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Determination of Diffusion Rates in the Proton Beam Halo of HERA

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Abstract
A method for estimating diffusion rates (diffusion constants) in the halo of the HERA proton beam is presented. Relaxation processes in the transverse particle distribution are measured, which are induced by a change in the boundary condition set by a collimator jaw. The counting rates of scattered protons, detected at the collimator, are analysed.
A simple diffusion model is evaluated to provide a fit-function for the time dependence of the background rates. As one of the fit-variables the desired diffusion constant is obtained.
Finally the results of measurements under different machine conditions are presented. Variations of the diffusion rate over two orders of magnitude have been observed.

1. Introduction

The HERA proton ring is equipped with a system of 12 collimator jaws, arranged in three blocks, to remove the beam halo and thereby reduce the proton induced background at the experiments. Beam loss monitors are used for the exact positioning of the jaws. The monitors are installed downstream of the collimators and detect hadronic showers from protons hitting the jaws. Each monitor contains two silicon PIN-diodes (1.6 cm^2) which are mounted close to each other. In order to suppress the high synchrotron radiation background in the HERA tunnel only coincident events in both diodes are counted. If an aperture-defining jaw is moved a characteristic pattern in the loss rate is observed (Fig. 1).

If the jaw is moved closer to the beam, the loss rate increases sharply. Keeping the collimator fixed in the new position, the increased loss rate decays with a time constant of typically some 10 seconds. Retracting the jaw from the beam leads to the inverse behaviour: the loss rate drops sharply and increases again afterwards. The observed changes in loss rate, following a collimator motion, are obviously related to relaxation processes in the transverse particle distribution of the proton beam. The loss rate is remarkably sensitive to the collimator position. Motions as small as 50 μm change the instantaneous rate by nearly an order of magnitude.

From these observations the idea arose to extract some useful information about the diffusion process in the beam halo, that is to compute a diffusion constant from the distance moved by the collimator jaw and the typical time development of the rates. The method is similar to a measurement which had been performed in 1990 at the SPS [2].

