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Wake Field Effects in the Final Focus Quadrupoles of a Linear Collider

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1 Introduction

The parameter list of Ref. (1) for a 500 GeV S-Band Linear Collider assumes a spot size of $169 \times 5.48 nm^2$ in order to achieve a luminosity of $2.4 \times 10^{33} cm^{-2} sec^{-1}$ with 7×10^9 particles per bunch at a repetition rate of 50 Hz. Such a small spot size with a design emittances $\varepsilon_x = 9.52 \times 10^{-12} m$ and $\varepsilon_y = 10^{-13} m$ requires careful examination of the wake field effects in the quadrupoles of the final focus system with betatron function values $\beta_x = 3mm$ and $\beta_y = 0.3mm$ at the interaction point.

To achieve these β values at the IP a strength of $k_\beta \sim 1\, m^{-1}$ is necessary for the last quadrupoles (length: $L \sim 1\, m$). This corresponds to a gradient of $G \sim 900\ T/m$ for an electron energy of E=250 GeV. Assuming permanent magnet quadrupoles with pole-tip field of the order of 1.2 T, the aperture required is very small (of the order $1mm$).

In this report the geometrical and resistive wake field effects in a quadrupole with aperture $1\, mm$ and a rms bunch length of $\sigma = 0.2\ mm$ are discussed. The model of the quadrupole for the wake fields calculation is a single collimator of circular cross section and long middle region.

The TBCI code [2] was used for the calculation for such a geometry. Direct use of this code for a real quadrupole with length $1m$ and such a short bunch length requires too many mesh points (of the order of a few million). However, from a physics point of view it is clear that the interaction of diffracted fields with the particles is dominated by the transition part of the radiation when a bunch enters a narrowing pipe and exits into a broadening pipe. We must also distinguish these two cases from a particles dynamics point of view due to the different properties of the external focusing field inside and outside the quadrupole.

The longitudinal and transverse wake field effects due to resistive walls will be compared and discussed.

2 The loss factor and longitudinal wake potential for a step

Consider an ultrarelativistic Gaussian bunch moving along the z axis of a circular waveguide with an abrupt change in the cross section from radius b to radius a and from radius a back to radius b (Fig.1). Wake fields will be generated when a bunch enters a narrowing pipe ("step-in") and a bunch exits into a broadening pipe ("step-out"). What we really want to know: the bunch travel distance z_{tr} at which the interaction of the particles within the bunch with its environment

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dominate and the magnitude of the wake potentials for given geometry ($b = 5mm, a = 1mm$) and bunch parameters (rms length $\sigma = 0.2mm$, total charge $Q = 1.14nC$).

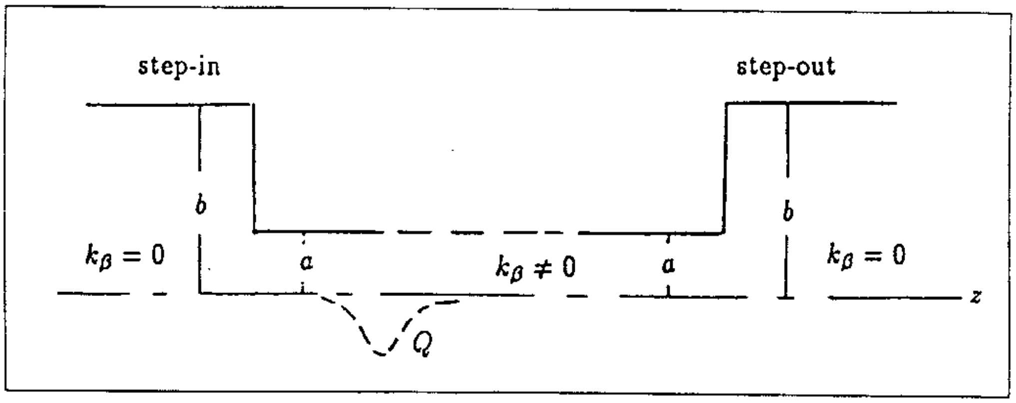

Figure 1: Geometry of the quadrupole for TBCI calculation ($b = 5mm, a = 1mm$).

The properties of the loss factor and impedance for a collimator and a step are discussed in Ref. 3. We start by a particle moving in free space.

For an ultrarelativistic particle moving in free space at a velocity v close to the speed of light c a good aproximation for the nonzero components of the field in the region $r < \gamma/k$ (γ-Lorentz factor, $k = \omega/c$) is

$$E_{\omega r} = \frac{2q}{cr} \exp(ikz)$$

$$H_{\omega \theta} = \frac{2q}{cr} \exp(ikz)$$

The field propagates synchronously with the charge. In the region $kr > \beta\gamma$ the field is exponentially small. The total field energy of the charge is given by:

$$W = 2\pi \int_{-\infty}^{\infty} dt \int_0^{\infty} rdr \frac{c}{4\pi}(E \times H)_z = 2\pi c \int_{r_{min}}^{\infty} rdr \int_0^{c\gamma/r} d\omega \mid E_{\omega r} \mid^2$$

For a rigid bunch of N particles the contribution of all particles

$$\mid E_{\omega r} \mid^2 = (\frac{2q}{cr})^2 \sum_{i,j} \exp(ik(z_i - z_j))$$

can be found by replacing the sum by the integral over the normalized distribution function $\rho(z)$

$$\mid E_{\omega r} \mid^2 = \mid \frac{2qN^2}{cr} \int dz_1 dz_2 \rho(z_1)\rho(z_2) \exp(ik(z_1 - z_2))$$

For a gaussian bunch with the rms length σ the energy per particle is

$$W = \frac{Nq^2}{\sqrt{\pi}\sigma} \ln \frac{\gamma\sigma}{r_{min}}$$

For a particle moving in a circular waveguide of radius a the energy of the synchronous component of the field moving with the particle for $\gamma\sigma > a$ is

$$W(a) = \frac{Nq^2}{\sqrt{\pi}\sigma} \ln \frac{a}{r_{min}}$$

In the simple example of an abrupt change in the cross section of a circular wavequide from radius a to radius b the change of the synchronous component of the field moving with the particle of a gaussian bunch is

$$\Delta W = \frac{Nq^2}{\sqrt{\pi}\sigma} \ln \frac{a}{b}$$

The same ammount of energy $\Delta\mathcal{E}_{rad}$ will be radiated. The total energy loss in "step-in" and "step-out" cases is

$$k_{in} = \Delta\mathcal{E}_{rad} - \Delta W + \delta w_{in}$$

$$k_{out} = \Delta\mathcal{E}_{rad} + \Delta W + \delta w_{out}$$

where $\delta w_{in}(\delta w_{out})$ is the contribution to the energy gain (or lost) by the bunch due to the interaction of the particles with the radiated fields. Figure 2 shows the electric field lines for three successive time steps calculated by the computer code TBCI.

