Dimension-7 operators in the $b \rightarrow s$ transitionG. Chalons^{1,*} and F. Domingo^{2,†}¹*Institut für Theoretische Teilchenphysik, Karlsruhe Institute of Technology, Universität Karlsruhe
Engesserstraße 7, 76128 Karlsruhe, Germany*²*Deutsches Elektronen Synchrotron, Notkestraße 85, 22607 Hamburg, Germany*

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We extend the low-energy effective field theory relevant for $b \rightarrow s$ transitions up to operators of mass dimension seven and compute the associated anomalous-dimension matrix. We then compare our findings to the known results for dimension-six operators and derive a solution for the renormalization group equations involving operators of dimension seven. We finally apply our analysis to a particularly simple case where the Standard Model is extended by an electroweak-magnetic operator and consider limits on this scenario from the decays $B_s \rightarrow \mu^+ \mu^-$ and $B \rightarrow K \nu \bar{\nu}$.

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I. INTRODUCTION

Flavor-violating processes are well known as a central test of the Standard Model (SM). The pattern conceded to flavor transitions is indeed particularly constrained in this model: the Cabibbo-Kobayashi-Maskawa (CKM) matrix encodes the only source of flavor-breaking effects, while only the charged currents of the weak interaction convey flavor violation at tree level. Consequently, flavor transitions also define closely watched observables in the quest for new physics and—due to the absence of positive deviations from the SM predictions—set serious constraints on the forms that physics beyond the SM (BSM) could take. One of the latest results is the observation by the LHCb collaboration of the decay $B_s^0 \rightarrow \mu^+ \mu^-$ [1], with a branching ratio that is very compatible with the SM expectations (refer to Ref. [2] for a recent summary),

$$\begin{aligned} \text{Br}(B_s^0 \rightarrow \mu^+ \mu^-)^{\text{exp}} &= (3.23 \pm 0.27) \times 10^{-9}, \\ \text{Br}(B_s^0 \rightarrow \mu^+ \mu^-)^{\text{SM}} &= (3.56 \pm 0.18) \times 10^{-9}. \end{aligned} \quad (1)$$

On the theoretical side, dedicated tools have been devised in order to study flavor-violating processes, in the form of low-energy effective field theories (EFT; refer to, e.g., Ref. [3] for a review). These EFTs allow for a separation of the long-distance, low-energy strong-interaction effects and the short-distance, flavor-changing physics. In the context of B physics, at the level of terms of dimension four and smaller the relevant EFT consists of a QED $\times$ QCD model with five quark flavors (u, d, c, s, b) and three charged leptons (e, μ, τ), as well as neutrinos. Then, in the low-energy processes involving these fields, at a scale $\mu_b \sim M_B$, the impact of higher-energy (top/electroweak/Higgs/BSM; we assume here that there are no further

low-energy “invisible” fields) physics can be essentially encoded within operators of dimension > 4 , collectively defining the “effective Hamiltonian.” The strong-interaction problem then consists in evaluating the S -matrix elements driven by these operators among physical (mesonic/baryonic) states: this question is answered either through lattice QCD, QCD sum rules, heavy-quark expansions, and other theoretical descriptions of the nonperturbative strong-interaction effects, or phenomenologically through an identification of the decay constants by comparison with a few standard channels. The short-distance problem is summarized within the couplings multiplying the operators. These must be matched at high energy with the predictions of the “more fundamental” theory (the Standard Model, supersymmetry-inspired models, etc.): scattering amplitudes in both the EFT and the “full theory” are equated at the matching scale $\mu_0 \gtrsim M_W$, and hence they define a boundary condition for the parameters of the EFT in terms of those of the underlying model.

To relate these two scales, μ_b and μ_0 , one relies on the renormalization group equations (RGEs) driven by the renormalization of the EFT: leading logarithmic contributions can thus be consistently resummed. We will mostly be interested in this part of the procedure in the following.

To fix notations, let us write the lagrangian density of the EFT under consideration,

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} + i \bar{f} [\gamma^\mu D_\mu - m_f] f - \mathcal{H}_{\text{eff}}^{(\text{dim} \geq 5)}, \quad (2)$$

with $f = u, d, c, s, b, e, \mu, \tau, \nu_{e,\mu,\tau}$, where $D_\mu = \partial_\mu - ie Q_f A_\mu - ig_S G_\mu^a T^a$ is the covariant derivative, m_f is the mass of the fermion f , and Q_f is its charge; $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $G_{\mu\nu}^a = G_\mu^a T^a - G_\nu^a T^a + g_S f^{abc} G_\mu^b G_\nu^c$ represent the electromagnetic and gluonic field tensors, respectively; T^a and f^{abc} denote the $\text{SU}(3)_c$ generators [0 for $\text{SU}(3)_c$ singlets; Gell-Mann matrices in the fundamental

*guillaume.chalons@kit.edu

†florian.domingo@desy.de

representation] and structure constants; e and g_S , respectively, stand for the elementary electric charge and the strong coupling constant; $\alpha \equiv \frac{e^2}{4\pi}$ and $\alpha_S \equiv \frac{g_S^2}{4\pi}$; and $\mathcal{H}_{\text{eff}}^{(\text{dim} \geq 5)}$ is the effective Hamiltonian.

The question arising at this point is that of the operators that one needs to consider in order to describe $b \rightarrow s$ transitions. Up to now, only operators of the lowest possible mass dimension have been considered, that is, dimension-six operators.¹ This approach is justified by the suppression factor of $\frac{m_b}{M_Z} \sim 5 \times 10^{-2}$, which is expected for higher-dimensional operators. Moreover, it was (with reason) regarded as sufficient to consider only the smallest subset of dimension-six operators that would close under renormalization and for which the classical models (SM, supersymmetry-inspired, etc.) would generate a nontrivial contribution,

$$\mathcal{H}_{\text{eff}}^{(\text{dim}=6)} = \sum_i C_i(\mu) O_i^{(\text{dim}=6)}(\mu), \quad (3)$$

where the list of operators O_i can be read in, e.g., Refs. [3–5] (with small variations; note that a factor $G_F/\sqrt{2}$ is conventionally factored out in the usual notations). The determination at leading order of the anomalous-dimension matrix for the four-quark operators of this subset is quite old [6–10]. The renormalization of the magnetic and chromomagnetic operators can be found in Ref. [11], while Ref. [12] included the mixing of these with the four-quark operators (a two-loop effect). One may refer to Refs. [13–15] for the analysis of the semileptonic operators. Later works have focused on next-to-leading order [$O(\alpha_S)$] [5,16–20], electroweak [$O(\alpha)$; see, e.g., Ref. [21]], and next-to-next-to-leading QCD [$O(\alpha_S^2)$; refer, e.g., to Ref. [22] and to the summary in Ref. [23]] effects: this formidable amount of work allows for a theoretical prediction, e.g., in the SM $\bar{B} \rightarrow X_s \gamma$ decay that is competitive with experimental bounds. Interestingly, Ref. [22] showed that QCD three-loop effects to the electromagnetic dipole operator can be largely absorbed in the mass factor multiplying the fields, thereby strengthening the argument that this term is a genuine dimension-six (rather than dimension-five) operator.

