



# Phenomenology of $k_T$ -factorization for inclusive Higgs boson production at LHC

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## ABSTRACT

We investigate the inclusive Higgs boson production in proton–proton collisions at high energies in the framework of  $k_T$ -factorization QCD approach. The attention is focused on the dominant off-shell gluon–gluon fusion subprocess  $g^*g^* \rightarrow H \rightarrow \gamma\gamma$ , where the transverse momentum of incoming gluons are taken into account. The transverse momentum dependent (or unintegrated) gluon densities of the proton are determined using the CCFM evolution equation as well as the Kimber–Martin–Ryskin prescription. We study the theoretical uncertainties of our calculations and perform the comparison with the results of traditional pQCD evaluations. Our predictions agree well with the first experimental data taken by the ATLAS Collaboration at the LHC. We argue that further studies of the Higgs boson production are capable of constraining the unintegrated gluon densities of the proton.

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In 2012, during the search for the SM Higgs boson at the LHC, the CMS and ATLAS Collaborations observed a new particle [1,2], undiscovered before. Some time later, with additional data and improved analysis strategy, a specific spin-2 hypothesis has been excluded with a confidence level above 99.9% [3]. Spin and relative production rates of the observed new particle in different decay modes conform to the SM expectations for the Higgs boson. So, the experimental detection of the Higgs particle has been claimed giving us the confidence in the physical picture of fundamental interactions which follows from the SM Lagrangian. Indeed, it has become a great triumph of the SM and marked new stage in high energy physics.

Very recently the ATLAS Collaboration has reported first measurements of the Higgs boson differential cross sections in the diphoton decay mode [4]. In particular, the distributions on the diphoton transverse momentum  $p_T^{\gamma\gamma}$ , rapidity  $|y^{\gamma\gamma}|$  and helicity angle  $|\cos\theta^*|$  have been presented.<sup>1</sup> These observables describe the fundamental kinematic properties of the discovered Higgs particle, probe its spin and test the corresponding theoretical calculations within the QCD, which are performed for basic gluon–gluon fusion subprocess  $gg \rightarrow H$  [5–10] at next-to-next-to-leading order (NNLO) [10–15] and matched with soft-gluon resummation carried out up to next-to-next-to-leading logarithmic accuracy (NNLL)

[16,17]. The measured cross sections are higher than the central SM expectations, although no significant deviations from the SM predictions are observed within the experimental and theoretical uncertainties [4].

In the present note we analyze recent ATLAS data using the  $k_T$ -factorization QCD approach [18,19]. This approach is based on the Balitsky–Fadin–Kuraev–Lipatov (BFKL) [20] or Ciafaloni–Catani–Fiorani–Marchesini (CCFM) [21] gluon evolution equations and provides solid theoretical grounds for the effects of initial gluon radiation and intrinsic gluon transverse momentum. A detailed description and discussion of the  $k_T$ -factorization formalism can be found, for example, in reviews [22]. Here we only mention that the soft gluon resummation formulas implemented in NNLL calculations [16,17] are the result of the approximate treatment of the solutions of CCFM equation [23]. Previously, the  $k_T$ -factorization formalism has been applied [23–26] to calculate transverse momentum distribution of the inclusive Higgs boson production. The effective Lagrangian [27,28] for the Higgs boson coupling to gluons (valid in the large top quark mass limit) has been used to calculate the amplitude of basic gluon–gluon fusion subprocess. However, the calculations [23,24] neglect the transverse momentum of initial gluons in the production amplitude, and the simplified solution of the CCFM equation in the single loop approximation (where the small- $x$  effects are omitted) has been applied in [23]. In [24] the transverse momenta of incoming gluons are generated at the last evolution step (the so-called Kimber–Martin–Ryskin prescription). The off-shell gluon–gluon fusion amplitude  $g^*g^* \rightarrow H$  has been evaluated independently in [25,29], and corresponding ex-

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<sup>1</sup> The helicity angle  $\theta^*$  is defined as the angle between the beam axis and the photons in the Collins–Soper frame of the Higgs boson.

pression [29] has been implemented [26] in a Monte Carlo event generator CASCADE [30]. The investigations [23–26,29] inspired further studies [31–33] of the inclusive Higgs boson production where finite mass effects in the triangle quark loop have been taken into account. The associated production of the Higgs boson and beauty or top quark pair at high energies in the  $k_T$ -factorization approach have been also investigated in [34].