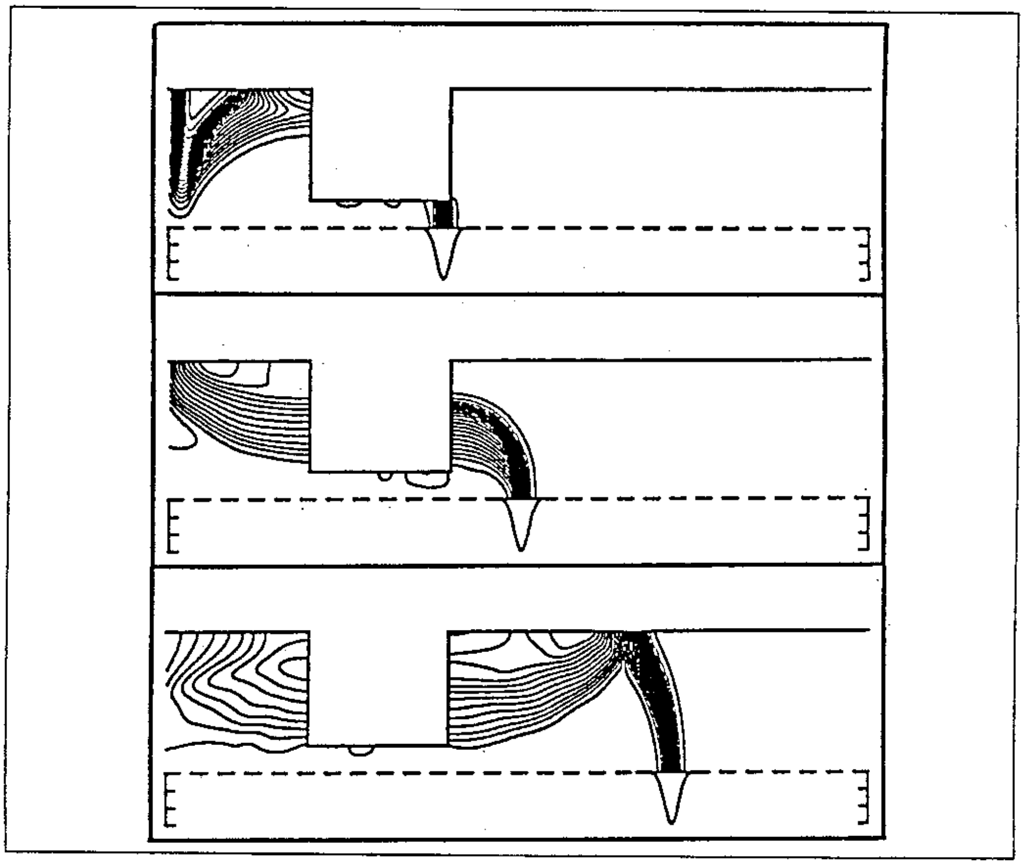

Figure 2: Wake fields generated by a Gaussian bunch passing an obstacle in the beam pipe.

Consider a "step-in" case , when particles enter a narrowing pipe (Fig.1a). For this case the radiated energy comes from the excess of the particle field energy in a wide pipe. The distance that the head charge must travel before the induced radiation reaches the axis at a distance s behind the head particle for $a/\gamma << s < a$ is

$$z_q \approx \frac{a^2 + s^2}{2s}$$

If s is equal to the length of the bunch S_b (usually for gaussian distribution $S_b = 4\sigma$) then for a distance z_q we can assume that $\delta w_{in} = 0$ and $k_{in} = 0$. In this region a total energy of the electromagnetic fields is equal to the sum of the radiated energy and the energy of EM fields moving synchronously with the particle

$$U_{tot} = \Delta\mathcal{E}_{rad} + W(b)$$

In the region $z > z_q$ particles of the bunch interact with secondary fields scattered from the step edge. Figure 3a shows the normalized longitudinal wake potential within the bunch for three succesive positions of the bunch in the quadrupole.

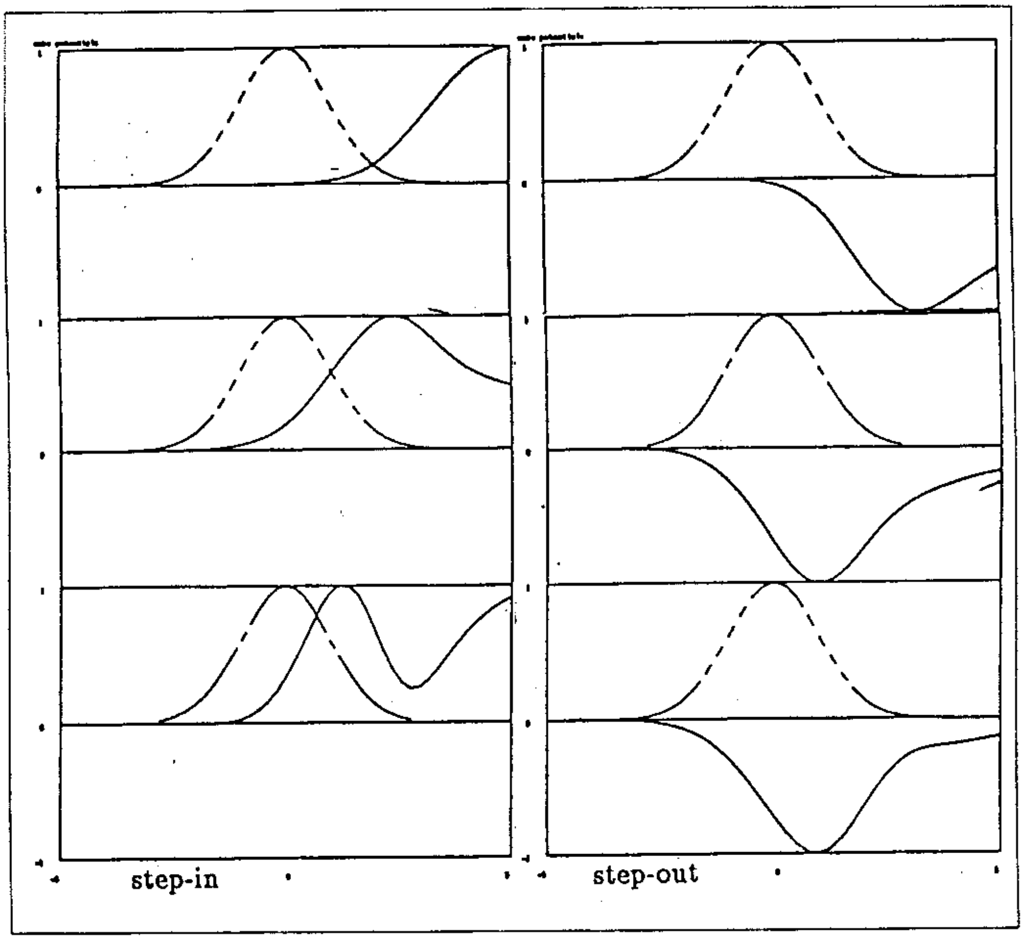

Figure 3: Longitudinal wake potential for bunch position $z_b = 1mm, 3mm, 8mm$ ("step-in") and $z_b = 1mm, 10mm, 20mm$ ("step-out").

For a bunch travel distance $z_q = a = 1mm$ it corresponds to the above mention situation, when particles within the bunch $(-2\sigma < s < 2\sigma)$ do not see any longitudinal potential although a test charge following at a distance $s > a$ behind the bunch will seen comparatively large accelerating or retarding potential. Note, that positive values of the longitudinal wake potential within the bunch for bunch travel distance $z > z_q$ corresponds to an energy gain for the particles, this means that part of the synchronous field energy is transformed by reflected fields into the kinetic energy of the particles.

In the "step-out" case, when particles exit the quadrupole, the field configuration is perturbed by the diffracted fields which have to restore new boundary condition. The restoration of the particle field energy take place. The energy for this is taken from the particle energy. Figure 3b shows the longitudinal wake potential for three positions of the bunch in a wide tube. It is negative within the bunch and corresponds to the energy lost by the particles. Hence, the loss factor is positive and comparatively large.