In this paper we choose to adopt a different, more unprejudiced (if somewhat more anecdotal) approach to the renormalization of the $b \rightarrow s$ EFT: we shall consider all possible (on-shell) operators up to mass dimension seven and compute the corresponding anomalous-dimension matrix at one-loop QCD order. As far as we know, no attempt has ever been made in this direction, due to the suppression of the order $\frac{m_b}{M_Z} \sim 5 \times 10^{-2}$ which one expects for higher-dimensional operators. In fact, if one considers

the matching conditions in the particular case of the SM, with its restricted flavor-changing currents, the suppression would be even larger. The inclusion of higher-dimension operators admittedly appears in this concrete case more as a curiosity than a compelling necessity. There are however several reasons why analyzing dimension-seven effects may not be completely irrelevant.

- (1) From the point of view of precision physics, one observes that $\alpha_S(M_Z) \sim \frac{m_b}{M_Z}$: with increasing precision in the SM evaluation as well as experimental measurements, observables shall eventually become sensitive to dimension-seven effects. As a comparison, we may consider the analysis of Ref. [22], showing that, when the operators are suitably normalized, $O(\alpha_S^2)$ corrections to the electromagnetic dipole operator affect $\Gamma(\bar{B} \rightarrow X_s \gamma)$ only at the percent level, which is comparable to what one would expect naively from dimension-seven effects.
- (2) Certain new physics contributions are actually “hidden” dimension-seven effects. A simple example lies in the famous Higgs-penguin contributions to dimension-six $bsll$ operators [24], which are relevant, e.g., in supersymmetric models at large $\tan \beta$; the corresponding coefficients actually contain a factor m_b , formally increasing their order to dimension seven. Note however that other dimension-seven effects are not expected to receive an equally large $\tan \beta$ enhancement, although they should be relevant already at subleading order [25].
- (3) New physics in dimension-six operators is already stringently constrained, essentially enforcing the usual “minimal flavor violation” condition. A possible strategy to account for this absence of new physics in flavor observables would be to reject its existence in operators of higher dimension. Similar proposals have been made in the neutrino sector to reconcile neutrino masses, baryon/lepton number, and lepton-flavor-violating processes [26]. Note that here we do not propose a mechanism that would ensure the suppression of new physics by rejecting it on operators of dimension ≥ 7 , but we merely mention this possibility as a motivation to consider such operators.
- (4) If one parametrizes new physics blindly in an expansion of SM dimension-six operators [27,28], it turns out that certain operators would only have a dimension-seven signature at low energy. Including dimension-seven operators for the $b \rightarrow s$ transition thus naturally affords an unprejudiced analysis of physics BSM.

In the next section, we shall derive a list of all the (on-shell) operators of mass dimension five, six, and seven which may intervene in the $b \rightarrow s$ EFT. We shall then detail the calculation of their UV divergences at one-loop QCD order, before we proceed to the renormalization and

¹In fact, the magnetic and chromomagnetic operators have mass dimension five. However, they always appear, e.g., in the SM, with an additional mass suppression, lowering their scale to an apparent dimension six.

establish the RGEs. The following section will be dedicated to the solution of these RGEs in some specific cases. Finally, we shall illustrate our discussion by presenting a concrete—if naive—case where our analysis of dimension-seven operators applies. A short conclusion will summarize our achievements.

II. OPERATORS OF DIMENSION FIVE, SIX, AND SEVEN IN THE $b \rightarrow s$ TRANSITION

A. List of operators

The first step of our analysis consists in establishing a list of all the operators of dimension five, six, and seven intervening in the $b \rightarrow s$ transition (but that are Lorentz + gauge invariant!). For simplicity, all the fields shall be taken on-shell, i.e., they satisfy their equation of motion: this will be sufficient, at least for the leading-order calculation that we aim at. For a discussion concerning the relevance of on-shell EFTs and in particular the question of applying only “naive” classical equations of motion (dismissing ghosts and gauge-fixing terms), we refer the reader to Ref. [29]. We can obviously distinguish among three categories of operators: four-quark, two-quark + two lepton (semileptonic), and $\bar{b}s$ -gauge operators.

Let us start with four-fermion operators: the fermion fields already account for a mass dimension of six, leaving room for at most one covariant (for gauge invariance) derivative, when one restricts to operators of mass dimension ≤ 7 . Considering chiral fermions (that is, we include projectors $P_{L,R} \equiv \frac{1 \pm \gamma_5}{2}$ in the fermion products), there are at most three ways to contract the spinor algebra: $\mathbb{1}$ provides scalar currents, γ^μ provides vector currents, and $\sigma^{\mu\nu} \equiv [\gamma^\mu, \gamma^\nu]$ provides tensor currents²; any higher combination of γ matrices can be reduced down to these three cases (through the use of the Levi-Civita tensor $\varepsilon_{\mu\nu\rho\sigma}$ and identities of the Dirac algebra). (Note that γ_5 only gives a sign, when applied on chiral fermions.) Moreover, we may always keep the b and s of the flavor transition within the same current: other combinations are made redundant by the Fierz identities (see, e.g., Appendix A).

These considerations allow us to construct the relevant three classes of dimension-six operators:

- (1) Scalar $(\bar{b}P_{L,R}s)(\bar{f}P_{L,R}f)$.
- (2) Vector $(\bar{b}\gamma^\mu P_{L,R}s)(\bar{f}\gamma_\mu P_{L,R}f)$.
- (3) Tensor $(\bar{b}\sigma^{\mu\nu} P_{L,R}s)(\bar{f}\sigma_{\mu\nu} P_{L,R}f)$. Note that only two chiral combinations are possible for the tensor currents: $L \times L$ and $R \times R$; the other combinations, $L \times R$ and $R \times L$, are identically zero.

One may then consider authentic dimension-seven operators by incorporating one covariant derivative in the fermion products. Two possibilities appear:

- (1) Contracting the Lorentz index of D_μ with a vector current.
- (2) Contracting it with a tensor current, with the second tensorial index being contracted with the second vector current.

