*Hierarchical Axions

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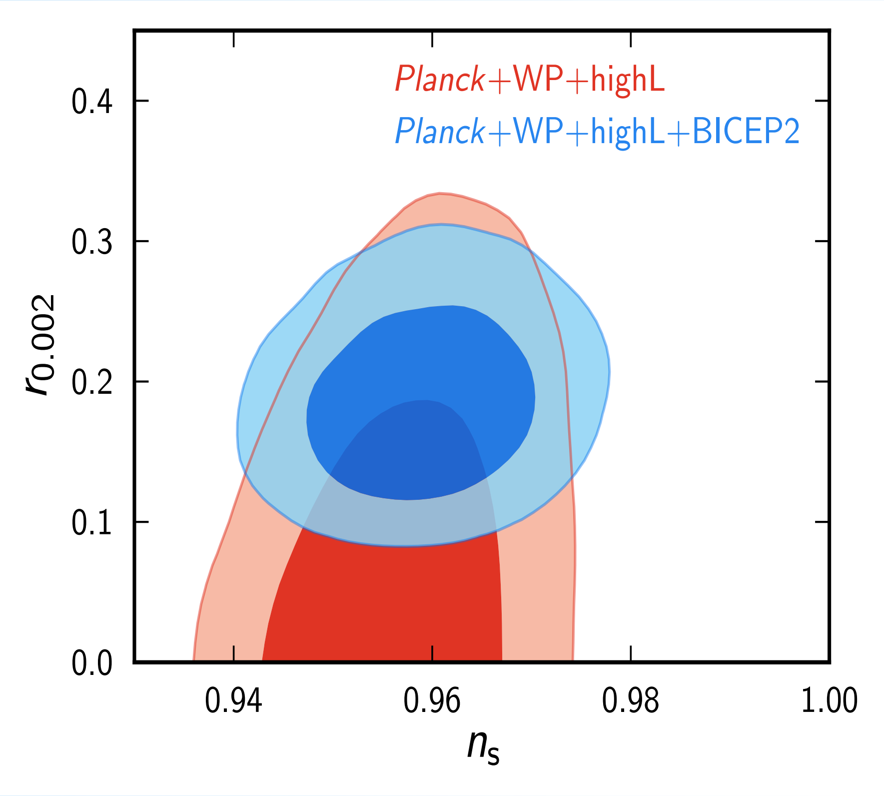
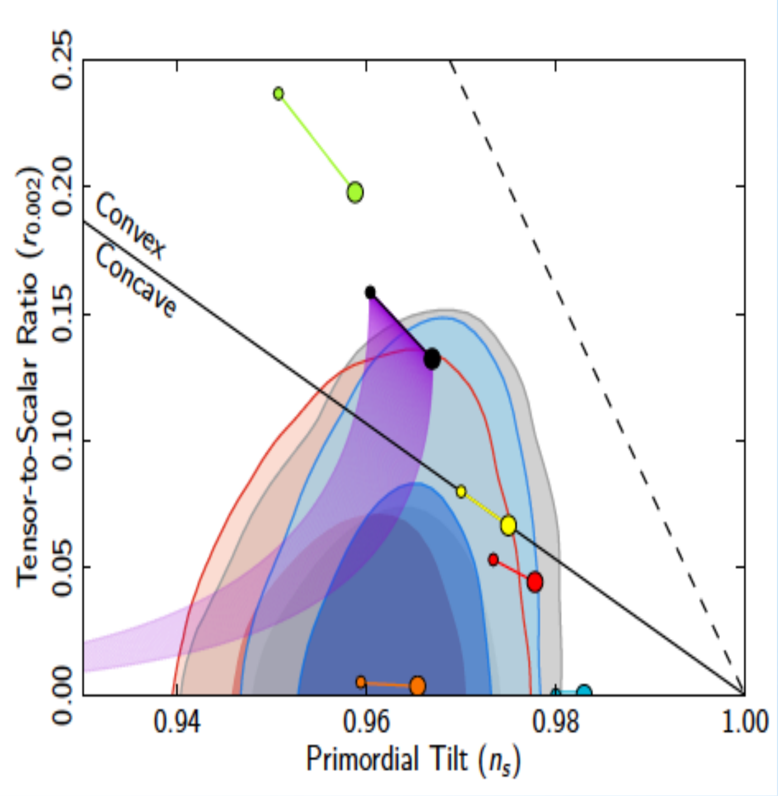
*Outline

* Axion/Natural Inflation
* Hierarchical Axion mechanism (Field Theory) 1403.7773
* String theory embedding (arXiv on Wednesday)
* Outlook: Underlying Mathematical structure - Monge-Ampere equation (WOP)





*Observational Status


GOOD BAD UGLY
SERGIO LEONE

*Natural Inflation – The Good!