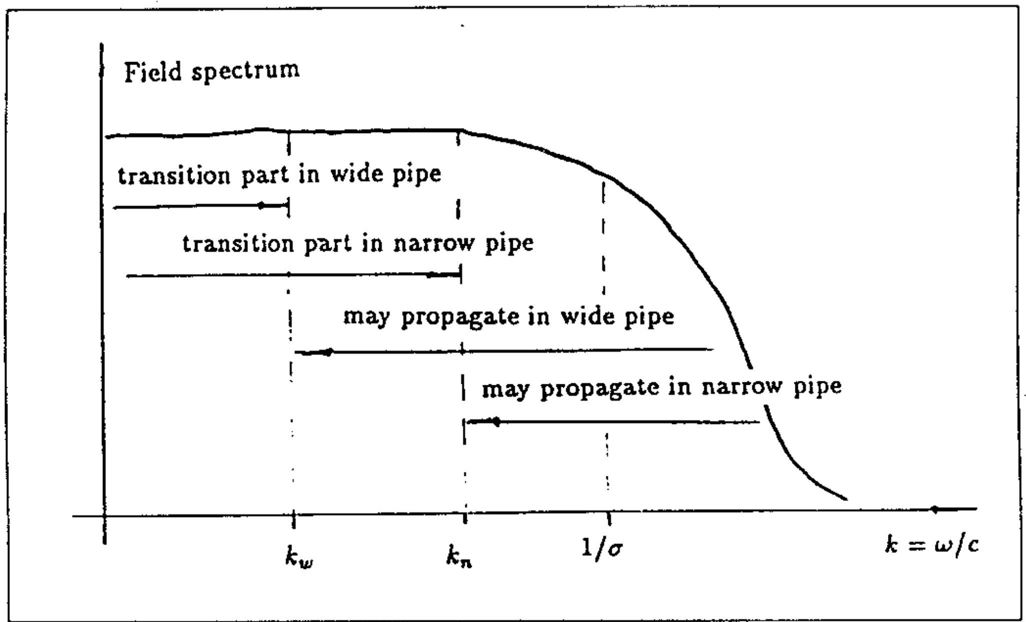

Figure 4: The properties of the field spectrum in a collimator (k_w - cut-off frequency of the wide tube, k_n - cut-off frequency of the narrow pipe).

Since the "effective size" of the bunch at the edge is $\sim (\sigma + a/\gamma)$ the spectrum of the diffracted fields extends for $\sigma >> a/\gamma$ up to frequencies of the order of c/σ as is shown schematically in Fig.4. Below the cut-off frequencies of the pipes the fields are exponentially damped and define the transition zone of the radiation. The high frequency part of the radiation above the cut-off frequency of the pipe may propagate in the direction of the particle motion with phase velocity higher than the velocity of light. Hence, the interaction of this part of the field spectrum with the particle is on average small (zero). The main part of the radiated fields (low frequency part of the spectrum) propagate in the opposite direction to the particle motion for the "step-in" case. Hence, the interaction of the field with the particles is small. For the "step-out" case these low frequency parts propagate in the same direction as the particle motion. Hence, there is a large radiation transition zone.

The dependence of the maximum longitudinal wake potential within the bunch for "step-in" and "step-out" cases is presented in Fig.5 where the above mentioned typical radiation forming zone is shown.

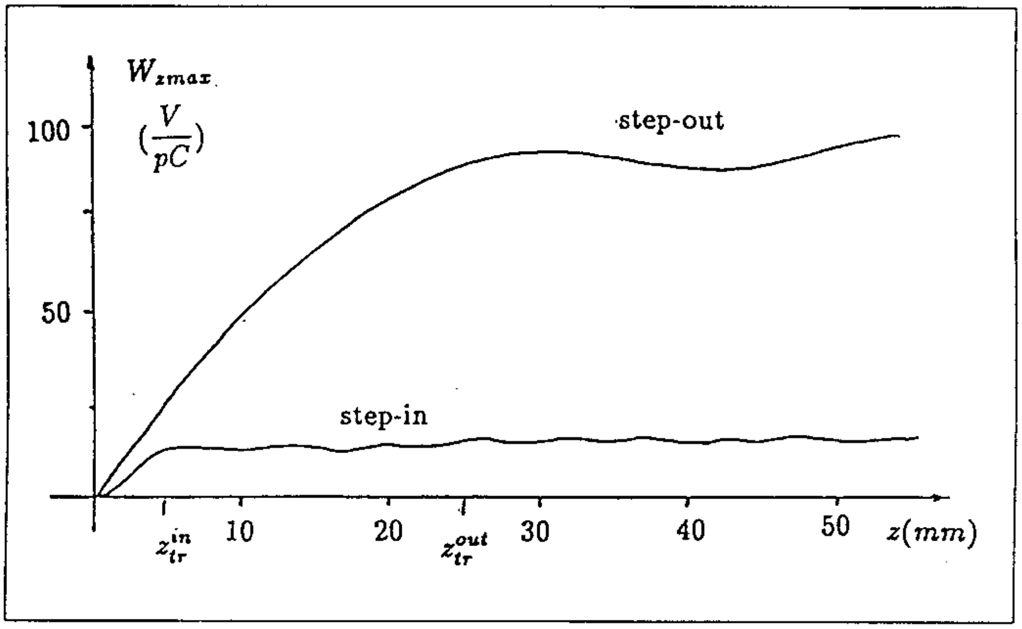

Figure 5: The dependence of the maximum longitudinal wake potential from bunch position for "step-in" and "step-out" cases.

For our geometry the main part of the energy spread of the bunch due to wakefields dominates at a bunch travel distance of

$$z_{tr}^{in} \sim 5a = 5mm$$

$$z_{tr}^{out} \sim \frac{b}{a} z_{tr}^{in} \sim 25mm$$

Note, that the maximum of the potential for the "step-out" case is an order of magnitude larger than for the "step-in" case.
The total longitudinal wake potential after the bunch exits the quadrupole is shown at Fig.6. At the maximum value of the potential, at $s = 2\sigma$ within the bunch, with total charge $Q = 1.14nC(\sim 7 \times 10^9 electrons)$ the maximum energy deviation is of an order of 90 keV.

3 Transverse wake potential

The transverse wake potential $W_\perp$is defined as the transverse momentum kick experienced by the test charge following at a distance s behind the exiting charge divided by the total charge Q of the bunch.

$$W_\perp(s, z_d) = \frac{1}{eQ}\int_0^{z_d} F_\perp(z, t = \frac{z+s}{c})dz = \frac{1}{Q}\int_0^{z_d} dz[E_\perp + (v \times B)_\perp]_{t=(z+s)/c}$$

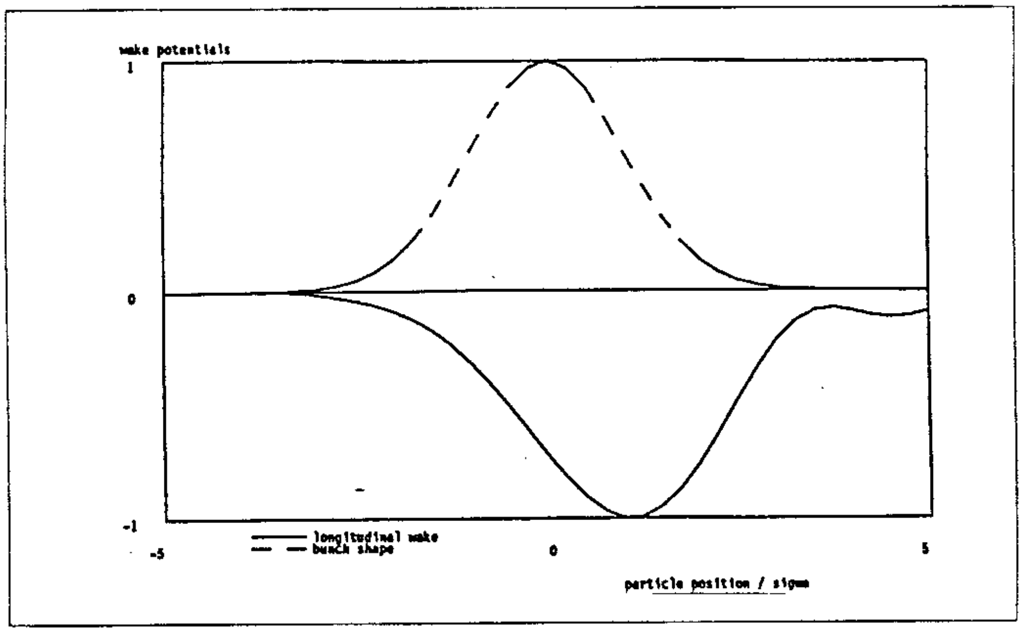

Figure 6: The normalized longitudinal wake potential after the bunch has left the quadrupole