Note however that by using the equations of motion on the fermions $[(i\not{D} - m_f)f = 0]$ and realizing partial integrations (i.e., adding a total derivative to the Lagrangian density), the resulting set of operators is largely redundant (together with the dimension-six operators). Indeed, the second possibility which we mentioned (“ $D^\mu \gamma^\nu \otimes \sigma_{\mu\nu}$ ”) can always be reduced down to operators of the first class (“ $D_\mu \otimes \gamma^\mu$ ”) plus dimension-six operators (multiplying fermion masses). Additionally, certain combinations of the operators of the first class reduce to dimension-six terms. We thus retain only two kinds of linearly independent operators:

- (1) $[\bar{b}i(\not{D} - \not{D})^\mu P_{L,R}s](\bar{f}\gamma_\mu P_{L,R}f)$.
- (2) $(\bar{b}\gamma^\mu P_{L,R}s)[\bar{f}i(\not{D} - \not{D})_\mu P_{L,R}f]$.

For semileptonic operators, i.e., when f is a lepton ($f = l$), the analysis is essentially over.

Let us therefore focus on four-quark operators ($f = q$). One should then also consider the contraction of the color indices. Two possibilities arise:

- (1) The product of two color-singlet currents: color indices contracted between the $\bar{b}$ and s on one side, and between $\bar{q}$ and q on the other.
- (2) The product of two octet currents: color indices contracted between the $\bar{b}$ and q on one side, and between $\bar{q}$ and s on the other.

In the special cases where $q = b, s$, however, Fierz identities make this distinction superfluous, and we may then consider only singlet products (refer to Appendix A). This is our final word for four-fermion operators.

We now turn to $\bar{b}s$ -gauge operators. The methodology follows that of Ref. [12] for the dimension-six operators: one may simply write all the possible ways to include covariant derivatives (at most four for operators of dimension ≤ 7) within the fermionic current. Using partial integration and the equations of motion, it turns out that all these covariant derivatives can be combined in field-strength tensors. One thus simply needs to consider the possible ways of combining the indices of these field-strength tensors with those of the fermionic current:

- (i) When only one field strength is present, it can only contract with a (Lorentz) tensor current, and, in the case of the QCD field strength $G_{\mu\nu}^a$, with an $SU(3)_c$ octet current.
- (ii) When two field strengths are present, we may either contract their Lorentz indices together—thus reducing the fermionic current to a scalar—via the metric or a $\varepsilon_{\mu\nu\rho\sigma}$ tensor, or contract two of their Lorentz indices with a tensor current (with the other two being contracted through the metric, or equivalently, $\varepsilon_{\mu\nu\rho\sigma}$). In the case where only one QCD field strength

²Note that the definition of $\sigma^{\mu\nu}$ which we adopt here for simplicity differs somewhat (by a factor $i/2$) from the one the reader may have encountered in the literature.

is involved (among the two), one needs again an $SU(3)_c$ octet on the fermionic side. When two QCD field strengths are involved ($G_{\mu\nu}^a G_{\rho\sigma}^b$), the color indices may be contracted together (δ^{ab} : color singlet) with a symmetric tensor ($\{T^a, T^b\}$) or with an antisymmetric tensor ($[T^a, T^b]$) (superposition of color-octets + singlet), depending on the compatibility of these structures with the Lorentz form of the fermionic current.

We now have all the ingredients to present the list of relevant operators. Note that their normalization is *a priori* free: our choice can be justified *a posteriori* to ensure that all the RGEs intervene at the same leading order in α_s . Note however that this choice may be slightly misleading—for instance, in the SM—for reasons that we will discuss in the next section when we solve the RGEs.

(i) $(\bar{b}s)(\bar{l}l)$ operators³:

$$\begin{cases} S_{L,RL,R}^l = \frac{\alpha}{\alpha_s} m_b (\bar{b} P_{L,R} s) (\bar{l} P_{L,R} l), \\ V_{L,RL,R}^l = \frac{\alpha}{\alpha_s} m_b (\bar{b} \gamma^\mu P_{L,R} s) (\bar{l} \gamma_\mu P_{L,R} l), \\ T_{L,RL,R}^l = \frac{\alpha}{\alpha_s} m_b (\bar{b} \sigma^{\mu\nu} P_{L,R} s) (\bar{l} \sigma_{\mu\nu} P_{L,R} l), \quad \sigma^{\mu\nu} \equiv [\gamma^\mu, \gamma^\nu], \\ H_{L,RL,R}^l = \frac{\alpha}{\alpha_s} [\bar{b} (i\vec{\partial} - i\vec{\partial} + 2Q_d eA + 2g_s T^a G^a)^\mu P_{L,R} s] (\bar{l} \gamma_\mu P_{L,R} l), \\ \tilde{H}_{L,RL,R}^l = \frac{\alpha}{\alpha_s} (\bar{b} \gamma^\mu P_{L,R} s) [\bar{l} (i\vec{\partial} - i\vec{\partial} + 2Q_l eA)_\mu P_{L,R} l]. \end{cases} \quad (4)$$

(ii) $(\bar{b}s)(\bar{q}q)$ operators⁴:

$$\begin{cases} S_{L,RL,R}^q = m_b (\bar{b} P_{L,R} s) (\bar{q} P_{L,R} q), \\ V_{L,RL,R}^q = m_b (\bar{b} \gamma^\mu P_{L,R} s) (\bar{q} \gamma_\mu P_{L,R} q), \\ T_{L,RL,R}^q = m_b (\bar{b} \sigma^{\mu\nu} P_{L,R} s) (\bar{q} \sigma_{\mu\nu} P_{L,R} q), \\ H_{L,RL,R}^q = [\bar{b} i(\vec{D} - \vec{D})^\mu P_{L,R} s] (\bar{q} \gamma_\mu P_{L,R} q), \\ \tilde{H}_{L,RL,R}^q = (\bar{b} \gamma^\mu P_{L,R} s) [\bar{q} i(\vec{D} - \vec{D})_\mu P_{L,R} q], \\ S_{L,RL,R}^q = m_b (\bar{b}_\alpha P_{L,R} s_\beta) (\bar{q}_\beta P_{L,R} q_\alpha), \\ \mathcal{V}_{L,RL,R}^q = m_b (\bar{b}_\alpha \gamma^\mu P_{L,R} s_\beta) (\bar{q}_\beta \gamma_\mu P_{L,R} q_\alpha), \\ \mathcal{T}_{L,RL,R}^q = m_b (\bar{b}_\alpha \sigma^{\mu\nu} P_{L,R} s_\beta) (\bar{q}_\beta \sigma_{\mu\nu} P_{L,R} q_\alpha), \\ \mathcal{H}_{L,RL,R}^q = [\bar{b}_\alpha i(\vec{D} - \vec{D})^\mu P_{L,R} s_\beta] (\bar{q}_\beta \gamma_\mu P_{L,R} q_\alpha), \\ \tilde{\mathcal{H}}_{L,RL,R}^q = (\bar{b}_\alpha \gamma^\mu P_{L,R} s_\beta) [\bar{q}_\beta i(\vec{D} - \vec{D})_\mu P_{L,R} q_\alpha]. \end{cases} \quad (5)$$