* Consider a single axion with a canonical kinetic term and a decay constant f.
* For certain values of f:

1. Protected by a shift symmetry.
2. Eternally inflates.
3. Graceful exit.
4. Large or small field model.
5. Fits observations.

$$V = \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f}\right)\right]$$

*Natural Inflation - The Bad!

$$f > \sim 5M_p$$

*Natural Inflation ST- The Ugly!

* Specifically in string theory $f << M_p$
* The axions are coupled to moduli that must be stabilized
* Full analysis of Inflation and Moduli Stabilization is required.

*Hierarchical Axions

* Generating f_{eff}>1 by a hierarchy in the axion decay constants. Originally all f_i<<1.
* Just 2 axions
* Non-perturbative effects only
* Least amount of tuning of the input parameters
* The trajectory is contained in a very small field domain

$$V = \Lambda_1^4 \left(1 - \cos\left(\frac{p_1}{\tilde{f}_1}\phi_1 + \frac{p_2}{\tilde{f}_2}\phi_2\right)\right) + \Lambda_2^4 \left(1 - \cos\left(\frac{q_1}{\tilde{f}_1}\phi_1 + \frac{q_2}{\tilde{f}_2}\phi_2\right)\right),$$

$$V = \Lambda_{eff.}^4 \left[1 - \cos(\phi_{eff.}/f_{eff.})\right]$$

*Hierarchical Axions

$$V = \Lambda_1^4 \left(1 - \cos\left(\frac{p_1}{\tilde{f}_1}\phi_1 + \frac{p_2}{\tilde{f}_2}\phi_2\right)\right) + \Lambda_2^4 \left(1 - \cos\left(\frac{q_1}{\tilde{f}_1}\phi_1 + \frac{q_2}{\tilde{f}_2}\phi_2\right)\right),$$

* If we write $V = V(\phi_1 + \phi_2) \Rightarrow \det V_{ij} = 0$

We have a flat direction (KNP, DI and HA)

* Set p_2=0, and turn on Λ_1
* No alignment, but we have a hierarchy if $p_1 << q_1$. $p_1 \to 0$ reproduces exact flatness.
* Around the origin – a stable minimum. $\frac{\Lambda_2}{\Lambda_1} > \sqrt{\frac{p_1}{q_1}}.$

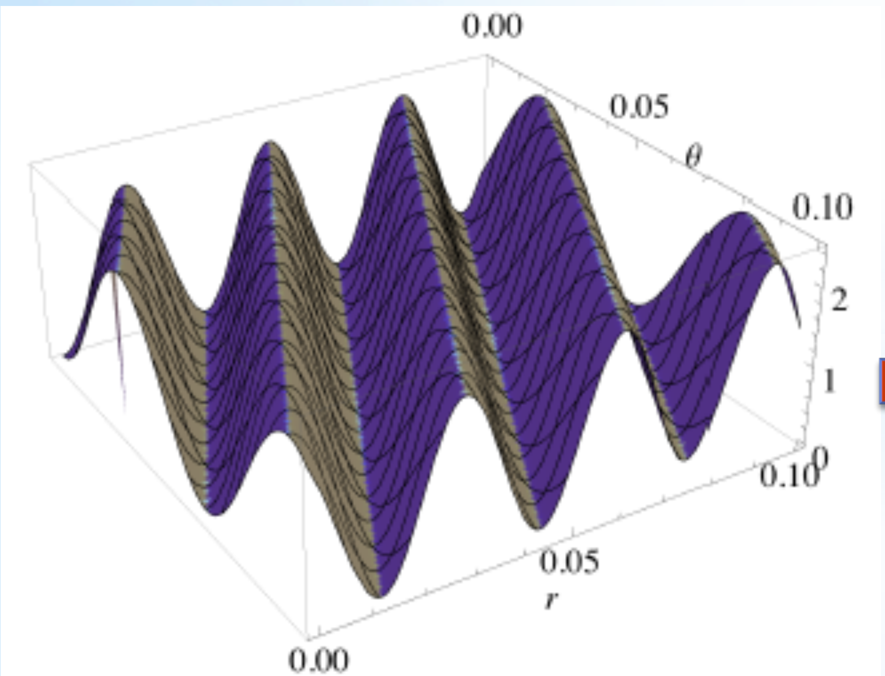
$$\det M^2 = \Lambda_1^4 \Lambda_2^4 \frac{p_1^2 q_2^2}{\tilde{f}_1^2 \tilde{f}_2^2},$$