where $\vec{F}_{\perp}(s,z)$ is the transverse Lorentz force acting on the test particle which follows at a distance s behind the bunch and at a position z along the structure with length z_d. Then for the Lorentz force we have

$$\boldsymbol{F}_{\perp}(s,z) = eQ\frac{d\boldsymbol{W}_{\perp}}{dz_d}\Big|_{z_d=z}$$

In a cylindrically symmetric structure all the modes depend on the azimuthal angle θ as $\exp(im\theta)$, where m is an integer. We arrange the axes so that the transverse position of the exciting charge is $r = r_0, \theta = 0$ (on the x-axis). The test charge also moves in the z direction but at transverse position (r,θ) and at a distance s behind the exciting charge. It can be shown that the m-pole component of the wakes experienced by the test particle can be written as [4]

$$\boldsymbol{W}_{\perp} = m\left(\frac{r_0}{a}\right)^m\left(\frac{r}{a}\right)^{m-1}(\boldsymbol{r}\cos m\theta - \boldsymbol{\theta}\sin m\theta)w_m(s,z_d)$$

where $w_m(s,z_d)$ is defined by the boundary condition and bunch distribution. Normally bunches remain near the axis and the factors $(r/a),(r_0/a)$ can be considered as small. Then the transverse wakefield is dominated by the dipole ($m = 1$) and quadrupole ($m = 2$) modes. Note that the transverse dipole wake is in the x direction and is independent of the test charge's transverse position, while it depends on the exciting charge as the first power of its offset. It acts on the test particle like a dipole magnet with force

$$F_x = eQ\frac{r_0}{a}\frac{d}{dz}w_1(s,z)$$

$$F_y = 0$$

This force is in the same direction as the transverse offset of the exciting charge. The quadrupole mode acts like a quadrupole magnet with a force of

$$f_x = eQ\left(\frac{r_0}{a}\right)^2 \frac{x}{a}\frac{dw_2(z)}{dz}$$

$$f_y = -eQ\left(\frac{r_0}{a}\right)^2 \frac{y}{a}\frac{dw_2(z)}{dz}$$

For $w_2'(z) > 0$ it produces a focusing field in the vertical plane and defocusing in the horizontal.

Usually the offset of the exciting charge in a linear accelerator is less than 0.1 of the aperture of the quadrupole. For our example all the transverse wakes are calculated for a gaussian bunch with offset $r_0 = 0.1mm$ and the potentials are integrated at a transverse position of $r = 0.1mm$.

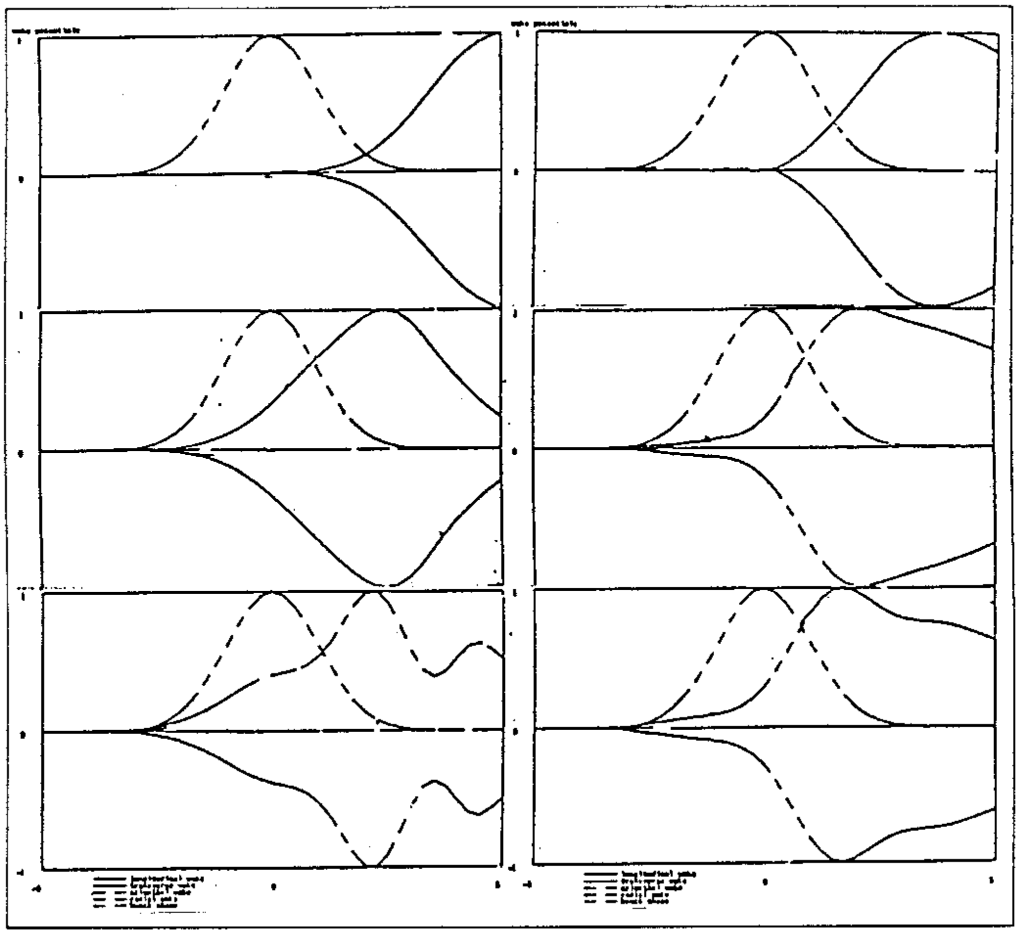

Figure 7: Transverse dipole mode wake potential for a bunch travel distance of $z = 1mm, 5mm, 10mm$ ("step-in") and $z = 1mm, 10mm, 20mm$ ("step-out").

Figure 7 shows the normalized dipole mode transverse wake potential inside the bunch for three succesive positions of the bunch for the "step-in" and "step-out"cases . Letters "a" and "r" mean azimuthal and radial transverse wake potentials respectively. In both cases it produces kicks in the same direction as the bunch offset.

Figure 8 shows the dependence of the maximum dipole mode and its value at position $s = 2\sigma$ within the bunch on the bunch position for "step-in" and "step-out" cases.