Our present consideration is based mainly on the study [25] where the inclusive and jet associated Higgs boson cross sections have been investigated. We calculate the off-shell amplitude of gluon–gluon fusion subprocess  $g^*g^* \rightarrow H \rightarrow \gamma\gamma$ , which is not provided by the previous papers [23–26,29,31–33], and evaluate the total and differential sections of the Higgs boson production (in the diphoton decay mode). Numerically, we apply the unintegrated, or transverse momentum dependent (TMD) gluon densities of the proton obtained from the numerical solution of CCFM evolution equation [35]. We analyze the transverse momentum  $p_T^{\gamma\gamma}$ , rapidity  $|y^{\gamma\gamma}|$  and helicity angle  $|\cos\theta^*|$  distributions measured by the ATLAS Collaboration and compare our predictions with the data [4]. Such calculations in the framework of  $k_T$ -factorization QCD approach are performed for the first time. Additional motivation of our study is that the transverse momentum of Higgs boson is strongly related to the initial gluon transverse momenta [25], and therefore such an observable could impose constraint on the TMD gluon density of the proton.<sup>2</sup>

Let us start from a short review of calculation steps. We describe first the evaluation of the off-shell  $g^*g^* \rightarrow H \rightarrow \gamma\gamma$  production amplitude. In the limit of large top quark mass  $m_t \rightarrow \infty$  the effective Lagrangian for the Higgs boson coupling to gluons reads [27,28]

$$\mathcal{L}_{ggH} = \frac{\alpha_s}{12\pi} (G_F \sqrt{2})^{1/2} G_{\mu\nu}^a G^{a\mu\nu} H, \quad (1)$$

where  $G_F$  is the Fermi coupling constant,  $G_{\mu\nu}^a$  is the gluon field strength tensor and  $H$  is the Higgs scalar field. The large  $m_t$  approximation is valid to an accuracy of  $\sim 5\%$  in the mass range  $m_H < 2m_t$ , and, of course, is applicable at the measured Higgs boson mass  $m_H \sim 125$  GeV [1–4]. The triangle vertex  $T_{ggH}^{\mu\nu,ab}(k_1, k_2)$  for two off-shell gluons having four-momenta  $k_1$  and  $k_2$  and color indexes  $a$  and  $b$  can be obtained from the effective Lagrangian (1):

$$T_{ggH}^{\mu\nu,ab}(k_1, k_2) = i\delta^{ab} \frac{\alpha_s}{3\pi} (G_F \sqrt{2})^{1/2} [k_2^\mu k_1^\nu - (k_1 \cdot k_2) g^{\mu\nu}]. \quad (2)$$

The two-photon decay process  $H \rightarrow \gamma\gamma$  in the SM proceeds through  $W$  boson and fermion loops. The corresponding  $H \rightarrow \gamma\gamma$  coupling is determined by the effective Lagrangian [27,28]

$$\mathcal{L}_{H\gamma\gamma} = \frac{\alpha}{8\pi} \mathcal{A} (G_F \sqrt{2})^{1/2} F_{\mu\nu} F^{\mu\nu} H, \quad (3)$$

where  $F_{\mu\nu}$  is the photon field strength tensor and

$$\mathcal{A} = \mathcal{A}_W(\tau_W) + N_c \sum_f Q_f^2 \mathcal{A}_f(\tau_f). \quad (4)$$