(iii) $(\bar{b}s)$ -gauge operators:

$$\begin{cases} E_{L,R} = \frac{ie m_b^2}{4\pi\alpha_s} \bar{b} \sigma^{\mu\nu} P_{L,R} s F_{\mu\nu}, \\ Q_{L,R} = \frac{im_b^2}{g_s} \bar{b} \sigma^{\mu\nu} T^a P_{L,R} s G_{\mu\nu}^a, \\ \mathcal{E}_{L,R}^S = \frac{\alpha}{\alpha_s} \bar{b} P_{L,R} s F_{\mu\nu} F^{\mu\nu}, \\ \tilde{\mathcal{E}}_{L,R}^S = i \frac{\alpha}{\alpha_s} \bar{b} P_{L,R} s F_{\mu\nu} \tilde{F}^{\mu\nu}, \\ \mathcal{E}_{L,R}^T = \frac{\alpha}{\alpha_s} \bar{b} \sigma^{\nu\rho} P_{L,R} s F_{\mu\nu} F_\rho^\mu, \\ \mathcal{H}_{L,R}^S = \frac{e}{g_s} \bar{b} T^a P_{L,R} s F_{\mu\nu} G^{a\mu\nu}, \\ \tilde{\mathcal{H}}_{L,R}^S = \frac{ie}{g_s} \bar{b} T^a P_{L,R} s F_{\mu\nu} \tilde{G}^{a\mu\nu}, \\ \mathcal{H}_{L,R}^T = \frac{e}{g_s} \bar{b} \sigma^{\nu\rho} T^a P_{L,R} s F_\nu^\mu G_{\mu\rho}^a, \\ Q_{L,R}^D = \bar{b} P_{L,R} s G_{\mu\nu}^a G^{a\mu\nu}, \\ \tilde{Q}_{L,R}^D = i \bar{b} P_{L,R} s G_{\mu\nu}^a \tilde{G}^{a\mu\nu}, \\ Q_{L,R}^S = \bar{b} \frac{\{T^a, T^b\}}{2} P_{L,R} s G_{\mu\nu}^a G^{b\mu\nu}, \\ \tilde{Q}_{L,R}^S = i \bar{b} \frac{\{T^a, T^b\}}{2} P_{L,R} s G_{\mu\nu}^a \tilde{G}^{b\mu\nu}, \\ Q_{L,R}^T = \bar{b} \sigma^{\nu\rho} \frac{[T^a, T^b]}{2} P_{L,R} s G_{\mu\nu}^a G_\rho^{b\mu}. \end{cases} \quad (6)$$

³We repeat that, for the tensor operators, only the $L \times L$ and $R \times R$ combinations are relevant, which may not be obvious from our notation.

⁴We discard $\mathcal{S}, \mathcal{V}, \mathcal{T}, \mathcal{H}, \tilde{\mathcal{H}}^q$ for $q = b, s$ since they reduce to $\mathcal{S}, \mathcal{V}, \mathcal{T}, \mathcal{H}, \tilde{\mathcal{H}}^q$ due to Fierz transformations, which are provided in Appendix A. The color indices α, β of the fermions are displayed when not trivially contracted. Note finally that the ambiguous notation “ $\bar{D}_\mu q_\alpha$ ” actually stands for $(D_\mu q)_\alpha \dots$

This list determines the effective Hamiltonian of our EFT (the generic notation O^i spans the whole list),

$$\mathcal{H}_{\text{eff}}^{(\dim \leq 7)} = \sum_i C_i(\mu) O^i(\mu) + \text{H.c.} \quad (7)$$

FIG. 1. Contributions from the $(\bar{b}s)(\bar{l}l)$ operators to the $\bar{b}s \rightarrow \bar{l}l$ amplitude. Here, as in the following plots, the grey blob represents a vertex associated with the operators of mass dimension five through seven. The subscript “ $H^l, \tilde{H}^l$ ” under some of the diagrams indicates that such diagrams are relevant only for these kinds of operators. Feynman diagrams are plotted with JAXODRAW [30].

Summing over all the flavors and chiralities, we count 251 terms.

For the purpose of illustration, let us sketch the typical matching conditions that one could expect for these operators in a minimal-flavor-violating model (the CKM matrix elements are written as $V_{qq'}$). Only the operators $(\bar{b}s)(\bar{q}q)$ with $q = c, u$ would receive a contribution at tree level: $C_{q=c,u} \sim \frac{G_F V_{qs} V_{qb}^*}{m_b}$ (the case $q = u$ may be neglected because of the CKM suppression). All the other contributions would arise at the loop level, $C_{\text{dim } 6} \sim \frac{\alpha_S}{4\pi} \frac{G_F V_{ts} V_{tb}^*}{m_b}$ and $C_{\text{dim } 7} \sim \frac{\alpha_S}{4\pi} \frac{G_F V_{ts} V_{tb}^*}{m_b} \frac{m_b}{M_W}$, for the operators present at dimension six and seven, respectively.

B. UV-divergent amplitudes involving the dimension-five, -six, and -seven operators

Before renormalizing the EFT, we compute the divergences associated with QCD loops. We perform this calculation in the most naive way possible: in the Feynman-t’Hooft gauge and with the dimensional regularization $D = 4 - 2\epsilon$; we keep only the divergent terms $[2 - \frac{D}{2}]^{-1}$. Note that we will assume that the couplings C_i come together with an electric charge factor e whenever a photon or a lepton appear in the external lines: in this

fashion $(\bar{b}s)(\bar{l}l)$ shall be regarded as electroweakly suppressed with respect to $(\bar{b}s)(\bar{q}q)$, relegating $(\bar{b}s)(\bar{l}l)$ UV contributions to $(\bar{b}s)(\bar{q}q)$ operators to a subdominant order. The computation can be organized in “blocks”:

- (i) $(\bar{b}s)(\bar{l}l)$ contributions to the $(\bar{b}s)(\bar{l}l)$ amplitude; see Fig. 1.
- (ii) $(\bar{b}s)(\bar{q}q)$ contributions to the $(\bar{b}s)$ -gauge amplitude. Such contributions involve a quark loop. While the external fermions are explicitly taken on-shell, through the application of their equations of motion explicit off-shell terms appear for the external photons and gluons: such terms must organize as an equation of motion for the gauge-boson line, which provides us with a cross-check of our calculation. They do not matter for the renormalization, so we will not keep them explicitly in the divergent amplitudes. However, the amplitudes $(\bar{b}s) - \gamma^*$ or $(\bar{b}s) - g^*$ are relevant for the divergent amplitudes $(\bar{b}s)(\bar{l}l)$, $(\bar{b}s)(\bar{q}'q')$, and $(\bar{b}s)(gg)$, which we will present later.