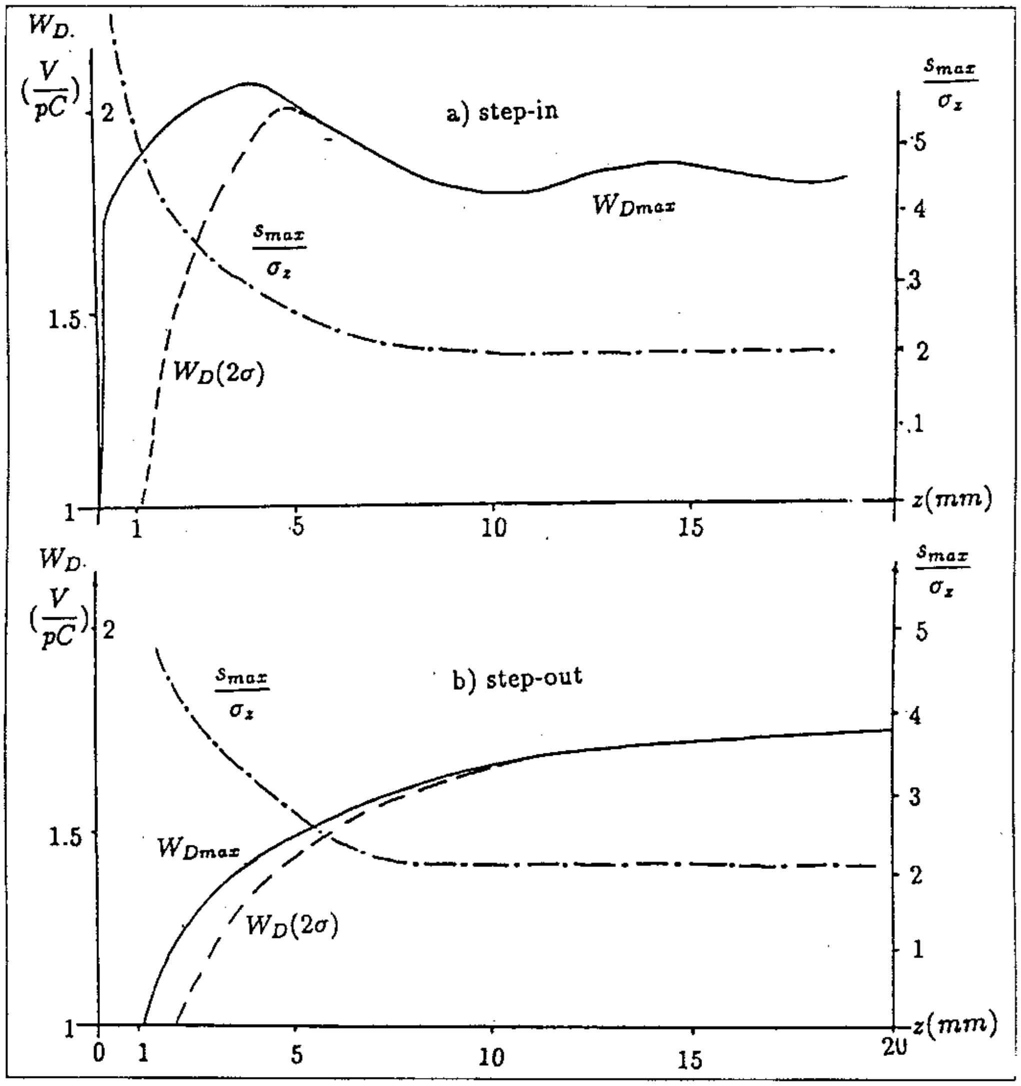

Figure 8: The dependence of the maximum dipole mode wake potential W_{Dmax} and its value at position $s = 2\sigma$ within the bunch $W_D(2\sigma)$ on the bunch position for "step-in" and "step-out" cases; s_{max} is the position of the particle within the bunch which seen the maximum transverse wake potential.

The situation is similar to the longitudinal wake case. Below the cut-off frequency of the pipes radiated fields defined a transition zone while above the cut-off frequency fields may propagate in the same direction as the bunch moving with a phase velocity higher than the velocity of light. This part of the wake field produces oscilating dipole fields in the bunch frame system. The maximum of the transverse dipole mode wake potential is of the same order for "step-in" and "step-out" cases. We will approximate the dependence of the transverse wake potential at position $s = 2\sigma$ within the bunch by the linear function

$$W_\perp(2\sigma, z) = W_d \frac{z}{z_{tr}} \qquad for \quad 0 \leq z \leq z_{tr}$$

where $W_d = W_\perp(2\sigma, z_{tr})$.

For the bunch travel distance $z > z_{tr}$ we will use a single mode approximation with typical values of the amplitude W_0 and wavelength λ from the numerical results for the transverse wake potential. In this assumption

$$W_\perp(2\sigma, z) = W_d - W_0 \sin\frac{2\pi}{\lambda}(z - z_{tr}) \qquad for \quad z \geq z_{tr}$$

In Table 1 the characteristic values of the transverse wake potential for a dipole mode are presented

	z_{tr} (mm)	W_{dmax} (V/pC)	W_d (V/pC)	W_0 (V/pC)	λ (mm)	W_{Qmax} (V/nC)
step-in	5	2.1	1.8	0.05	10	17.7
step-out	20	1.8	1.7	0.1	50	10.8

The normalized transverse quadrupole mode wake potential is presented in Fig. 9. The maximum of this quadrupole wake potential is $W_{Qmax} = 17.7V/nC$ and $WQmax = 10.8V/nC$ for the "step-in" and "step-out" cases respectively. Note, that the quadrupole wake is two order of magnitude less than dipole mode wake potential. The maximum of this quadrupole field is seen by the particles at $s \sim \sigma$ within the bunch.

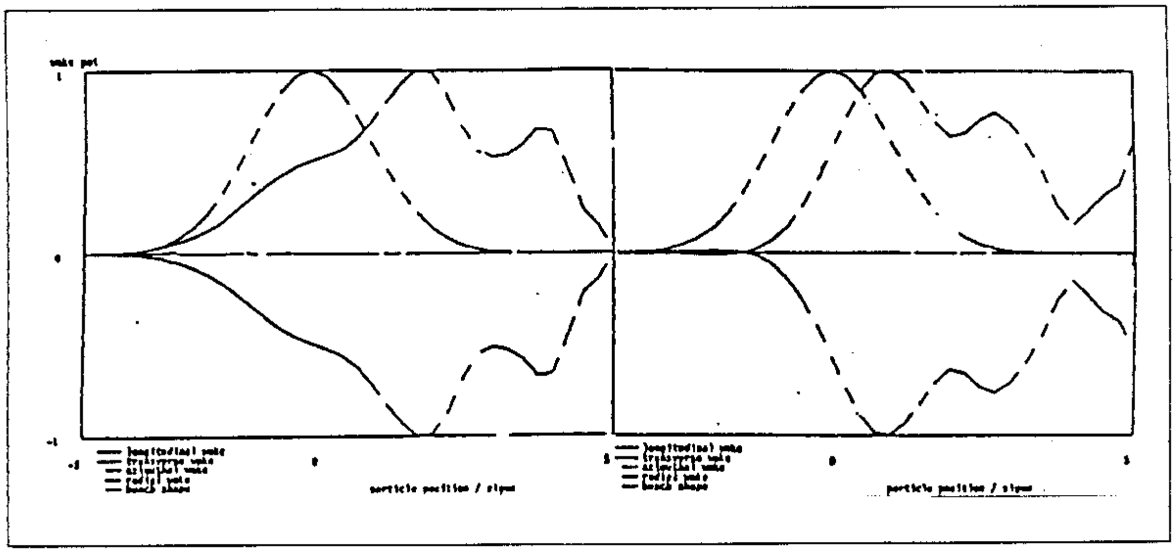

Figure 9: The normalized transverse quadrupole mode wake potential for a bunch travel distance of $z = 1mm$ (""step-in") and $z = 5mm$ ("step-out")