Here  $N_c$  is the color factor,  $Q_f$  is the (fractional) electric charge of the fermion  $f$  and the scaling variables  $\tau_f$  and  $\tau_W$  are defined by

$$\tau_f = \frac{m_H^2}{4m_f^2}, \quad \tau_W = \frac{m_H^2}{4m_W^2}, \quad (5)$$

where  $m_W$  and  $m_f$  are the  $W$  boson and fermion masses, respectively. The amplitudes  $\mathcal{A}_f$  and  $\mathcal{A}_W$  are expressed as [27,28]

$$\begin{aligned} \mathcal{A}_f(\tau) &= 2[\tau + (\tau - 1)f(\tau)]/\tau^2, \\ \mathcal{A}_W(\tau) &= -2[2\tau^2 + 3\tau + 3(2\tau - 1)f(\tau)]/\tau^2, \end{aligned} \quad (6)$$

where the function  $f(\tau)$  is given by

$$f(\tau) = \begin{cases} \arcsin^2 \sqrt{\tau}, & \tau \leq 1 \\ -\frac{1}{4} \left[ \log \frac{1+\sqrt{1-1/\tau}}{1-\sqrt{1-1/\tau}} - i\pi \right]^2, & \tau > 1. \end{cases} \quad (7)$$

The effective vertex  $T_{H\gamma\gamma}^{\mu\nu}(p_1, p_2)$  for two photons having four-momenta  $p_1$  and  $p_2$  obtained from the effective Lagrangian (3) reads

$$T_{H\gamma\gamma}^{\mu\nu}(p_1, p_2) = i \frac{\alpha}{2\pi} \mathcal{A} (G_F \sqrt{2})^{1/2} [p_2^\mu p_1^\nu - (p_1 \cdot p_2) g^{\mu\nu}]. \quad (8)$$

Taking into account the non-zero transverse momenta of initial gluons  $k_1^2 = -\mathbf{k}_1^2 \neq 0$  and  $k_2^2 = -\mathbf{k}_2^2 \neq 0$  and using the effective vertices (2) and (8), one can easily derive the off-shell amplitude of gluon–gluon fusion subprocess  $g^*g^* \rightarrow H \rightarrow \gamma\gamma$ . We have obtained

$$|\bar{\mathcal{M}}|^2 = \frac{1}{1152\pi^4} \alpha^2 \alpha_s^2 G_F^2 |\mathcal{A}|^2 \frac{\hat{s}^2 (\hat{s} + \mathbf{p}_T^2)^2}{(\hat{s} - m_H^2)^2 + m_H^2 \Gamma_H^2} \cos^2 \phi, \quad (9)$$

where  $\Gamma_H$  is the Higgs boson full decay width,  $\hat{s} = (k_1 + k_2)^2$ , the transverse momentum of the Higgs particle is  $\mathbf{p}_T = \mathbf{k}_{1T} + \mathbf{k}_{2T}$  and  $\phi$  is the azimuthal angle between the transverse momenta of initial gluons. Here we take the propagator of the intermediate Higgs boson in the Breit–Wigner form to avoid any artificial singularities in the numerical calculations. The summation over the polarizations of produced on-shell photons is carried with the usual formula  $\sum \epsilon^\mu \epsilon^{*\nu} = -g^{\mu\nu}$ . In accordance to the  $k_T$ -factorization prescription [18,19], the summation over the polarizations of incoming off-shell gluons is carried with  $\sum \epsilon^\mu \epsilon^{*\nu} = \mathbf{k}_T^\mu \mathbf{k}_T^\nu / \mathbf{k}_T^2$ . In the collinear limit, when  $|\mathbf{k}_T| \rightarrow 0$ , this expression converges to the ordinary one after averaging on the azimuthal angle. In all other respects the evaluation follows the standard QCD Feynman rules. We note that the expression (9) is fully consistent with one derived in [25] (where the subsequent Higgs decay  $H \rightarrow \gamma\gamma$  was not considered).