The contributions to the amplitudes $\bar{b}s \rightarrow \gamma/g$ are shown in Fig. 2. The diagrams intervening in $\bar{b}s \rightarrow 2\gamma/\gamma g/2g$ are depicted in Figs. 3, 4, and 5. Note that in the case of the dimension-six four-quark operators, no contribution to the $\bar{b}s - VV'$ (where $V, V' \in \{\gamma, g\}$) operators is expected: the dimension-six basis is stable

FIG. 2. Contributions from the $(\bar{b}s)(\bar{q}q)$ operators to the $\bar{b}s \rightarrow \gamma/g$ amplitude. Again, as in the following figures, the label “ $H^q, \tilde{H}^q, \mathcal{H}^q, \tilde{\mathcal{H}}^q$ ” indicates that the corresponding diagrams intervene only for these operators.

$q = b, s$, coincide with $S^q, V^q, T^q, H^q, \tilde{H}^q$ via Fierz identities. Instead of computing the new “ t -channel” contributions,⁵ we thus use our generic result for the $S^q, \mathcal{V}^q, T^q, \mathcal{H}^q, \tilde{\mathcal{H}}^q$ operators and project it onto $S^q, V^q, T^q, H^q, \tilde{H}^q$ with the Fierz matrices $V_{\text{Fierz}}^{b,s}$ which exchange the two bases (as defined in Appendix A). A factor -1 appears in this process, corresponding to the anticommutation of the two fermion fields and/or the transformation of an open fermion loop into a closed one.

⁵In fact, we checked this relation explicitly.

 FIG. 5. Contributions from the $(\bar{b}s)(\bar{q}q)$ operators to the $\bar{b}s \rightarrow 2g$ amplitude.

 (iii) $(\bar{b}s)(\bar{q}q)$ contributions to the $(\bar{b}s)(\bar{f}f)$ amplitude.

The contributions to the $\bar{b}s \rightarrow \bar{l}l$ amplitude proceed only from the exchange of an off-shell photon, as depicted in the first diagram of Fig. 6. For the $\bar{b}s \rightarrow \bar{q}'q'$ amplitude, contributions originate similarly from the exchange of an off-shell gluon (note that the photon exchange would be of a higher order). Additional “diagonal” contributions are obtained for $q = q'$ through the dressing of the $(\bar{b}s)(\bar{q}q)$ vertex by a gluon line. All the corresponding diagrams are depicted in Fig. 6.

 (iv) $(\bar{b}s)$ -gauge contributions to the $(\bar{b}s)$ -gauge amplitudes.

Once again, off-shell photons and gluons appear within the calculation of these amplitudes, offering a nice

cross-check of the result, given that they must combine into an equation of motion for the corresponding field. Although such terms will not matter for the renormalization, $\bar{b}s \rightarrow \gamma^*$ and $\bar{b}s \rightarrow g^*$ will be relevant for $\bar{b}s \rightarrow \bar{l}l/\bar{q}q/gg$. Another check, particularly in the case where two gauge bosons appear in the final state, is simply the projectability of the result on the basis of operators that we have defined; indeed, the results typically reach this form only after the summation of several diagrams, the coefficients and color factors of which must combine in the appropriate way. Note that only “authentic” dimension-seven operators need to be included in the amplitudes $\bar{b}s \rightarrow \gamma\gamma/\gamma g/gg$: the

 FIG. 6. Contributions from the $(\bar{b}s)(\bar{q}q)$ operators to the $\bar{b}s \rightarrow \bar{l}l$ and $\bar{b}s \rightarrow \bar{q}q$ amplitudes. The grey blob with a quark loop stands for all possible $(\bar{b}s)(\bar{q}q)$ contributions to an off-shell photon/gluon; see Fig. 2.

FIG. 7. Contributions from the $(\bar{b}s)$ -gauge operators (E , Q , and $\mathcal{H} \leftrightarrow \mathcal{H}^S, \tilde{\mathcal{H}}^S, \mathcal{H}^T$) to the $\bar{b}s \rightarrow \gamma \mu$ amplitude. We also define the $b \rightarrow s$ self-energy associated with the operators Q (and $Q \leftrightarrow \tilde{Q}^D, \tilde{Q}^S, \tilde{Q}^T$), which is depicted in this figure (and those that follow) by a fermion line with a grey blob and a gluon bubble.

FIG. 8. Contributions from the $(\bar{b}s)$ -gauge operators (Q and $\tilde{Q}$) to the $\bar{b}s \rightarrow g \mu, a$ amplitude.

presence of E or Q in the divergences of such amplitudes (while subtracting the counterterms of E and Q) would signal an instability of the dimension-six basis, which would require dimension-seven counterterms. This evidently does not occur. The corresponding diagrams are depicted in Figs. 7–11.

- (v) $(\bar{b}s)$ -gauge contributions to the $(\bar{b}s)(\bar{f}f)$ amplitudes. Such contributions proceed from off-shell photon or gluon exchanges or a gluon “pseudobox”⁶ and are depicted in Fig. 12.

Finally, we may cast all these divergent (on-shell) amplitudes into the following matrix form:

$$\text{div} = \frac{t}{4\pi(2 - \frac{D}{2})} (C) \mathcal{M}(\Gamma), \quad (8)$$

where $(C) = (C_{S'}^{L,RL,R}, \dots)$ is a row vector⁷ collecting all the couplings of the effective Hamiltonian and (Γ) is a column vector constituted by all the form factors associated to the operators and intervening in the amplitudes that we

⁶By which we mean the last diagram of Fig. 12.

⁷We refrain from writing this row vector with a transposition symbol [i.e., $(C)^T$] to keep already heavy notations (which will appear in the following sections) from becoming completely unreadable.

FIG. 9. Contributions from the $(\bar{b}s)$ -gauge operators ($\mathcal{E} \leftrightarrow \mathcal{E}^S, \mathcal{E}^{\bar{S}}, \mathcal{E}^T$, and $\mathcal{H}$) to the $\bar{b}s \rightarrow 2\gamma$ amplitude. The black blobs signal a counterterm from the $\bar{b}s\gamma$ or $\bar{b}s g$ amplitudes.

have just presented (we only need one form factor per operator since we have not considered redundant amplitudes, such as $\bar{b}s \rightarrow \bar{f}f\gamma$, etc.). $\mathcal{M}$ depends only on fermion masses, gauge couplings, and charges.

C. Renormalization of the operators

1. Basics of the renormalization of the EFT in the minimal subtraction scheme

Let us recall a few basic ingredients⁸ of the renormalization procedure for our EFT in the minimal subtraction scheme.