4 The transverse dynamic of the single bunch.

In a continuous distribution of charge, each particle is affected by the transverse wakes from all other particles in the bunch which are ahead of the particle. We start by the horizontal equation of motion for a test particle with a constant energy E_0 at longitudinal position s within the bunch, and at position z at the focusing quadrupole with strength k_β

$$\frac{d^2x}{dz^2} + k_\beta^2 x = \frac{1}{E_0}\sum_m \int_{-\infty}^{s} \rho(s')F_{mx}(s-s',z)\frac{x^m(s',z)}{r_0^m}ds'$$

where $F_{mx}(s,z)$ is the x-component of the m pole Lorentz force produced by a gaussian bunch with offset r_0 whichis connected with the transverse wake potential by

$$\int_0^z F_x(z,s)dz = eQW_x(z,s)$$

The simplest model that still contains the essentials of the situation is a two particle model for the bunch[5]. Consider a model for a gaussian bunch of total charge Q in which a head charge Q/2 is located at $s' = -2\sigma$ and a tail charge $Q/2$ is located at $s' = 2\sigma$. For a uniform external focusing field of strength k_β the head particle moves on an orbit described by $x_0 \cos k_\beta z$. For our case the radiation forming zone z_{tr} is small compared with the quadrupole length. For the "step-in" case $k_\beta z_{tr}^{in} << 1$, for "step-out"case $k_\beta = 0$. We will assume that the head particle has an offset of x_0. The transverse force acting on the tail particle due to the dipole wake is then

$$F_1 = \frac{eQx_0}{2r_0 z_{tr}}W_d(2\sigma) \qquad for \quad 0 < z < z_{tr}$$

$$F_1 = \frac{eQx_0}{2r_0}W_0\frac{2\pi}{\lambda}\cos\frac{2\pi}{\lambda}(z - z_{tr}) \qquad for \quad z > z_{tr}$$

The displacement x_1 for the tail particle obeys the transverse equation of motion

$$x_1'' + k_\beta^2 x_1 = \frac{F_1(z)}{E_0}$$

Assuming $x_1 = x_1' = 0$ at $z = 0$,the solution for $z < z_{tr}$ is

$$x_1 = \frac{C}{2}z^2$$

$$x_1' = Cz$$

where

$$C = \frac{eQx_0W_D(2\sigma)}{2r_0 z_{tr}E_0}$$

For "step-in" and "step-out" cases the quantity of C is

$$C_{in} = [3.5\times 10^{-3}m^{-2}]x_0$$

$$C_{out} = [1.2\times 10^{-3}m^{-2}]x_0$$

Then the tail particle will change its transverse position and angle at a distance z_{tr} by

$$x_{1in} = 4.4\cdot 10^{-8}x_0 \qquad x_{1out} = 3.75\cdot 10^{-7}x_0$$

$$x'_{1in} = [1.75 \cdot 10^{-5} m^{-1}]x_0 \qquad x'_{1out} = [3 \cdot 10^{-5} m^{-1}]x_0$$

The solution for the equation of motion for $z > z_{tr}$,assuming $x_1 = x'_1 = 0$ at $z = z_{tr}$, is

$$x_1 = \frac{Bk}{k_\beta^2 - k^2}[\cos k(z - z_{tr}) - \cos k_\beta(z - z_{tr})]$$

$$x'_1 = \frac{Bk}{k_\beta^2 - k^2}[k_\beta \sin k_\beta(z - z_{tr}) - k \sin k(z - z_{tr})]$$

where

$$B = \frac{eQx_0}{2r_0E_0}W_0$$

$$k = \frac{2\pi}{\lambda}$$

The maximum amplitudes of x and x', assuming k_β/k is small, are

$$\mid x_1 \mid = \frac{B}{k}$$

$$\mid x'_1 \mid = B$$

For the "step-in" and "step-out" cases we have the following estimates of these quantities

$$x_{1in} \sim 4 \cdot 10^{-9} x_0 \qquad x_{1out} \sim 10^{-7} x_0$$

$$x'_{1in} \sim [2 \cdot 10^{-6} m^{-1}]x_0 \qquad x'_{1out} \sim [10^{-5} m^{-1}]x_0$$

The quadrupole mode produces an additional focusing (defocusing) field with strength of the order of

$$\Delta k = \left(\frac{eQW_{Qmax}}{2E_0 r z_{tr}}\right)^{\frac{1}{2}} \frac{x_0}{r_0}$$

For the maximum focusing strength deviation for our case we have

$$\Delta k_{in} = [1.85 \cdot 10^2 m^{-2}]x_0$$

$$\Delta k_{out} = [1.4 \cdot 10^2 m^{-2}]x_0$$

The transverse kicks in the horizontal and vertical plane at the distance $z = z_{tr}$ due to the quadrupole mode wake, assuming $x_1 = x_{10}, x'_1 = 0$ at $z = 0$ are

$$\mid \Delta x_1 \mid \sim \frac{\Delta k^2 z_{tr}^2}{2} x_{10}$$

In Table 2 the previous quantities for the beam offset $x_0 = \sigma_x$ are shown

	β_x (m)	$x_0 = \sigma_x$ (mm)	x_1/σ_x	$x_1\prime$ (rad)	Δk (m^{-1})	$\frac{\Delta x_1}{x_{10}}$
step-in	1600	0.2	$4.4 \cdot 10^{-8}$	$3.5 \cdot 10^{-9}$	$3.7 \cdot 10{-}2$	$6.8 \cdot 10^{-10}$
step-out	0.003	$0.15 \cdot 10^{-3}$	$3.75 \cdot 10^{-7}$	$4.5 \cdot 10^{-12}$	$2.1 \cdot 10^{-5}$	$5.5 \cdot 10^{-13}$

5 Wake fields due to resistive walls

The wake field effects of an ultrarelativistic bunch moving in a cylindrical pipe with resistive walls, one of analitically solved problems, have been presented in many Refs.[6–9]. It is well known that the behaviour of the wake potentials in a cylindrical tube of radius a and conductivity σ defined by the characteristic distance

$$s_0 = \left(\frac{a^2}{Z_0\sigma}\right)^{\frac{1}{3}}$$

where $Z_0 = 4\pi/c = 377\Omega$ is the vacuum impedance. For copper ($\sigma = 5.9 \times 10^7\Omega^{-1}m^{-1}$): $s_0 \approx 3.5\mu m$ for a tube radius of $a = 1mm$.

The wake potentials of a normalized Gaussian charge distribution $\rho(s)$ with rms- length σ_z are known when the bunch length is large compared to the characteristic length s_0 ($s_0/\sigma_z \sim 0.01$). The monopole term for the longitudinal and the dipole term for the transverse wake potentials can be written as .

$$W_z(s) = -\frac{c}{2\pi a\sigma^{3/2}}\sqrt{\frac{Z_0}{2\pi\sigma}}\exp(-\frac{u^2}{4})D_{+\frac{1}{2}}(-u)$$

$$W_\perp(s) = \frac{r_0 c}{\pi a^3\sigma_z^{1/2}}\sqrt{\frac{Z_0}{2\pi\sigma}}\exp(-\frac{u^2}{4})D_{-\frac{1}{2}}(-u)$$

with $u = s/\sigma_z$; $D_{+\frac{1}{2}}$ the parabolic cylinder functions.