The cross section of inclusive Higgs boson production in the  $k_T$ -factorization approach is calculated as a convolution of the off-shell partonic cross section and the TMD gluon densities of the proton. Our master formula reads:

$$\begin{aligned} \sigma &= \int \frac{|\bar{\mathcal{M}}|^2}{16\pi (x_1 x_2 s)^2} f_g(x_1, \mathbf{k}_{1T}^2, \mu^2) \\ &\quad \times f_g(x_2, \mathbf{k}_{2T}^2, \mu^2) d\mathbf{p}_{1T}^2 d\mathbf{k}_{1T}^2 d\mathbf{k}_{2T}^2 dy_1 dy_2 \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi}, \end{aligned} \quad (10)$$

where  $f_g(x, \mathbf{k}_T^2, \mu^2)$  is the TMD gluon density of the proton,  $s$  is the total energy,  $y_1$  and  $y_2$  are the center-of-mass rapidities of decay photons,  $\phi_1$  and  $\phi_2$  are the azimuthal angles of the initial gluons having the fractions  $x_1$  and  $x_2$  of the longitudinal momenta of the colliding protons. From the conservation law one can easily obtain the following relations:

$$\mathbf{k}_{1T} + \mathbf{k}_{2T} = \mathbf{p}_{1T} + \mathbf{p}_{2T}, \quad (11)$$

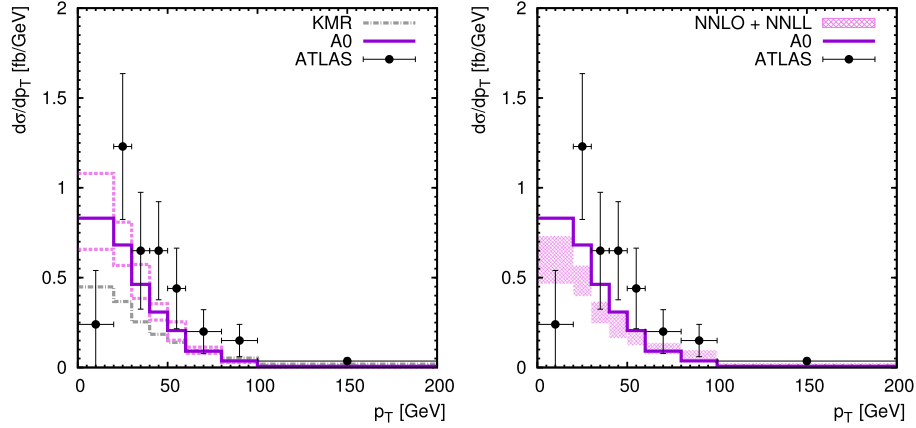
$$x_1 \sqrt{s} = |\mathbf{p}_{1T}| e^{y_1} + |\mathbf{p}_{2T}| e^{y_2}, \quad (12)$$

$$x_2 \sqrt{s} = |\mathbf{p}_{1T}| e^{-y_1} + |\mathbf{p}_{2T}| e^{-y_2}, \quad (13)$$

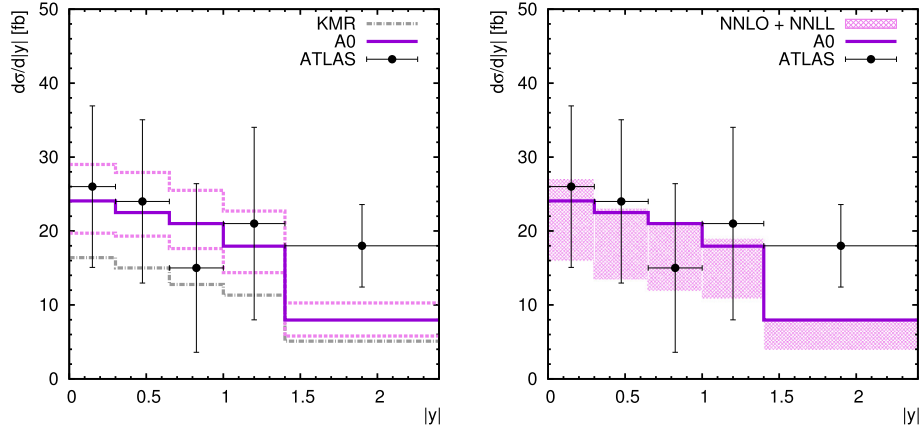
where  $\mathbf{p}_{1T}$  and  $\mathbf{p}_{2T}$  are the transverse momenta of final photons.