Dismissing the effective Hamiltonian for the time being, the ($\dim \leq 4$) EFT is a renormalizable QED $\times$ QCD model. We neglect the QED loop effects here (since $\alpha \ll \alpha_S$) and focus on a QCD renormalization only. The fermion wavefunction and mass renormalization constants, Z_f and Z_{m_f} , respectively, are determined by requiring that the counterterm contribution to the f self-energy (with external momentum p), $\Sigma_f^{\text{CT}}(p) = i(Z_f - 1)\not{p} - i(Z_f Z_{m_f} - 1)$, cancels the divergent loop contributions: $\Sigma_f^{\text{loop div}}(p) = 0$ (leptons) or $-\frac{iC_2(3)\alpha_S}{4\pi(2-\frac{D}{2})}[-\not{p} + 4m_f]$ (quarks), where $C_2(3) = \frac{4}{3}$ denotes the Casimir operator of $\text{SU}(3)_c$ in the fundamental representation,

$$Z_l = 1 = Z_{m_l}, \quad \begin{cases} Z_q = 1 - \frac{C_2(3)\alpha_S}{4\pi(2-\frac{D}{2})} + O(\alpha_S^2), \\ Z_{m_q} = 1 - \frac{3C_2(3)\alpha_S}{4\pi(2-\frac{D}{2})} + O(\alpha_S^2). \end{cases} \quad (9)$$

⁸These are well-known basics of the QCD renormalization and are available in countless textbooks. We mention them only for the sake of completeness as well as clarity in our notations.

The gluon self-energy (involving gluons, quarks, and ghosts) similarly determines the gluon renormalization constant: in the Feynman gauge, $Z_3 \approx 1 - \frac{\alpha_S}{4\pi(2-\frac{D}{2})}[\frac{2}{3}n_F - \frac{5}{3}N_c]$, where $n_F = 5$ is the number of quark flavors and $N_c = 3$ is the number of colors. From the quark-gluon vertex, one renormalizes the strong coupling constant g_S , resulting in the RGE (from the scale independence of the bare coupling) for $\alpha_S \equiv \frac{g_S^2}{4\pi}$, with the renormalization scale μ ,

$$\begin{aligned} \frac{d\alpha_S}{d \ln \mu} &= -2 \left(2 - \frac{D}{2} \right) \alpha_S + \beta(\alpha_S), \\ \beta(\alpha_S) &= -\frac{2\beta_0}{4\pi} \alpha_S^2 + O(\alpha_S^3), \\ \beta_0 &= \frac{11N_c - 2n_F}{3} = \frac{23}{3} \\ \Rightarrow \alpha_S(\mu) &\approx \frac{\alpha_S(\mu_0)}{1 + \alpha_S(\mu_0) \frac{2\beta_0}{4\pi} \ln \frac{\mu}{\mu_0}}. \end{aligned} \quad (10)$$

The bare fields $Z_f f$ as well as the bare masses $Z_{m_f} m_f$ must be scale independent (note that the μ dependence enters Z_f and Z_{m_f} indirectly through α_S). One then deduces the quark mass running,

$$\begin{aligned} \frac{dm_q}{d \ln \mu} &= -6C_2(3) \frac{\alpha_S}{4\pi} m_q + O(\alpha_S^2) \\ \Leftrightarrow \frac{dm_q}{d \ln \alpha_S} &= \frac{3C_2(3)}{\beta_0} m_q + O(\alpha_S) \\ \Rightarrow \frac{m_q(\mu)}{m_q(\mu_0)} &= \left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)} \right)^{\frac{3C_2(3)}{\beta_0}}. \end{aligned} \quad (11)$$

Let us come back to the effective Hamiltonian. The loopdivergences in the amplitudes (8) must be

FIG. 10. Contributions from the $(\bar{b}s)$ -gauge operators ($\mathcal{H}$ and $\mathcal{Q}$) to the $\bar{b}s \rightarrow \gamma g$ amplitude. The (...) stand for additional diagrams involving a gluon tadpole, which give vanishing contributions.

compensated by the operator counterterm contributions: $\iota(C)[Z^C - 1]Z_\Gamma(\Gamma)$, where Z^C is the operator-renormalization matrix and (the diagonal) Z_Γ is the renormalization constant of the fields (and normalization) entering the form factor Γ . This determines $Z^C = [1 - \frac{\mathcal{M}}{4\pi(2-\frac{D}{2})}]Z_\Gamma^{-1}$. The scale independence of the bare couplings $(C)Z^C$ (due to the fact that $Z_\Gamma, Z_\Gamma O_\Gamma$ are scale independent) leads to the RGE for the couplings (C) ,

$$\frac{d(C)}{d \ln \mu} = (C)\gamma_C, \quad \gamma_C \equiv -\frac{d[Z^C]}{d \ln \mu} (Z^C)^{-1},$$

$$Z^C = \left[1 - \frac{\mathcal{M}}{4\pi(2-\frac{D}{2})} \right] Z_\Gamma^{-1}, \quad (12)$$

where the μ dependence of Z^C originates from the dependence of $\mathcal{M}$ on g_s and the quark masses, as well as from the renormalization constants in Z_Γ . We have introduced the anomalous-dimension matrix γ_C .

Considering the normalization of the operators that we introduced at the beginning of this section, one may harmoniously factor out $C_2(3)\alpha_s$ from the matrix $\mathcal{M}$, leading to

$$\text{div} = \frac{\iota C_2(3)\alpha_s}{4\pi(2-\frac{D}{2})} (C)\tilde{\mathcal{M}}(\Gamma), \quad (13)$$

where $\tilde{\mathcal{M}}$ is a constant matrix (depending only on the quark mass ratios $\frac{m_u}{m_b}$).

At this leading order in QCD, by using Eq. (10), Eq. (12) may be written explicitly as

FIG. 11. Contributions from the $(\bar{b}s)$ -gauge operators ($\mathcal{Q}$) to the $\bar{b}s \rightarrow 2g$ amplitude. The grey blob with an inscribed gluon bubble and that is attached to a photon/gluon line stands for all the contributions to off-shell photon/gluon production; see to Figs. 7 and 8.

$$\begin{aligned}
 \frac{d(C)}{d \ln \mu} &= \frac{2\alpha_S C_2(3)}{4\pi} (C) [\tilde{\mathcal{D}} - \tilde{\mathcal{M}}] + O(\alpha_S^2) \\
 \Leftrightarrow \frac{d(C)}{d \ln \alpha_S} &= -\frac{C_2(3)}{\beta_0} (C) [\tilde{\mathcal{D}} - \tilde{\mathcal{M}}] + O(\alpha_S) \\
 \Rightarrow \gamma_C &= \frac{2\alpha_S C_2(3)}{4\pi} [\tilde{\mathcal{D}} - \tilde{\mathcal{M}}] + O(\alpha_S^2), \quad (14)
 \end{aligned}$$

where we have introduced the diagonal matrix $\frac{2\alpha_S C_2(3)}{4\pi} \tilde{\mathcal{D}} \equiv \frac{dZ_\Gamma}{d \ln \mu}$. The corresponding scaling may be extracted from the field renormalization factors ($Z_q^{1/2}$ for each external quark and $Z_3^{1/2}$ for the external gluons) and the normalization factors (inserting g_S, α_S, m_b).