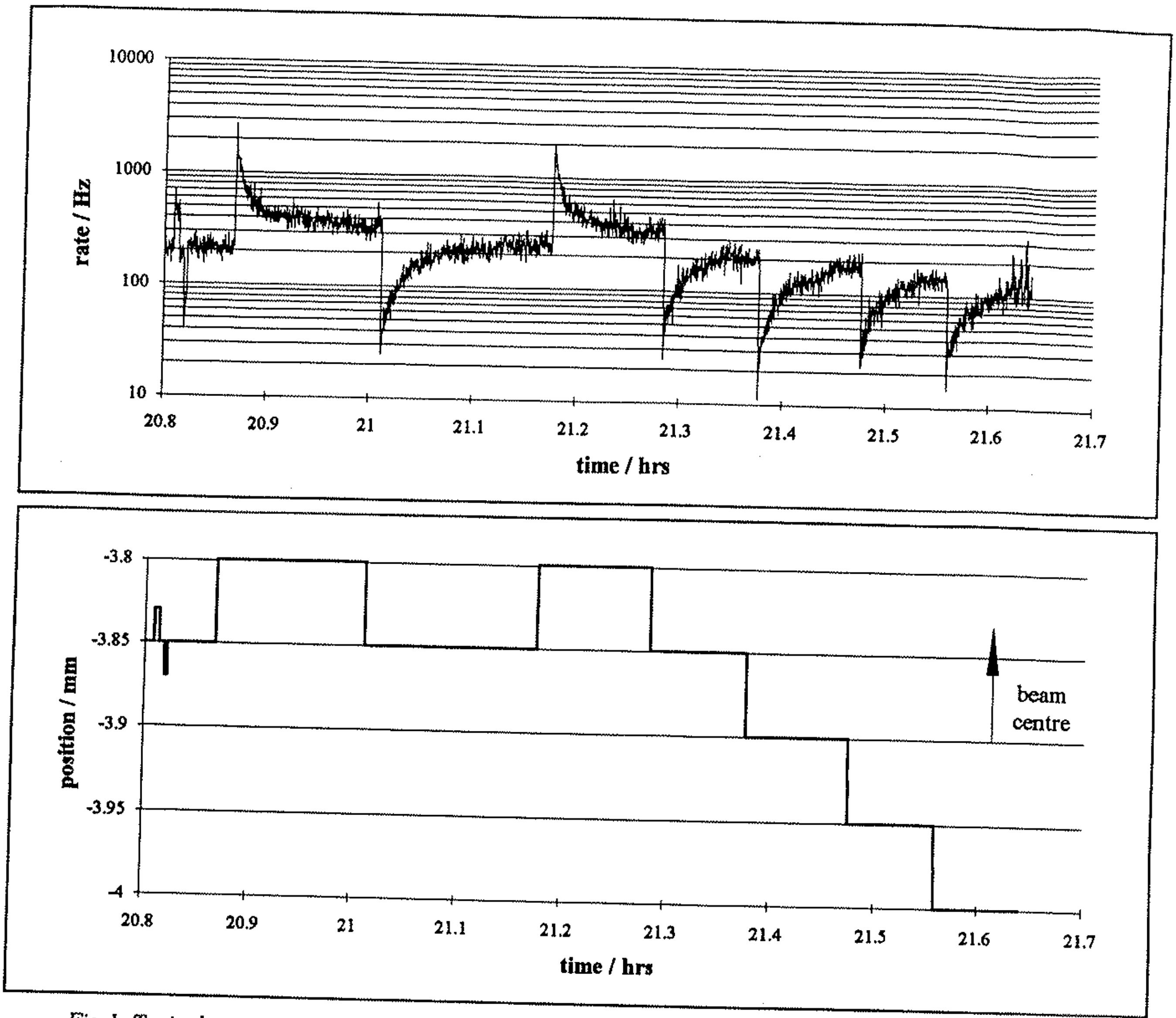

Fig.1: Typical counting rates after stepwise movements of a collimator jaw (jaw position on the lower plot)

The diffusion rate in the beam halo has a large influence on the mean impact parameter (distance of hit from jaw edge, see Fig. 2) of protons, hitting a collimator jaw and therefore on the efficiency of a collimation system. The knowledge of the diffusion constant at the normal transverse position of the collimator jaws is important for a better understanding and modelling of the collimation system.

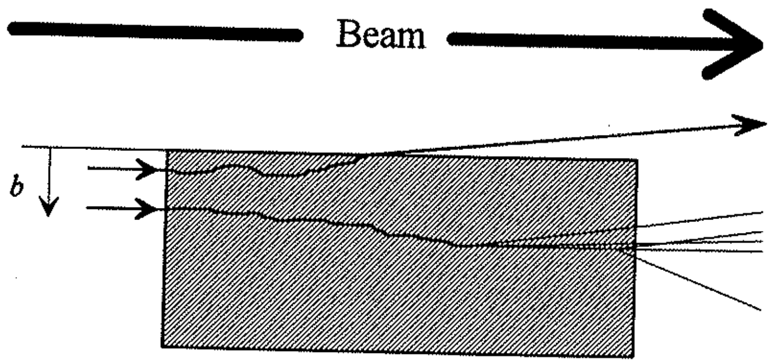

Fig. 2: The impact parameter b depends very sensitively on the transverse diffusion rate and has a large influence on the scraping efficiency of a collimator jaw (the smaller the impact parameter the higher the probability of elastic back scattering).

2. A SIMPLE DIFFUSION MODEL

We consider here the diffusion equation in the form

$$\frac{\partial}{\partial t} f(\vec{r},t)=C\nabla^2 f(\vec{r},t), \quad \vec{r}=(x,p_x), \tag{1}$$

where $\vec{r}$ is a transverse phase space vector, $f(\vec{r},t)$ the transverse distribution function and C the diffusion coefficient. For simplicity C is assumed to be a scalar constant as it is true for multiple scattering with residual gas atoms. The diffusion equation can be obtained by the assumption that the flux of particles is proportional to the gradient of the distribution function and inserting this relation into the continuity equation.

$$\vec{j}=-C\nabla f, \quad \frac{\partial f}{\partial t}+\nabla\vec{j}=0 \tag{2}$$

The following derivations are taken from [1]. In accelerator physics the particle motion is often described in terms of the Courant Snyder Invariant $W = (x^2 + p_x^2)/\beta = (x^2+(\alpha x+\beta x')^2)/\beta \equiv r^2/\beta$, which leads to the equation

$$\frac{\partial}{\partial t} f(W,t) = \frac{\partial}{\partial W}\left(D(W)\frac{\partial}{\partial W} f(W,t)\right), \text{ with } D(W) = \frac{4C}{\beta}W. \tag{3}$$