The longitudinal and transverse wake potentials are plotted in Figs. 10 and 11. The maximum of the longitudinal retarding wake potential induced , per unit length, by the resistive walls on a Gaussian bunch with $\sigma_z >> s_0$ travelling on axis ($r = 0$) can be written as

$$W_{zmax} = -\frac{0.85c}{4\pi a\sigma_z^{3/2}}\sqrt{\frac{Z_0}{2\sigma}}$$

The rms energy spread σ_E and the longitudinal loss factor k_z per unit length on a Gaussian bunch with $\sigma_z >> s_0$ are also well known

$$\sigma_E = eQ\left(\int_{-\infty}^{\infty} ds\rho(s)W_z^2(s) - k_z^2\right)^{1/2} = \frac{0.46eQc}{2\pi^2 a\sigma_z^{3/2}}\left(\frac{Z_0}{\sigma}\right)^{1/2}$$

$$k_z = \int_{-\infty}^{\infty} \rho(s)W_z(s)ds = \frac{\Gamma(3/4)c}{4\pi^2 a\sigma_z^{3/2}}\sqrt{\frac{Z_0}{2\sigma}}$$

Putting in constants, this becomes

$$W_{zmax} = [2.78 \times 10^8 \frac{V \cdot m}{C \cdot \Omega^{1/2}}]\frac{1}{a\sigma_z^{3/2}\sigma^{1/2}}$$

$$\sigma_E = [1.35 \times 10^8 \frac{V \cdot m}{C \cdot \Omega^{1/2}}]\frac{eQ}{a\sigma_z^{3/2}\sigma^{1/2}}$$

$$k_z = [1.28 \times 10^8 \frac{V \cdot m}{C \cdot \Omega^{1/2}}]\frac{1}{a\sigma_z^{3/2}\sigma^{1/2}}$$

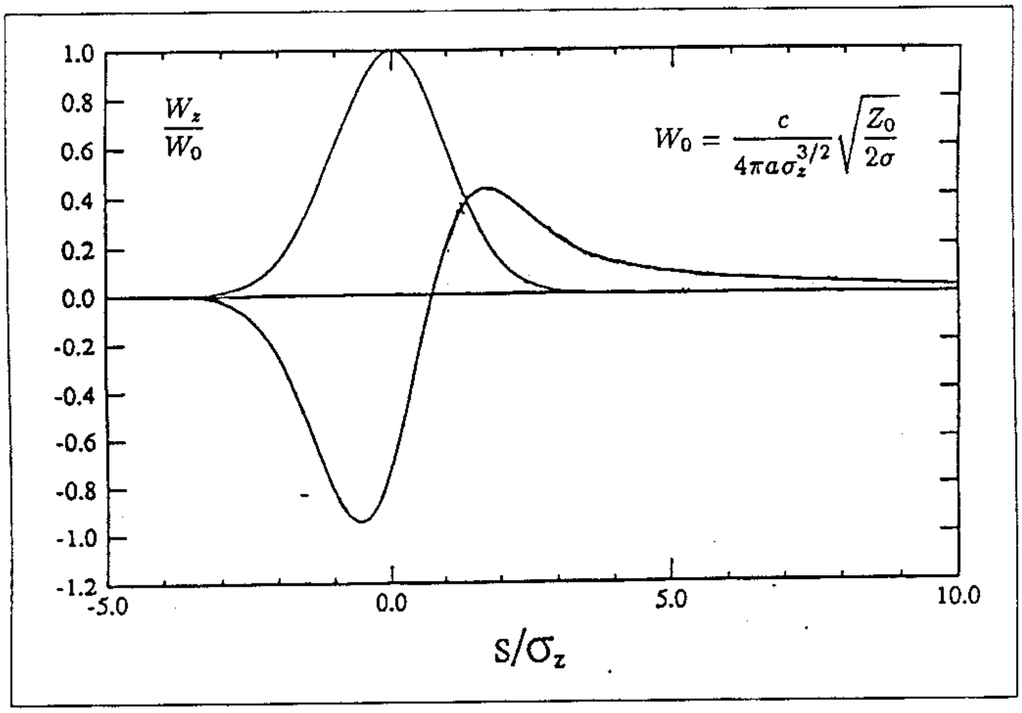

Figure 10: The Gaussian longitudinal resistive wake potential.

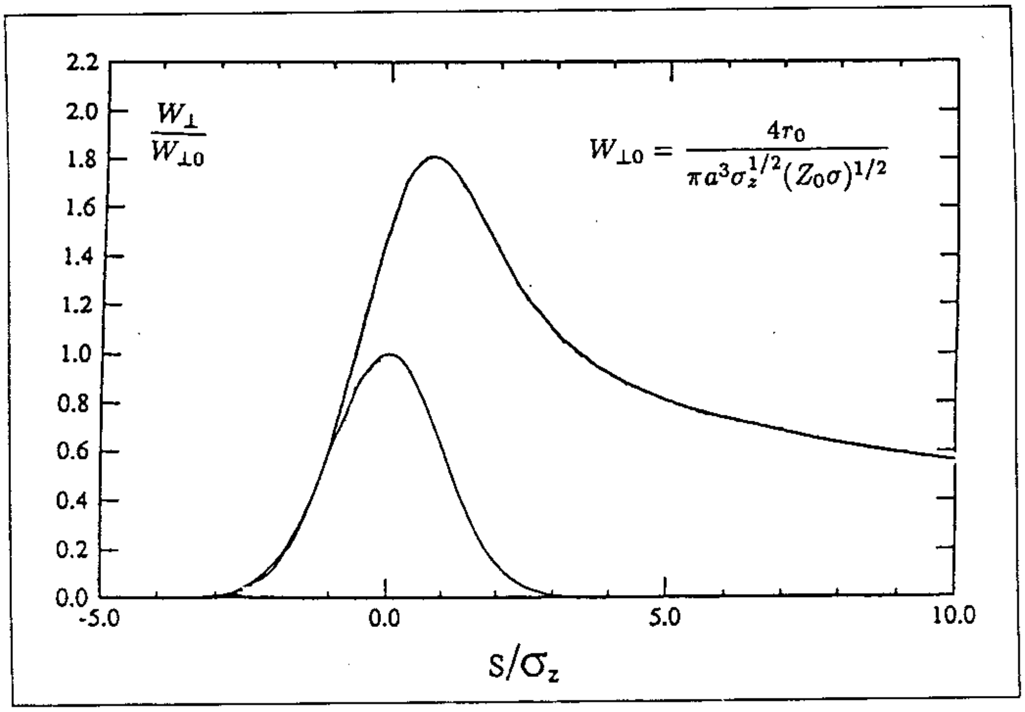

Figure 11: The Gaussian transverse resistive wake potential.

For our bunch parameters this corresponds to an energy lost per $1m$ of quadrupole for the particles at $s \sim -\sigma_z/2$ within the bunch by an order of 14.3 keV. The rms energy spread $\sigma_E = 6.96KeV$ and energy loss $eQk_z = 6.59KeV$ for a quadrupole aperture $a = 1mm$. Thus the energy spread due to resistive longitudinal wake fields can be neglected.

The maximum of the transverse dipole mode wake potential seen by the particles at $s \sim \sigma_z$ within the bunch is

$$W_{\perp max} = \frac{4 \cdot 1.8 r_0}{\pi a^3 \sigma_z^{1/2} (Z_0 \sigma)^{1/2}}$$

or putting in constants

$$W_{\perp max} = [1.06 \cdot 10^9 \frac{V \cdot m}{C \cdot \Omega^{1/2}}] \frac{r_0}{a^3 \sigma_z^{1/2} \sigma^{1/2}}$$

For our bunch parameters these corespond to the dipole Lorentz force

$$F_r = eQW_{\perp max} = [1.09 \cdot 10^7 \frac{eV}{m^2}] r_0$$

Scaling the results of the geometrical and resistive transverse wake potentials per unit length we obtain effects of the same order. However, for the final focus system, transverse resistive wake fields dominate due to the long region of the interaction of the particles within the bunch with the induced fields in the quadrupole vacuum chamber.