2. Renormalization group equations for the operator basis

We can now explicitly extract the RGEs for the operators under consideration. The operators are ordered as above in Eqs. (4), (5), and (6). Moreover, as far as the chiralities are concerned, we use the ordering LL, LR, RR, RL for four-fermion operators (LL, RR for the tensors) and L, R for the $\bar{b}s$ -gauge operators: we will not write them down explicitly in the following, so as to keep notations as tractable as possible, but we recall that the coefficients come together with one or two chirality indices. The anomalous-dimension matrix can be split into several blocks, corresponding to the various types (four-quark, semileptonic, $\bar{b}s$ -gauge) of operators,

FIG. 12. Contributions from the $(\bar{b}s)$ -gauge operators ($\mathcal{Q}, \mathcal{H}$, and $\mathcal{Q}$) to the $\bar{b}s \rightarrow \bar{l}l/\bar{q}q$ amplitude.

$$\gamma_C = \frac{2\alpha_s C_2(3)}{4\pi} \begin{bmatrix} \tilde{\mathcal{D}}^{ll} - \tilde{\mathcal{M}}^{ll} & 0 & 0 \\ -\tilde{\mathcal{M}}^{ql} & \tilde{\mathcal{D}}^{qq} - \tilde{\mathcal{M}}^{qq} & -\tilde{\mathcal{M}}^{qg} \\ -\tilde{\mathcal{M}}^{gl} & -\tilde{\mathcal{M}}^{gq} & \tilde{\mathcal{D}}^{gg} - \tilde{\mathcal{M}}^{gg} \end{bmatrix}, \quad (C) = (C^l, C^q, C^g), \quad (15)$$

where the meaning of the subindices l , q , and g should be transparent. For the sake of clarity, we choose to present the various blocks along with the specific RGE that they affect, separating the diagonal scaling contributions from the UV-divergent diagrams of Sec. II B. However, should one wish to implement the whole 251×251 γ_C matrix directly without considering our detailed study, the corresponding results are gathered in Appendix B.

(i) $(\bar{b}s)(ll)$ couplings⁹:

$$\frac{d(C^l)}{d \ln \alpha_s} = -\frac{C_2(3)}{\beta_0} \left\{ (C^l) \left[-\frac{1}{4} \text{diag}(71, 71, 71, 191, 191) - \tilde{\mathcal{M}}^{ll} \right] - (C^g) \tilde{\mathcal{M}}^{gl} - (C^q) \tilde{\mathcal{M}}^{ql} \right\} + O(\alpha_s). \quad (16)$$

The matrices $\tilde{\mathcal{M}}^{ll, gl, ql}$ must be extracted from our calculation of the divergent contributions,¹⁰

$$\tilde{\mathcal{M}}^{ll} = \begin{bmatrix} 4\mathbb{1} & 0 & 0 & 0 & 0 \\ 0 & \mathbb{1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -2(1 + \frac{m_s}{m_b} \Sigma) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbb{1} \end{bmatrix} \quad \Sigma \equiv \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad (17)$$

$$\tilde{\mathcal{M}}^{gl} = -\frac{1}{3} Q_l \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ & \dots & & & \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \Xi + \frac{m_s}{m_b} \Phi & 0 & 0 & 0 \\ 0 & 2[\Xi' + \frac{m_s}{m_b} \Phi'] & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ & \dots & & & \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \leftarrow E \\ \dots \\ \leftarrow \mathcal{E}^T \\ \leftarrow \mathcal{H}^S \\ \leftarrow \tilde{\mathcal{H}}^S \\ \leftarrow \mathcal{H}^T \\ \dots \\ \leftarrow \mathcal{Q}^T \end{matrix} \quad \left\{ \begin{matrix} \Xi \equiv \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \\ \Xi' \equiv \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 1 \\ -1 & -1 & 0 & 0 \end{bmatrix} \\ \Phi \equiv \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ -1 & -1 & 0 & 0 \end{bmatrix} \end{matrix} \right. \quad (18)$$

$$\tilde{\mathcal{M}}^{ql} = \frac{3}{2} Q_l Q_q [1; -\tilde{V}_{\text{Fierz}}^q]_{(q=b,s)} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -(1 + \tilde{\Sigma}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(1 + \tilde{\Sigma}) & 0 \\ 0 & \frac{m_q}{m_b} (1 + \tilde{\Sigma}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{3} (1 + \tilde{\Sigma}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{3} (1 + \tilde{\Sigma}) & 0 \\ 0 & \frac{m_q}{3m_b} (1 + \tilde{\Sigma}) & 0 & 0 & 0 \end{bmatrix} \quad \tilde{\Sigma} \equiv \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \quad (19)$$

⁹Note that operators with different lepton flavors do not mix, justifying the use of only one index l .

¹⁰To make our notations more transparent, each entry in the matrices $\tilde{\mathcal{M}}$ corresponds to a 4×4 , 4×2 , 2×4 , or 2×2 block in chirality space corresponding to the type of coefficient it multiplies (row index) and the one it affects (column index). These blocks involve the 4×4 or 2×2 identities, $\mathbb{1}$, as well as several other matrices (Σ , Ξ , Φ , etc.), which we define alongside their first appearance. When we mention operators alongside the matrix $\tilde{\mathcal{M}}^{gl}$ in Eq. (18), it is simply in order to clarify which type of operator corresponds to the given row (since we cut the matrix to skip numerous 0 entries).

We define the object $[1; -\tilde{V}_{\text{Fierz}}^q]_{(q=b,s)}$ as follows: when $q \neq b, s$, it is simply the (36×36) identity in the space defined by the corresponding $(\bar{b}s)(\bar{q}q)$ operators; when $q = b, s$, then it is a (18×36) matrix built with the (18×18) identity in the left-hand sub-block and the (18×18) transition matrix $-\tilde{V}_{\text{Fierz}}^q$, defined in Appendix A, in the right-hand sub-block. Its origin is related to the fact that, for $q = b, s$, Fierz identities shorten the list of independent operators (compared to the cases $q \neq b, s$), and that the additional diagrams “in the t -channel,” which open for $q = b, s$, can be related to contributions that the redundant (omitted) operators would have “in the s -channel”: refer to our discussion in Sec. IIB.