As a consequence of the change in the variables the diffusion coefficient $D(W)$ is no longer independent of the transverse betatron amplitude of the particle. Furthermore we introduce the dimensionless variable $Z = W/W_c = 0...1$ as a "normalised" one particle emittance. $W_c = x_c^2/\beta_c$ is the machine acceptance, defined by the collimator. These changes in the variables result in the following equation:

$$\frac{\partial}{\partial t} f(Z,t) = R\frac{\partial}{\partial Z}\left(Z\frac{\partial}{\partial Z} f(Z,t)\right), \tag{4}$$

where the new coefficient R is related to C via $R = 4C/\beta W_c$. The factor $RZ = RW/W_c = D(W)/W_c^2$ in equation (4) can be interpreted as an amplitude dependent diffusion coefficient. The boundary conditions of the partial differential equation are those of a reflecting barrier at $Z = 0$ (no particle can diffuse to negative emittances) and an absorbing barrier at $Z = 1$:

$$j(Z=0,t) = -RZ\frac{\partial}{\partial Z} f(Z,t)\Big|_{Z=0} = 0, \quad f(Z=1,t) = 0.$$

The solution of (4) can be obtained by an expansion in terms of eigenfunctions:

$$f(Z,t) = \sum_n g_n(Z)\, e^{-k_n t},$$

where the $g_n(Z)$ and k_n are the eigenfunctions and eigenvalues of the equation

$$R\frac{\partial}{\partial Z}\left(Z\frac{\partial}{\partial Z} g_n(Z)\right) + k_n\, g_n(Z) = 0.$$

The solution is then found to be a series of Bessel-functions:

$$f(Z,t) = \sum_n c_n\, J_0\left(\lambda_n\sqrt{Z}\right) e^{-\lambda_n^2\frac{R}{4}t} \tag{5}$$

with $$c_n = \frac{1}{J_1^2(\lambda_n)}\int_0^1 f_0(Z)\, J_0(\lambda_n\sqrt{Z})\, dZ. \tag{6}$$

The λ_n are the n'th zeros of $J_0(Z)$ and $f_0(Z)$ is the initial particle distribution. The solution (5) predicts an asymptotic beam lifetime of

$$\tau_\infty = 4 / R\lambda_0^2 , \qquad (7)$$

which is the time constant of the first term in the series. The lifetime of the second term is already a factor of $\lambda_0^2/\lambda_1^2 = 0.19$ smaller.

We can now make use of the distribution function (5) to compute a formula for the time dependence of the rates. The counting rate is proportional to the flux of particles at the aperture restriction, set by the collimator.

$$\dot{N} \propto j(Z=1,t) = -RZ\frac{\partial}{\partial Z} f(Z,t)\Big|_{Z=1} \qquad (8)$$

For the calculation of the coefficients c_n we consider two cases:

case 1: The collimator is moved towards the beam centre, from a position $Z_0 > 1$ to a final position $Z = 1$.

case 2: The collimator is moved outwards, from a position $Z_0 < 1$ to a position $Z = 1$.

In order to obtain the initial particle distribution we assume a stationary state which can be approximated with the first term of (5). The aperture defining collimator is now at Z_0 instead of 1, i.e. we have to replace Z by Z/Z_0. Furthermore the distribution should be normalised to an area of 1. After solving some integrals (see Appendix A) we obtain the c_n's for both cases:

$$c_n^{(1)} = \frac{\lambda_0 \lambda_n \sqrt{Z_0}\, J_0(\lambda_0/\sqrt{Z_0})}{J_1(\lambda_n) J_1(\lambda_0/\sqrt{Z_0})(Z_0\lambda_n^2 - \lambda_0^2)} \qquad c_n^{(2)} = \frac{J_0(\lambda_n\sqrt{Z_0})}{J_1^2(\lambda_n)(1 - Z_0\lambda_n^2/\lambda_0^2)} \qquad (9)$$

Fig. 3 shows distribution functions for both cases. One expects a qualitative time dependence of the background rates as shown in Fig. 4.

