} , which is always smaller in **NMSSMCALC**, and the inclusion of the MSSM results for the $\mathcal{O}((\alpha_t + \alpha_b)^2)$ corrections in **FlexibleSUSY**, **NMSSMTools** and **SOFTSUSY** but not in **NMSSMCALC**. For MSSM-like points, the MSSM results provide a good approximation to the calculation implemented in **SPheno** which includes the NMSSM specific effects. On the other hand, this approximation usually fails for large values of λ and/or if light singlet states are present. In these cases, the results of **FlexibleSUSY**, **NMSSMTools** and **SOFTSUSY** have to be taken with care because it might be that the MSSM approximation has even the wrong sign compared to the full NMSSM calculation [39]. For these cases **SPheno**, with the additional NMSSM specific contributions, is expected to provide the most reliable prediction, with the caveat that if states in the loop become very light, however, the calculation can be plagued by intrinsic problems of the effective-potential calculation [118, 119, 120]. **NMSSMCALC**, on the other hand, is the only code that includes two-loop $\mathcal{O}(\alpha_t \alpha_s)$ corrections to v , which become non-negligible for large values of λ .

It has been demonstrated with one example, that the differences can be much more pronounced for GUT scenarios with values of λ close to the perturbativity limit. The differences in the threshold calculations among

the codes can cause huge discrepancies in particular for light singlets and the predicted masses can differ by 10 GeV and more.

In general, comparing the predictions of different codes may provide a ballpark for the theoretical uncertainty of the Higgs mass calculation. However, even if these codes agree very well, one should not necessarily assume that the uncertainty is small: it is known from the MSSM that spectrum generators performing a $\overline{\text{DR}}$ calculation (`SOFTSUSY`, `SPheno`, `Suspect` [121]) can agree quite well, while sizeable differences to the OS calculation of `FeynHiggs` exists. The differences are assumed to come from the missing electroweak corrections and momentum dependence at two-loop level as well as from the dominant three-loop corrections. These are the effects that underlie the often-quoted estimate of a 3 GeV uncertainty for the SM-like Higgs mass in the MSSM [67, 103]. To get an estimate of the remaining theoretical uncertainty in the NMSSM, a comparison between different renormalization schemes will therefore be necessary. This is deferred to future work, when further publicly available codes will have implemented OS renormalization. However, even the validity of such an estimate is under discussion already in the MSSM, in particular for a heavier SUSY scale of 2 TeV and larger. Calculations based on the effective field theory approach, which become valid in this mass range, usually predict a SM-like Higgs mass that is lower by a few GeV compared to the prediction from dedicated tools for the MSSM [122, 123, 124].

With this work an important first step has been taken to understand the large differences in the Higgs mass computations of the various public NMSSM codes. For the investigated codes these differences are now completely understood and it has been shown that discrepancies of more than 3 GeV are not unusual taking into account the different corrections implemented. To further pin down the residual theoretical uncertainty the next natural step will be to extend this investigation to the comparison of different renormalization schemes.

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A: Used options in input files

A.1: FlexibleSUSY

```

1 Block MODSEL                                # Select model
2     6     0                                # flavour violation
3     12    SCALE                             #
4 Block FlexibleSUSY
5     0     1.000000000e-04                  # precision goal
6     1     0                                # max. iterations (0 = automatic)
7     2     0                                # algorithm (0 = two_scale, 1 = lattice)
8     3     0                                # calculate SM pole masses
9     4     2                                # pole mass loop order
10    5     2                                # EWSB loop order
11    6     2                                # beta-functions loop order
12    7     1                                # threshold corrections loop order
13    8     1                                # Higgs 2-loop corrections O(alpha_t alpha_s)
14    9     1                                # Higgs 2-loop corrections O(alpha_b alpha_s)
15    10    1                                # Higgs 2-loop corrections O(alpha_t^2 + ↪
      alpha_t alpha_b + alpha_b^2)

```

```

16 11 1 # Higgs 2-loop corrections  $O(\alpha_{\tau}^2)$ 

```

A.2: NMSSMCALC

```

1 Block MODSEL
2   3 1 # NMSSM
3   5 0 # 0: CP-conserving; 2: general CP-violation
4   6 2 # loop level 1: one 2: two
5   7 1 # 1: DRbar scheme for top/stop-sector; 2: OS scheme for ↵
      top/stop-sector