6 Resistive wake field effects in the final focus douplet

All existing linear collider projects include an FD doublet for the final focus at the IP. The final focus doublet, considerd by the DESY/THD S-band linear collider study group [(10)] with quadrupole parameters, betatron functions and bunch transvers size at the entrance of the quadrupoles is presented at Fig. 12.

As mentioned in the previous section the energy spread within the bunch due to longitudinal wake fields can be neglected. We will disscuss the transverse wake field effects and the coresponding kicks on the tail particles at the IP . For the offset of the bunch at the entrance of the first quadrupole we will assume one sigma (σ_x for horizontal and σ_y for vertical planes).

Since a magnetic field of one Tesla gives the same bending force as an electric field of $300MV/m$ for relativistic particles, the tail particles at the maximum of the transverse wake potential at the first quadrupole will bend an angle $\alpha = L/\rho$ with a bending radius for electrons of

$$\rho = \frac{10^9 E}{W_{\perp max} Q} \quad (GeV, m, \frac{V}{m})$$

As an example, for electrons with an energy of $250GeV$, one sigma bunch offset at the entrance of the first quadrupole with aperture $1mm$ corresponds to a bending radius of $\rho = 8 \cdot 10^7 m$ and an angle at the exit of the first quadrupole of 12.5 $nrad$, which gives a displacement of the tail particle of an order of $\Delta x \sim 30\,nm$ at the IP. The transverse dipole field also causes a nonzero dispersion function of the same order at the IP.

In the two particle model with the head particle moving in the focusing quadrupole on an orbit described by $r_0 = x_0 \cos k_\beta z$ in the horizontal plane ($r_0 = y_0 \cosh k_\beta z$) the displacement

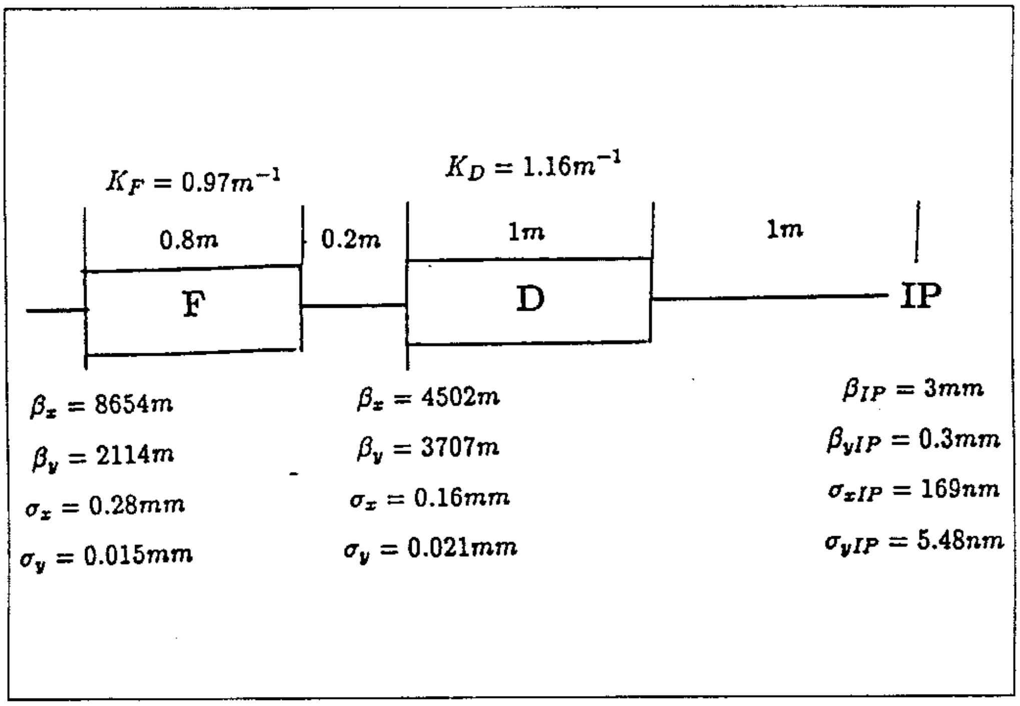

Figure 12: Final Focus Doublet for the $500GeV$ $S-$ Band Linear Collider.

$x_1(y_1)$ for the tail particle obeys the equation of motion

$$x_1'' + k_\beta^2 x_1 = \frac{F_r}{2E_0 r_0} x_0 \cos k_\beta z$$

$$y_1'' - k_\beta^2 y_1 = \frac{F_r}{2E_0 r_0} y_0 \cosh k_\beta z$$

In the horizontal plane we have the equation for the amplitude of a lossless harmonic oscilator driven at resonance. Assuming $x_1 = x_1' = 0 (y_1 = y_1' = 0)$ at $z = 0$ the solution is

$$x_1 = \frac{C x_0}{2k_\beta}(z \sin k_\beta z)$$

$$y_1 = \frac{C y_0}{2k_\beta}(z \sinh k_\beta z)$$

with $C = F_r/2E_0 r_0$.

If $x_F(y_F), x_D(y_D)$ are the offsets of the head particle at the entrance of the focusing and defocusing quadrupoles , the tail particle will have the following displacement at the IP

$$x_{IP} = [7.58 \cdot 10^{-5} mm^3] \frac{x_F}{a_F^3} + [4.61 \cdot 10^{-5} mm^3] \frac{x_D}{a_D^3}$$

$$y_{IP} = [1.72 \cdot 10^{-5} mm^3] \frac{y_F}{a_F^3} + [2.14 \cdot 10^{-5} mm^3] \frac{y_D}{a_D^3}$$

where a_F, a_D are the apertures of the quadrupoles. It is natural to use the same type of quadrupole magnets for the final focus system with the same aperture. Then for bunch offsets of one sigma and $a = 1mm$ the displacement of the tail particle at the IP are

$$x_{IP} = 28.6nm \qquad x'_{IP} = 16.8nrad$$
$$y_{IP} = 0.72nm \qquad y'_{IP} = 0.21nrad$$

with a dispersion function of the same order.

For 1% luminosity reduction the bunch offset at the entrance of the quadrupoles should be better controlled than

$$\frac{\Delta x}{\sigma_x} < 0.06$$
$$\frac{\Delta y}{\sigma_y} < 0.08$$

Finally , the results of the resistive wake field effect for the parameters of Ref. (1) are summerized in the following table:

Q (nC)	$Energy$ (TeV)	σ_z (mm)	$F_r/r_0(10^7)$ (eV/m^2)	$\Delta x_{IP}(10^{-3})/\sigma_x$
1.12	0.5	0.2	1.09	0.13

The wake fields are produced at first quadrupole with an aperture of $1mm$,a length of $1m$, at distance of $3m$ from IP by a bunch with a offset $r_0 = \sigma_x(\sigma_y)$. $\Delta x_{IP}(\Delta y_{IP})$ is the displacement of the tail particle at IP.

7 Conclusion

The longitudinal wake fields, due to both the abrupt change of radius and resistive wall of the vacuum chamber of the final focus quadrupoles, produce very small energy spread within the bunch. This effect is of order 10^{-7} and in comparison with other sources of energy spread it may be neglected.

The results of the geometrical and resistive transverse wake field effects scaled *per unit length* are of the same order. However, the transverse resistive wake field effects dominate due to the long region of the interaction of the particles within the bunch with the induced fields in the quadrupole vacuum chamber.

8 Reference

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