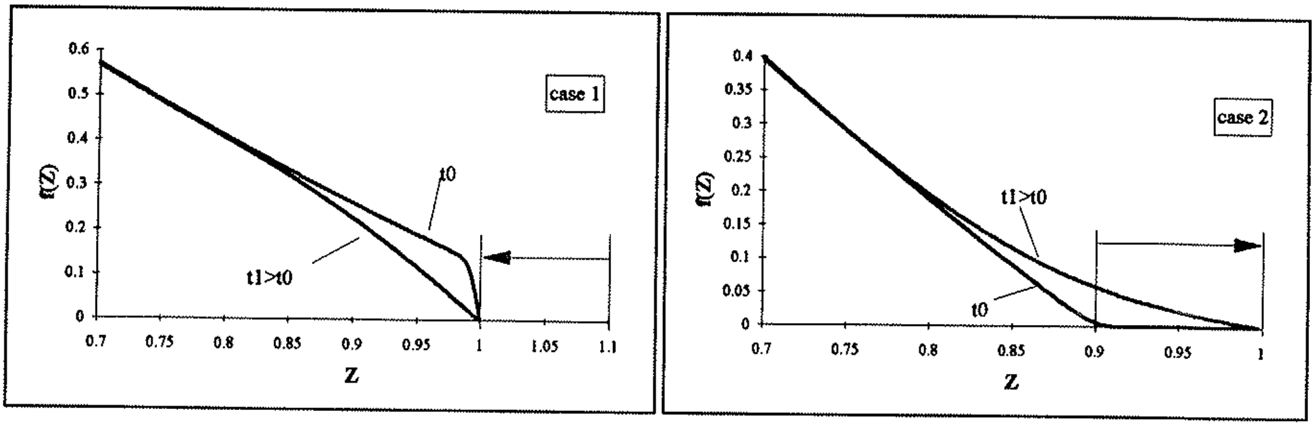

Fig. 3: Distribution functions at the beam edge for two fixed times after a collimator movement of $\Delta Z = 0.1$

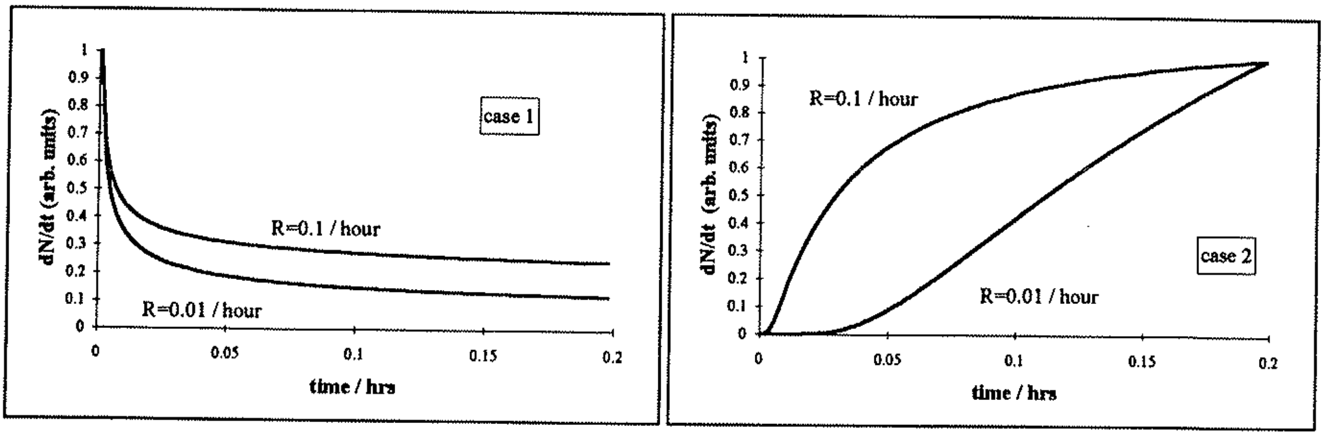

Fig. 4: Qualitative behaviour of the loss rates for two different diffusion constants

Finally we derive a fit function for the counting rates from the equations (5) and (8).

$$\dot{N}(t) \;=\; a_0 + a_1 \sum_n c_n^{(1,2)} \lambda_n J_1(\lambda_n) e^{-\frac{\lambda_n^2 R}{4}(t-t_0)} \qquad (10)$$

In this fit function we have introduced 3 fit variables a_0, a_1 and R. An offset a_0 is necessary because one has to take into account an additive rate from inelastic beam gas events and activation of the collimator jaws. The factor a_1 is a constant of proportionality between the flux of protons and the measured counting rate. One should note the dependence of (10) on the distance of the collimator movement which is hidden in the coefficients c_n. The diffusion coefficient (see eq. (3)) is then obtained from the fit variable R:

$$D(W{=}W_c) \;=\; R W_c^2 \;=\; R\, x_c^4 / \beta_c^2 .$$

In theoretical treatments of diffusion due to nonlinear magnetic fields often a strong dependence of the diffusion constant on the transverse emittance is found. For the HERA proton ring at injection energy a dependence $D(W) \propto W^{15}$ is predicted in [3] for the chaotic regions of phase space. Although our model is based on quite a different dependence, namely $D \propto W$, it may be applicable at injection energy for a local determination of $D(W \approx W_c)$ because the collimator is retracted only by a small distance. Immediately after the collimator retraction only those particles which feel the local diffusion coefficient $D(W_c)$ can contribute to the measured counting rate. The diffusion coefficient obtained from the fit calculation is therefore averaged over the range of the collimator movement.