In the numerical calculations we have tested a few different sets of TMD gluon densities. First of them, namely CCFM A0 set, is commonly recognized at present and widely used in the phenomenological applications. It has been obtained [35] from the

<sup>2</sup> Recently a program of QCD measurements at the LHC has been proposed where the Higgs boson is considered as a gluon trigger [36].



**Fig. 1.** The differential cross section of the Higgs boson production in  $pp$  collisions at the LHC as a function of diphoton transverse momentum. Left panel: the solid and dash-dotted histograms correspond to the CCFM A0 and KMR predictions, respectively; and the upper and lower dashed histograms correspond to the scale variations in the CCFM-based calculations, as it is described in the text. Right panel: the solid histogram corresponds to the CCFM A0 predictions, and the hatched histogram represent the NNLO + NNLL predictions obtained in the collinear QCD factorization (taken from [4]). The experimental data are from ATLAS [4].



**Fig. 2.** The differential cross section of the Higgs boson production in  $pp$  collisions at the LHC as a function of diphoton rapidity. Notation of all histograms is the same as in Fig. 1. The NNLO + NNLL pQCD predictions are taken from [4]. The experimental data are from ATLAS [4].

numerical solution of the CCFM gluon evolution equation where all input parameters have been fitted<sup>3</sup> to describe the proton structure function  $F_2(x, Q^2)$ . Beside the CCFM-evolved TMD gluon density, we will use the one obtained from the Kimber–Martin–Ryskin (KMR) prescription [38]. The KMR approach is a formalism to construct the TMD quark and gluon densities from well-known conventional ones. For the input, we have used leading-order Martin–Stirling–Thorn–Watt (MSTW) set [39].

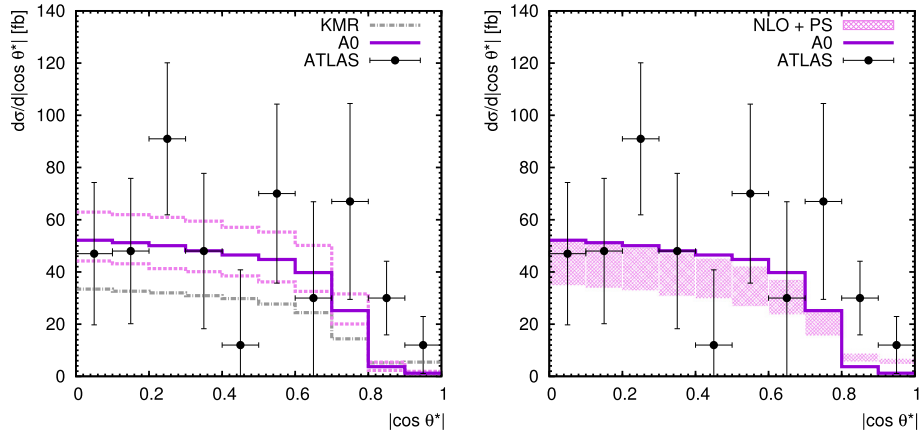
We now are in a position to present our numerical results and discussion. After we fixed the TMD gluon densities of the proton, the Higgs boson cross section (10) depends on the renormalization and factorization scales  $\mu_R$  and  $\mu_F$ . Numerically, we set them to be equal to  $\mu_R = \mu_F = \xi m_H$ . To estimate the scale uncertainties of our calculations we vary the parameter  $\xi$  between 1/2 and 2 about the default value  $\xi = 1$ . We set  $m_H = 126.8$  GeV, the central value of the measured mass in the diphoton channel [4]. Following to [40], we set  $\Gamma_H = 4.3$  MeV and use the LO formula for the strong coupling constant  $\alpha_s(\mu^2)$  with  $n_f = 4$  active quark flavors at  $\Lambda_{\text{QCD}} = 200$  MeV, so that  $\alpha_s(m_Z^2) = 0.1232$ . Note also that we

use the running QED coupling constant  $\alpha(\mu^2)$ . The multidimensional integration in (10) have been performed by the means of Monte Carlo technique, using the routine VEGAS [41]. The corresponding C++ code is available from the authors on request.<sup>4</sup>