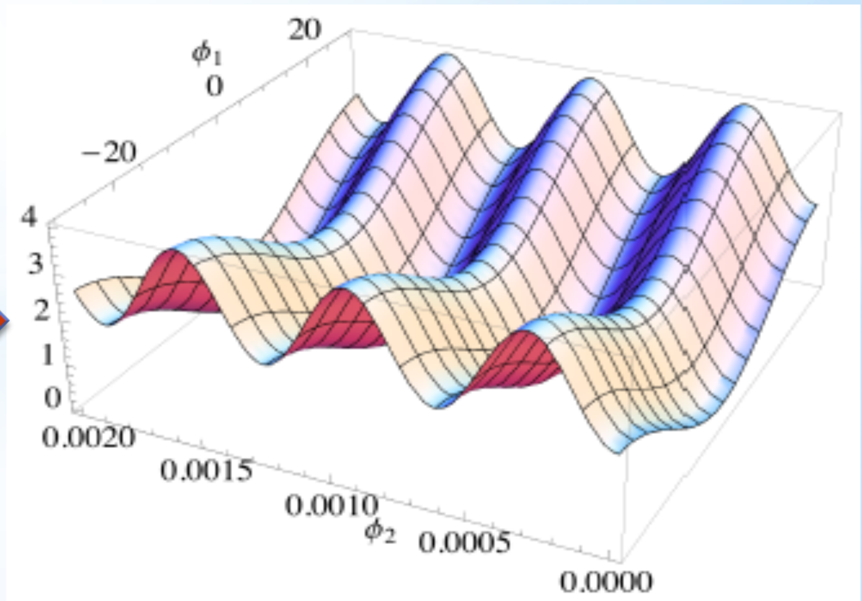
Hierarchical Axions

* Diagonalizing the mass matrix: $m_1 << m_2$

$$m_1^2 = \frac{\Lambda_1^4}{\tilde{f}_2^2}\left(\frac{p_1 q_2}{q_1}\right)^2, \qquad m_2^2 = \frac{\Lambda_1^4 p_1^2}{\tilde{f}_1^2} + \Lambda_2^4\left(\frac{q_1^2}{\tilde{f}_1^2} + \frac{q_2^2}{\tilde{f}_2^2}\right)$$

* Integrating out the heavier axion gives $f_{eff} = \tilde{f}_2 \frac{q_1}{q_2 p_1} > 1$





* Combine HA (and KNP) with moduli stabilization. Analyze both KKLT and LVS scenarios.

$$K = -2\ln\mathcal{V}, \quad W = W_0 + Ae^{-aT} + Pe^{-p_1 G_1 - p_2 G_2} + Qe^{-q_1 G_1 - q_2 G_2},$$

$$\mathcal{V} = c_L \left(T_L + \bar{T}_L - i\frac{1}{2(\tau-\bar{\tau})}\kappa_{L\beta\gamma}(G-\bar{G})^\beta (G^\gamma - \bar{G}^\gamma)\right)^{3/2}$$

$$- \sum_{i=2\ldots h_+^{1,1}} c_i \left(T_i + \bar{T}_i - i\frac{1}{2(\tau-\bar{\tau})}\kappa_{i\beta\gamma}(G-\bar{G})^\beta (G^\gamma - \bar{G}^\gamma)\right)^{3/2} .$$

* String theory embedding

* For a tractable analysis, we construct a hierarchy $V_0 >> V_1 >> V_2$.
* V_0 is the modified KKLT/LVS potential due to the existence of the additional b and c axions
* V_1 (and V_2), will uplift the remaining flat directions and will be the inflationary potential.
* The same hierarchy needed for the original moduli stabilization gives the hierarchy needed for inflation.