3. Connection between Diffusion Equation and Fokker Planck Equation

The diffusion equation (3) is a special case of the Fokker Planck equation:

$$\frac{\partial}{\partial t} f(W,t) \;=\; -\frac{\partial}{\partial W}\left[A(W) f(W,t)\right] + \frac{1}{2}\frac{\partial^2}{\partial W^2}\left[B(W) f(W,t)\right] \qquad (11)$$

The coefficients A and B are known to have the following meanings [4]:

$$A(W) \;=\; \frac{\langle \Delta W \rangle}{\Delta t}, \qquad B(W) \;=\; \frac{\langle \Delta W^2 \rangle}{\Delta t}.$$

If the relation $$A(W) \;=\; \frac{1}{2}\frac{\partial}{\partial W} B(W) \qquad (12)$$

is valid, (11) can be written as a diffusion equation with $D(W) = B(W)/2$. Indeed one can show that (12) is generally valid for Hamiltonian systems (see [5] and [3], Appendix F). Therefore we find that we can calculate the coefficients of the Fokker Planck Equation from our fit to the background data in the following way:

$$\frac{\langle \Delta W \rangle}{\Delta t} \;=\; R W_c \;=\; R x_c^2 / \beta_c , \qquad \frac{\langle \Delta W^2 \rangle}{\Delta t} \;=\; 2 R W_c^2 \;=\; 2 R x_c^4 / \beta_c^2 . \qquad (13)$$

If we are interested in the solution of the Fokker Planck equation (11) for short times Δt it is possible to treat the coefficients $A(W)$ and $B(W)$ as constants and solve the following equation:

$$\frac{\partial}{\partial t} f \;=\; -A \frac{\partial}{\partial W} f \;+\; B \frac{\partial^2}{\partial W^2} f . \qquad (14)$$

To find a statistical description of the motion of a single particle we assume further a sharply defined initial position and therefore $f(W, t{=}t_0) \;=\; \delta(W - W_0)$. The solution is then found to be:

$$f(W, \Delta t) \;=\; \frac{1}{\sqrt{2\pi B \Delta t}}\, e^{-\frac{(W - W_0 - A\Delta t)^2}{2B\Delta t}} . \qquad (15)$$

Equation (15) describes a Gaussian distribution with a mean value $A\Delta t$ and a width $\sqrt{B\Delta t}$. For short times we can therefore simulate the particle drift in W with the following equation:

$$W(t+\Delta t) \;=\; W(t) + A\,\Delta t + \sqrt{B\Delta t}\;\xi, \tag{16}$$

where ξ is a Gaussian distributed term with unit standard deviation. Equation (16) is closely related to the concept of stochastic differential equations [4]. If we apply (16) to a one turn transfer we can compute the relative change of the Courant Snyder Invariant per turn:

$$\left.\frac{\Delta W}{W}\right|_{W\approx W_c} \;=\; R\,\Delta t \;+\; \sqrt{2R\Delta t}\;\xi\,. \tag{17}$$

Here R is the fit value from equation (10), corresponding to the transverse position $W = W_c$, and Δt can be taken as the revolution time. Equation (17) permits to calculate an estimate of the relative change of the one particle emittance per turn due to stochastic processes. Under normal operating conditions in HERA, R is of the order $5\cdot 10^{-6}\ s^{-1}$. So it turns out that the systematic drift of the particle, which is represented by the first term in (17), is of the order 10^{-10}. The random diffusion term has a rms-width of $\sim 10^{-5}$ and is therefore much larger. The first term becomes only important if one considers longer times (in the order of hours) because it scales with t, whereas the second term scales with $t^{1/2}$.
At this point we have to remark that the random term ξ in equation (17) represents white noise. In other words the equation cannot describe correlations of step sizes in the emittance change in successive turns. In a real machine one has to consider effects like tune drift and resonance crossing of the tunes and then the step sizes of consecutive turns are correlated. Therefore the model can only give a rough approximation of the microscopic step size which is nevertheless based on measured data. The knowledge of this one turn step size can be used for estimations of impact parameters of protons hitting a collimator jaw.

4. Fit Calculations and Tests of the Model

The fit calculations were carried out on a 486 PC with a C++ program, using the Levenberg-Marquardt algorithm for nonlinear parameter fitting from the Mathpak '87 library [8].
As a first consistency check we perform the measurement of "moving in" and "moving out" under the same machine conditions. If the model is applicable the fit-curves of both cases should result in the same value of R. Fig. 5 shows the fits of both measurements which were made under normal operating conditions (820 GeV protons colliding with 26.6 GeV electrons). The fitted values of $9.1\cdot 10^{-6}\ s^{-1}$ resp. $8.6\cdot 10^{-6}\ s^{-1}$ for R indicate a good agreement for both cases. Because of the smaller time dependence fits of type 2 are better suited for the determination of the diffusion coefficient.