```

A.3: NMSSMTools

```

1 BLOCK MODSEL
2   3 1 # NMSSM particle content
3   1 0 # IMOD (0=general NMSSM, 1=mSUGRA, 2=GMSB)
4   10 0 # ISCAN (0=no scan, 1=grid scan, 2=random ↵
      scan, 3=MCMC)
5   9 0 # |OMGFLAG|=0: no (default), =1: relic ↵
      density only,
6 # =2: dir. det. rate, =3: indir. det. rate, ↵
      =4: both,
7 # OMGFLAG>0: 0.107<OMG<0.131, <0: OMG<0.131
8   8 2 # Precision for Higgs masses (default 0: as ↵
      before,
9 # 1: full 1 loop + full 2 loop from top/bot ↵
      Yukawas
10 # 2: as 1 + pole masses - 182 by courtesy ↵
      of P. Slavich)
11 13 0 # 1: Sparticle decays via NMSDECAY (default 0)
12 11 0 # Constraints on (g-2)_muon (1=yes, 0=no, ↵
      default=1)
13 14 0 # 0: H-> VV, VV* (default); 1: H->VV, VV*, V*V*
14 15 0 # Precision for micromegas (default=0):
15 # +0/1: fast computation on/off
16 # +0/2: Beps=1d-3, 1d-6
17 # +0/4: virtual Ws off/on
18 #
19
20 BLOCK MINPAR
21 0 SCALE # MSUSY (If != SQRT(2*MQ1+MU1+MD1)/2)
22 #

```

A.4: SOFTSUSY

```

1 # Example NMSSM input in SLHA2 format
2 Block MODSEL # Select model
3   6 0 # flavour violation

```

```

4      1      0      # mSUGRA
5      3      1      # NMSSM
6      12 SCALE      #
7 Block SOFTSUSY      # SOFTSUSY specific inputs
8      1      1.000000000e-04      # tolerance
9      2      2.000000000e+00      # up-quark mixing (=1) or down (=2)
10     5      1.000000000E+00      # 2-loop running
11     3      1.000000000E+00      # printout
12     4      SCALE      #
13     7      4.0
14     15     1.000000000E+00      # NMSSMTools compatible output (default: 0)
15     16     0.000000000E+00      # Call micrOmegas (default: 0 = no,
16                                     # 1 = relic density only,
17                                     # 2 = direct detection + relic density,
18                                     # 3 = indirect detection + relic density,
19                                     # 4 = all)
20     17     1.000000000E+00      # Sparticle decays via NMSDECAY (default: 0)
21     18     1.000000000E+00      # use soft Higgs masses as EWSB output

```

A.5: SPheno

```

1 Block MODSEL      #
2 1 0      # 1/0: High/low scale input
3 2 1      # Boundary Condition
4 6 0      # Generation Mixing
5 12 SCALE      #
6 Block SPhenoInput      # SPheno specific input
7 1 -1      # error level
8 2 0      # SPA conventions
9 7 0      # Skip 2-loop Higgs corrections
10 8 3      # Method used for two-loop calculation
11 9 1      # Gaugeless limit used at two-loop
12 10 0      # safe-mode used at two-loop
13 11 0      # calculate branching ratios
14 13 1      # include 3-Body decays
15 12 1.000E-04      # write only branching ratios larger than this value
16 15 1.000E-30      # write only decay if width larger than this value
17 31 -1      # fixed GUT scale (-1: dynamical GUT scale)
18 32 0      # Strict unification
19 34 1.000E-04      # Precision of mass calculation
20 35 80      # Maximal number of iterations
21 36 40      # Minimal number of iterations before discarding point
22 37 1      # Set Yukawa scheme
23 38 2      # 1- or 2-Loop RGEs
24 50 1      # Majorana phases: use only positive masses
25 51 0      # Write Output in CKM basis
26 52 1      # Write spectrum in case of tachyonic states
27 55 1      # Calculate one loop masses
28 57 0      # Calculate low energy constraints
29 65 1      # Solution tadpole equation

```

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