$$V_{inf} < V_{barrier} \sim \frac{W_0^2}{\mathcal{V}^2}$$

String Theory Embedding

* Chiral fields and geometric fields. D5 generated superpotential

$$\Psi = \{T_1, T_2, G_1, G_2, S\},\ \psi = \{\tau_1, \rho_1, \tau_2, \rho_2, b_1, b_2, c_1, c_2, s, C_0\}$$

$$K = -2\log\left[\Sigma_1^{3/2} - \Sigma_2^{3/2} + \xi\ (S+\bar{S})^{3/2}\right] - \log\left[S+\bar{S}\right],$$

$$\Sigma_1 \equiv T_1 + \bar{T}_1 + \frac{k_{11}}{2(S+\bar{S})}(G_1+\bar{G}_1)^2 + \frac{k_{12}}{2(S+\bar{S})}(G_2+\bar{G}_2)^2,$$

$$\Sigma_2 \equiv T_2 + \bar{T}_2 + \frac{k_{21}}{2(S+\bar{S})}(G_1+\bar{G}_1)^2 + \frac{k_{22}}{2(S+\bar{S})}(G_2+\bar{G}_2)^2.$$

* Simplest way to avoid ghosts and a stable minimum at $b_1=b_2=0$:

$$k_{ij} < 0\ \&\quad |k_{11}| > |k_{21}|\sqrt{\tau_2/\tau_1}\quad ,\quad |k_{12}| > |k_{22}|\sqrt{\tau_2/\tau_1}$$

LVS example

* The large and small cycle moduli are stabilized at the standard location, $b_1=b_2=0$.

$$V_{LVS} = \frac{9\left(b_1^2k_{11}+b_2^2k_{12}\right)^2W_0^2}{64g_s\tau_1^5} + \frac{3\xi W_0^2}{32\sqrt{g_s}\tau_1^{9/2}} - \frac{a^2A^2\left(b_1^2k_{12}k_{21}^2+b_2^2k_{11}k_{22}^2\right)}{12k_{11}k_{12}\tau_1^2}e^{a\left(\frac{b_1^2k_{21}+b_2^2k_{22}}{g_s}-2\tau_2\right)}$$
$$+\frac{a^2A^2g_s\sqrt{\tau_2}}{6\tau_1^{3/2}}e^{a\left(\frac{b_1^2k_{21}+b_2^2k_{22}}{g_s}-2\tau_2\right)} - \frac{g_saAW_0\tau_2}{4\tau_1^3}e^{a\left(\frac{b_1^2k_{21}+b_2^2k_{22}}{2g_s}-\tau_2\right)}.$$

$$\tau_1^{3/2} = \frac{3W_0\sqrt{\tau_2}\left(-1+a\tau_2\right)}{aA\left(-1+4a\tau_2\right)}e^{a\tau_2} \sim \frac{3W_0\sqrt{\tau_2}}{4aA}e^{a\tau_2}\left(1-\frac{3}{4a\tau_2}\right),$$

$$\tau_2^{3/2} = \frac{\xi\left(-1+4a\tau_2\right)^2}{16g_s^{3/2}a\tau_2\left(-1+a\tau_2\right)} \sim \frac{\xi}{g_s^{3/2}}\left(1+\frac{1}{2a\tau_2}\right).$$

* LVS Examle

* The inflationary part:

$$V_1 = \frac{3PW_0\xi}{64a\sqrt{g_s}\tau_1^{9/2}\tau_2}\cos\left[c_1p_1 + c_2p_2\right] + \frac{3QW_0\xi}{64a\sqrt{g_s}\tau_1^{9/2}\tau_2}\cos\left[c_1q_1 + c_2q_2\right]$$

$$V_2 = -\frac{PQ}{\tau_1^2}\frac{(k_{12}p_1q_1 + k_{11}p_2q_2)}{6k_{11}k_{12}}\cos\left[c_1p_1 + c_2p_2 - c_1q_1 - c_2q_2\right]$$
$$-\frac{P^2}{\tau_1^2}\frac{\left(k_{12}p_1^2 + k_{11}p_2^2\right)}{12k_{11}k_{12}} - \frac{Q^2}{\tau_1^2}\frac{\left(k_{12}q_1^2 + k_{11}q_2^2\right)}{12k_{11}k_{12}}.$$