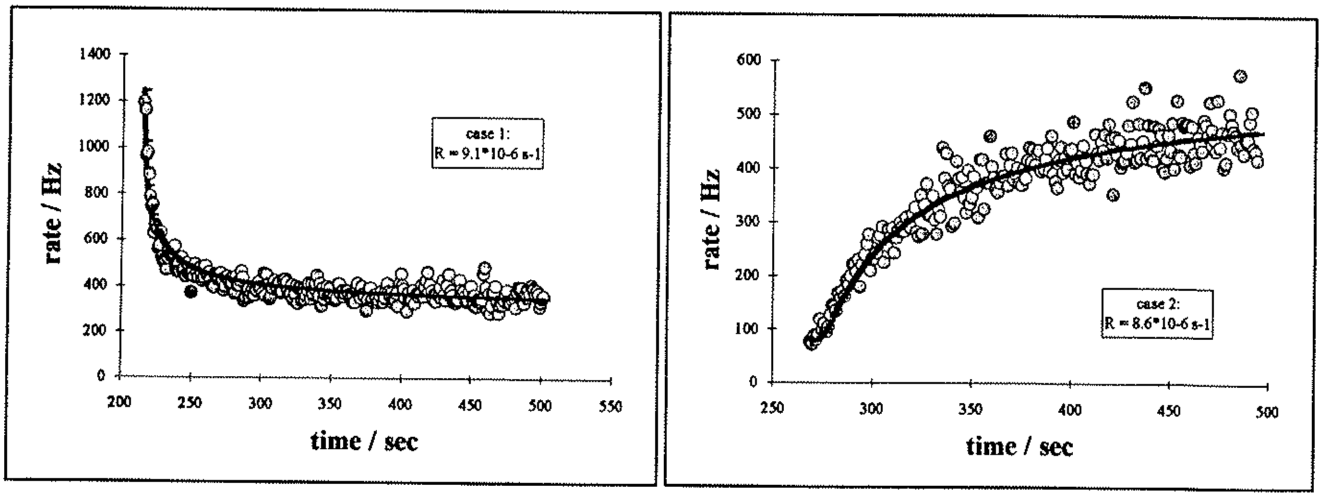

Fig. 5: Comparison of fits to the cases 1 and 2 at 820 GeV with beam-beam interaction, $\Delta x_c = 50\mu m$

As a second check we can vary the length of the collimator retraction Δx_c and compare the fit results. Again we should get the same R value from different measurements. For short times the diffusion term in the Fokker Planck Equation is dominant. Therefore we expect the involved typical times to scale with Δx_c^2. Fig. 6 shows the result of an appropriate experiment which was made at the injection energy of 40 GeV. The comparison of the fitted R - values shows a good agreement ($54 \cdot 10^{-6}\ s^{-1}$ resp. $58 \cdot 10^{-6}\ s^{-1}$).