The ATLAS Collaboration have performed the measurements in the kinematical region defined by  $|\eta^\gamma| < 2.37$ ,  $105 < M < 160$  GeV and  $E_T^\gamma/M > 0.35$  (0.25) for the leading (subleading) photon, where  $M$  is the invariant mass of produced photon pair [4]. The results of our calculations are presented in Figs. 1–3 in comparison with the ATLAS data. In left panels, the solid histograms are obtained with the CCFM A0 gluon density by fixing both the factorization and renormalization scales at the default value, whereas the upper and lower dashed histograms correspond to the scale variation as described above. The dash-dotted histograms correspond to the predictions obtained with the KMR gluon distribution. Note that to estimate the scale uncertainties we have used the set A0+ and A0− instead of the default gluon density function A0. These two sets represent a variation of the scale used in  $\alpha_s$  in the off-shell amplitude. The A0+ stands for a variation of  $2\mu_R$ , while set A0− reflects  $\mu_R/2$  (see also [35] for more information). We find that the ATLAS data are reasonably well described by the  $k_T$ -factorization approach supplemented with the CCFM-evolved gluon density. The predictions based on the KMR gluon

<sup>3</sup> Very recently new CCFM-evolved TMD gluon density function (the so-called JH-2013 set) has been presented [37]. The corresponding initial parameters have been derived from the numerical description of latest precision HERA data on the proton structure functions  $F_2(x, Q^2)$  and  $F_L(x, Q^2)$ . We plan to test the JH-2013 gluon density in future phenomenological applications.

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**Fig. 3.** The differential cross section of the Higgs boson production in  $pp$  collisions at the LHC as a function of  $|\cos \theta^*|$ . Notation of all histograms is the same as in Fig. 1. The NLO pQCD predictions are taken from [4]. The experimental data are from ATLAS [4].

distribution tend to underestimate the data. The sensitivity of predicted cross sections to the TMD gluon densities (at least, in the overall normalization) is clearly visible for all considered kinematical variables, that supports the previous conclusion [25].

In right panels of Figs. 1–3 we plot the matched NNLO + NNLL pQCD predictions [16,17] (or NLO ones for  $|\cos \theta^*|$  distribution) taken from [4] in comparison with our results and the ATLAS data. One can see that the measured cross sections are typically higher than the collinear QCD predictions, although no significant deviation within the theoretical and experimental uncertainties is observed, as it was claimed [4]. However, the  $k_T$ -factorization predictions at the default scale (with A0 gluon density) are rather similar to upper bound of collinear QCD results, providing us a better agreement with the ATLAS data. We note also that the main part of collinear QCD higher-order corrections (namely, NLO + NNLO + N<sup>3</sup>LO + ... contributions which correspond to the  $\log 1/x$  enhanced terms in perturbative series) is presented in our calculations as a part of the CCFM evolution of TMD gluon densities. These corrections are known to be large: their effect increases the leading-order cross section by about 80–100% [11,12]. So that, Figs. 1–3 illustrate the main advantage of  $k_T$ -factorization approach: it is possible to obtain in a straightforward manner the analytic description which reproduces the main features of rather cumbersome high-order pQCD calculations.

To conclude, in the present note we apply the  $k_T$ -factorization QCD approach to the analysis of first experimental data on the inclusive Higgs production taken by the ATLAS Collaboration at the LHC in the diphoton decay channel. Using the off-shell amplitude of gluon–gluon fusion subprocess  $g^*g^* \rightarrow H \rightarrow \gamma\gamma$  and CCFM-evolved gluon density in a proton, we obtained a reasonably well agreement between our predictions and the ATLAS data. We demonstrated that further theoretical and experimental studies of Higgs boson production at high energies could impose a constraints on the TMD gluon densities of the proton.

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