$$V_{LVS} \sim \mathcal{O}\left(-\frac{W_0^2}{\mathcal{V}^3\log\mathcal{V}}\right), \qquad V_1 \sim \mathcal{O}\left(\frac{QW_0 + PW_0}{\mathcal{V}^3\log\mathcal{V}}\right), \qquad V_2 \sim \mathcal{O}\left(\frac{PQ + P^2 + Q^2}{\mathcal{V}^{4/3}}\right).$$

$$P, Q \ll \frac{W_0}{\mathcal{V}^{5/6}\sqrt{\log\mathcal{V}}} \qquad \text{ED3/ED1 case:} \quad P, Q \ll \frac{1}{\sqrt{\log\mathcal{V}}},$$

* LVS example

	W_0	A	a	P	Q	p_1	q_1	r	δ	k_{11}	k_{12}	k_{21}	k_{22}	g_s	ξ
KNP_1	1	1	π	10^{-4}	10^{-3}	$\frac{\pi}{2}$	$\frac{\pi}{2}$	1	0.01	-3	-3	-3	-3	0.3	0.5
KNP_2	8	2	$\frac{2\pi}{5}$	0.01	0.01	$\frac{\pi}{5}$	$\frac{\pi}{5}$	0.8	0.1	-6	-6	-6	-6	0.3	1

Table 5. Input parameters for the LVS string embedding.

	W_0	A	a	P	Q	p_1	p_2	q_1	q_2	k_{11}	k_{12}	k_{21}	k_{22}	g_s	ξ
HA_1	1	1	$\frac{2\pi}{5}$	10^{-3}	10^{-3}	0	$\frac{\pi}{11}$	$\frac{\pi}{11}$	π	-1	-1	-1	-1	0.4	1.5
HA_2	10	2	π	5×10^{-3}	5×10^{-3}	0	$\frac{2\pi}{15}$	$\frac{2\pi}{25}$	π	-10	-10	-1	-1	0.5	0.7

Table 6. Input parameters for the LVS string embedding.

	τ_1	τ_2	$\Lambda_1^4\times10^{12}$	$\Lambda_2^4\times10^{12}$	$\Lambda_3^4\times10^{12}$	f_{eff}
KNP_1	48.2	2.22	0.016	0.16	11.8	13.4
KNP_2	51.3	3.64	30	30	709	5.0
HA_1	19.3	3.59	41	41	443	5.1
HA_2	42.3	1.70	21	21	306	11.23

Table 7. Compactification features and inflationary parameters. Dimensionful quantities in units of M_P.

*Numerical Example

ED3/ED1 Reduce the tuning by a factor of 10-100

* We fully embedded the HA (and KNP) variant of natural inflation in string theory with moduli stabilization.
* Just 2 axions
* Non-perturbative effects only
* Least amount of tuning of the input parameters
* The trajectory is contained in a very small field domain. Sheds light on the small vs. large field discussion. (The “argument” is only relevant for fundamental fields. It’s trivial for low energy effective ones)
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Executive Summary

*Outlook

* KNP and HA have the same origin:

$$V = V(\phi_1 + \phi_2) \Rightarrow \det V_{ij} = 0$$

* So called "Monge-Ampere equation":
* Novel classification of models:

1. Det V_{ij} =V_i=0 at a point => Inflection/Hilltop/Accidental/ Small field models.

2. Det V_{ij} =0 everywhere (or submanifold) => guaranteed "Almost" inflationary trajectory. If at a point V_i=0 and the other eigenvalues are positive => Inflation.

3. A full model is constructed by adding a subdominant positive potential a la HA/DI $\delta V = \delta V(\phi_1)\,, \delta V_{11} > 0$

* Can use the mathematical literature on solutions of the Monge-Ampere equation.

Hull et al. 2010, Barbieri et al. 1984

*Outlook example

* Another 2-field solution of the MA equation, f is an arbitrary function:

$$V = (c_1 x + c_2 y) f\left(\frac{x}{y}\right)$$

* Add $\delta V = c_3 x^2$

* With f''>0, we can construct an inflationary model $\delta V'' > 0$