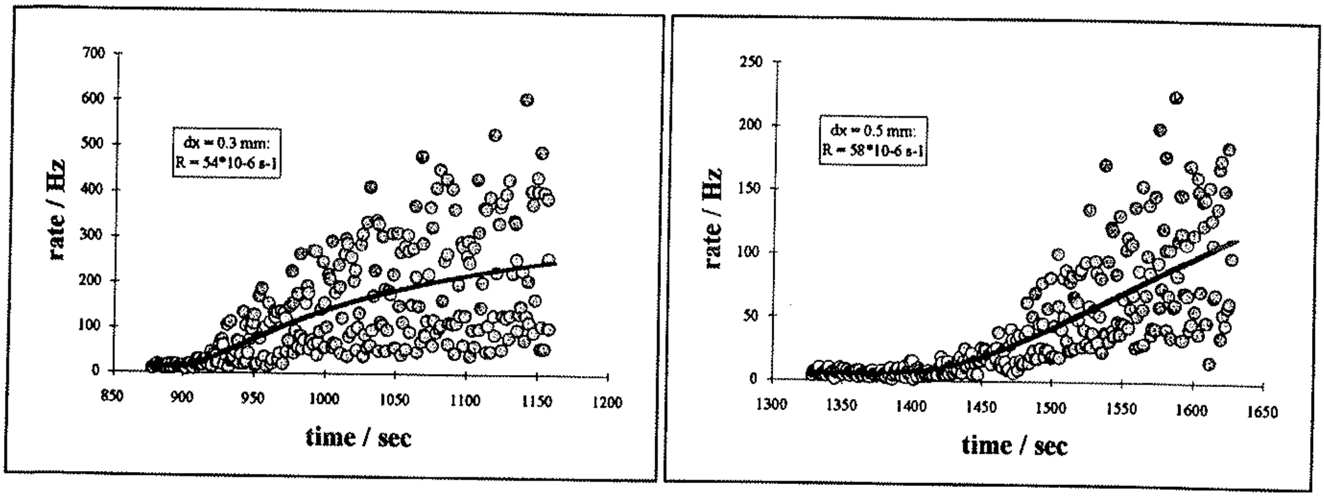

Fig. 6: Fits with two different retraction distances of Δx_c = 0.3 mm and 0.5 mm

Contrary to the other examples this measurement had been performed during daytime and the large fluctuations in the rates may be due to adiabatic beam position oscillations, induced by power supply ripples or ground motion. We often observed a much smaller spread of the rates in night shifts when the effects of ground motion are known to be weaker (see [6] and [7] for measurements of ground motion).

5. RESULTS

We performed diffusion measurements under 3 different operating conditions, but only in the horizontal plane:
case 1: E_p = 40 GeV, no beam-beam interaction
case 2: E_p = 820 GeV, no beam-beam interaction
case 3: E_p = 820 GeV, with beam-beam interaction
All the measurements have been done simultaneously to luminosity runs or in breaks caused by failures in the injectors or the electron machine. Unfortunately we have up to now only a small number of data points, in case 2 only one, and therefore a bad statistics. It turned out that the diffusion coefficient (and also the beam lifetime) varies in the three cases over more than two orders of magnitude. Fig. 7 shows fits of the cases 1 and 2. One should note the large difference in the collimator retraction length, which is 500μm in the 40 GeV case and 50μm in the 820 GeV case.

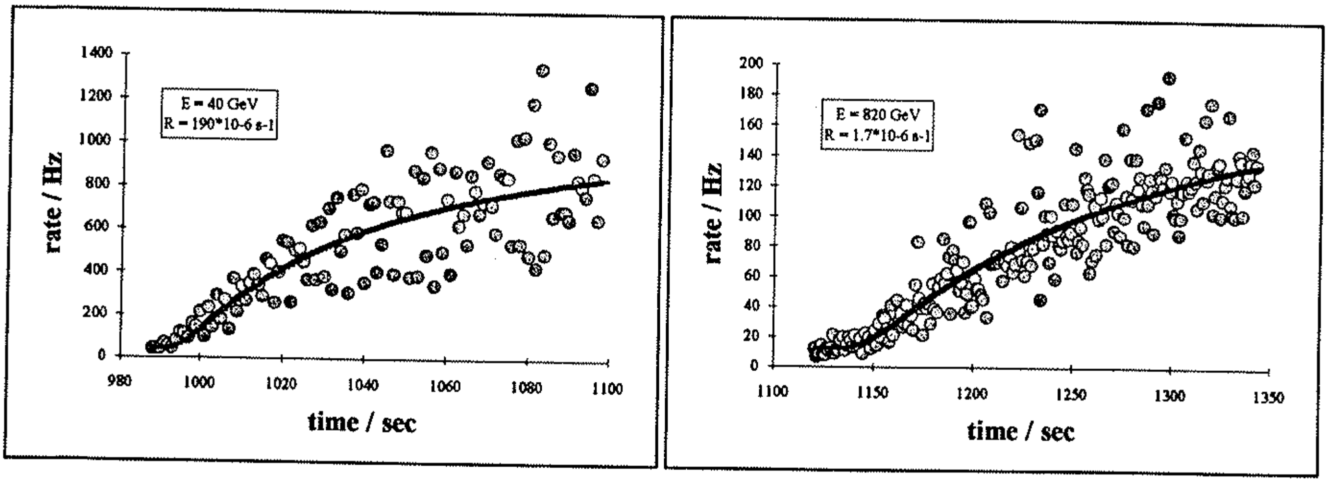

Fig. 7: Examples of collimator retraction experiments at 40 GeV and 820 GeV

An example of case 3 has already been shown in Fig. 5. In table 1 the complete results of the measurements are listed. The beam-σ value, given in table 1 was measured with a residual gas monitor and has been rescaled to the optics at the collimator position. The beam centre position has been determined by detecting the beam edges with collimator jaws from both sides of the beam via counting rates of the shower detectors. The collimator mechanics has an uncertainty in the jaw position of $\delta x_c < 5\ \mu m$. The β - function at the collimator location has a value of $70.0\ m$.

date of the measurement	E_p [GeV]	beam-beam?	I_p [mA]	τ_p (measured) [h]	τ_∞ (calc. from R) [h]	σ_x (at coll.) [mm]	x_k [mm],[σ]	R [s^{-1}]	$\langle\Delta\varepsilon\rangle/\Delta t$ [mm mrad s^{-1}]	$\langle\Delta\varepsilon^2\rangle/\Delta t$ [(mm mrad)^2 s^{-1}]
13.10.92	40	no	1.0	0.5...3	1.0	2.6	7.9, 3.0	$190\cdot10^{-6}$	$170\cdot10^{-6}$	$300\cdot10^{-6}$
11.11.92	40	no	1.0	??	3.5	2.8	7.4, 2.7	$55\cdot10^{-6}$	$43\cdot10^{-6}$	$67\cdot10^{-6}$
29.7.92	820	no	0.7	>100	115	1.6	4.7, 2.9	$1.7\cdot10^{-6}$	$0.5\cdot10^{-6}$	$0.34\cdot10^{-6}$
9.10.92	820	yes	1.5	~30	22	0.9	4.1, 4.3	$8.6\cdot10^{-6}$	$2.0\cdot10^{-6}$	$1.0\cdot10^{-6}$
15.10.92	820	yes	1.3	~50	70	0.7	3.8, 5.3	$2.7\cdot10^{-6}$	$0.6\cdot10^{-6}$	$0.23\cdot10^{-6}$
15.10.92	820	yes	1.0	~30	23	0.8	3.8, 4.8	$8.3\cdot10^{-6}$	$1.7\cdot10^{-6}$	$0.7\cdot10^{-6}$

Table 1: results of diffusion measurements at HERA-p in 1992

The smallest diffusion coefficient was measured at top energy without beam-beam interaction. Unfortunately it was not possible to measure the beam lifetime precisely in the short time of the experiment, and so we can give only a lower limit of τ_p. Under luminosity conditions the lifetime is mainly determined by the nonlinear fields of the beam-beam interaction. The largest diffusion rates have been observed at the injection energy of 40 GeV. This could be due to the stronger nonlinearities in the superconducting magnets, caused by persistent current effects or to the larger effect of residual gas scattering.
The asymptotic beam lifetimes, predicted by the solution of the diffusion equation and calculated from the measured diffusion rates (see equation (7)) are also given in table 1. A comparison with the measured lifetimes shows that these numbers are not completely different. This may be a hint that the diffusion coefficient has a weak emittance-dependence in the range of the measurements.
For the 1993-run of HERA we intend to perform more measurements and get an improved statistic. For a better understanding of the diffusion mechanism in the machine it would be interesting to measure the transverse emittance dependence of the diffusion coefficient.

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[8] Mathpak '87 Reference Manual, © 1989 by Precision Plus Software

APPENDIX A

Calculation of the coefficients c_n:

First we have to choose appropriate initial distributions for both cases. The distributions are taken at a time immediately *after* a collimator movement from an initial position Z_0 to a final position $Z=1$. We assume that all particles with amplitudes larger than the collimator-defined acceptance will be scraped away in a short time < 1 sec.

case 1 ("moving in"):

$$f_0^{(1)}(Z) = \frac{\lambda_0}{2\sqrt{Z_0}\,J_1(\lambda_0 / Z_0)}\, J_0\left(\lambda_0 \sqrt{Z/Z_0}\right)\, \Theta(1-Z)$$

case 2 ("moving out"):

$$f_0^{(2)}(Z) = \frac{\lambda_0}{2Z_0 J_1(\lambda_0)}\, J_0\left(\lambda_0 \sqrt{Z/Z_0}\right)\, \Theta(Z_0 - Z)$$

The Θ - Function has the following meaning:

$$\Theta(x) = \begin{cases} 0 \; for \; x<0 \\ 1 \; for \; x>0 \end{cases}.$$

The initial distributions are normalised:

$$\int_0^1 f_0^{(1,2)}(Z)\,dZ = 1.$$

The coefficients c_n can now be obtained by inserting the initial distributions in (6) and solving the integral. The results are given in (9). For the computation of the integrals we used the following expressions:

$$\int_0^k J_0\left(a\sqrt{x}\right)dx = \frac{2}{a}\sqrt{k}\, J_1\left(a\sqrt{k}\right)$$

$$\int_0^k J_0\left(a\sqrt{x}\right) J_0\left(b\sqrt{x}\right)dx = 2\sqrt{k}\,\frac{a\,J_0\left(b\sqrt{k}\right)J_1\left(a\sqrt{k}\right) - b\,J_0\left(a\sqrt{k}\right)J_1\left(b\sqrt{k}\right)}{a^2 - b^2}.$$