(ii) $(\bar{b}s)(\bar{q}q)$ couplings¹¹:

$$\frac{d(C^q)}{d \ln \alpha_S} = -\frac{C_2(3)}{\beta_0} \left\{ (C^{q'}) \left[\delta^{q'q} [1, 0]_{(q'=b,s)} \text{diag}(5\mathbb{1}, 5\mathbb{1}, 5\mathbb{1}, 2\mathbb{1}, 2\mathbb{1}, 5\mathbb{1}, 5\mathbb{1}, 5\mathbb{1}, 2\mathbb{1}, 2\mathbb{1}) \begin{bmatrix} \mathbb{1} \\ 0 \end{bmatrix}_{(q=b,s)} - \tilde{\mathcal{M}}^{q'q} \right] - (C^g) \tilde{\mathcal{M}}^{gq} \right\} + O(\alpha_S). \quad (20)$$

The objects $[1, 0]_{(q=b,s)}$ and $\begin{bmatrix} \mathbb{1} \\ 0 \end{bmatrix}_{(q=b,s)}$ are again defined as the (36×36) identity when $q \neq b, s$ and the (18×36) and (36×18) (respectively) matrices defined by the two (18×18) sub-blocks corresponding to the identity and the null matrix, when $q = b, s$; their meaning should be clear considering the shortened base in the case $q = b, s$, due to Fierz identities. From our calculation of the divergent contributions, the matrices $\tilde{\mathcal{M}}^{q'q,gq}$ read as follows:

$$\tilde{\mathcal{M}}^{q'q} = \delta^{q'q} [1, 0]_{q'=b,s} \tilde{\mathcal{M}}_{\text{diag}}^{q'q} \begin{bmatrix} \mathbb{1} \\ 0 \end{bmatrix}_{q=b,s} + [1; -\tilde{V}_{\text{Fierz}}^{q'}]_{q'=b,s} \tilde{\mathcal{M}}_{\text{univ}}^{q'q} \begin{bmatrix} \mathbb{1} \\ -\tilde{V}_{\text{Fierz}}^q \end{bmatrix}_{q=b,s}, \quad (21)$$

$$\tilde{\mathcal{M}}_{\text{diag}}^{q'q} = \begin{bmatrix} 8\mathbb{1} & 0 & \frac{1}{32}\Delta & 0 & 0 & 0 & 0 & -\frac{3}{32}\Delta & 0 & 0 \\ 0 & 2\mathbb{1} + \frac{3}{4}\Sigma_3 & 0 & 0 & 0 & 0 & -\frac{9}{4}\Sigma_3 & 0 & 0 & 0 \\ 24\Delta^T & 0 & 0 & 0 & 0 & -72\Delta^T & 0 & 0 & 0 & 0 \\ A_H & B_H & 0 & \mathbb{1} & 0 & C_H & D_H & 0 & 0 & 0 \\ A_{\bar{H}} & B_{\bar{H}} & 0 & 0 & \mathbb{1} & C_{\bar{H}} & D_{\bar{H}} & 0 & 0 & 0 \\ \frac{9}{4}\mathbb{1} & 0 & -\frac{3}{64}\Delta & 0 & 0 & \frac{5}{4}\mathbb{1} & 0 & -\frac{7}{64}\Delta & 0 & 0 \\ 0 & -\frac{9}{8}(\mathbb{1} + \Sigma_3) & 0 & 0 & 0 & 0 & \frac{1}{8}(43\mathbb{1} - 21\Sigma_3) & 0 & 0 & 0 \\ -36\Delta^T & 0 & -\frac{9}{4}\mathbb{1} & 0 & 0 & -84\Delta^T & 0 & \frac{27}{4}\mathbb{1} & 0 & 0 \\ A_{\mathcal{H}} & B_{\mathcal{H}} & 0 & 0 & 0 & C_{\mathcal{H}} & D_{\mathcal{H}} & 0 & \mathbb{1} & 0 \\ A_{\bar{\mathcal{H}}} & B_{\bar{\mathcal{H}}} & 0 & 0 & 0 & C_{\bar{\mathcal{H}}} & D_{\bar{\mathcal{H}}} & 0 & 0 & \mathbb{1} \end{bmatrix}$$

$$\tilde{\mathcal{M}}_{\text{univ}}^{q'q} = \frac{1}{4} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3}(\mathbb{1} + \tilde{\Sigma}) & 0 & 0 & 0 & 0 & -(\mathbb{1} + \tilde{\Sigma}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3}(\mathbb{1} + \tilde{\Sigma}) & 0 & 0 & 0 & 0 & -(\mathbb{1} + \tilde{\Sigma}) & 0 \\ 0 & -\frac{m_q}{3m_b}(\mathbb{1} + \tilde{\Sigma}) & 0 & 0 & 0 & 0 & \frac{m_q}{m_b}(\mathbb{1} + \tilde{\Sigma}) & 0 & 0 & 0 \end{bmatrix}$$

¹¹Note that operators with different quark flavors mix, justifying the use of two quark indices q and q' .

$$\begin{cases}
A_H = \frac{m_q}{4m_b} [3(\mathbb{1} + \tilde{\Sigma}) + \Sigma_3 + E] \\
A_{\tilde{H}} = \frac{1}{4} [3\mathbb{1} + \Sigma_3 + \frac{m_s}{m_b} (3\Sigma + \tilde{E})] \\
A_{\mathcal{H}} = -\frac{3}{8} \frac{m_q}{m_b} [3(\mathbb{1} + \tilde{\Sigma}) + \Sigma_3 + E] \\
A_{\tilde{\mathcal{H}}} = -\frac{3}{8} (3\mathbb{1} + \Sigma_3) - \frac{3}{8} \frac{m_s}{m_b} (3\Sigma + \tilde{E}) \\
B_H = \frac{1}{4} [-8\mathbb{1} + \Sigma_3 + \frac{m_s}{m_b} (-8\Sigma + \tilde{E})] \\
B_{\tilde{H}} = \frac{m_q}{4m_b} [-8(\mathbb{1} + \tilde{\Sigma}) + \Sigma_3 + E] \\
B_{\mathcal{H}} = -\frac{3}{8} (3\mathbb{1} + \Sigma_3) - \frac{3}{8} \frac{m_s}{m_b} (3\Sigma + \tilde{E}) \\
B_{\tilde{\mathcal{H}}} = -\frac{3}{8} \frac{m_q}{m_b} [3(\mathbb{1} + \tilde{\Sigma}) + \Sigma_3 + E] \\
C_H = -\frac{3}{4} \frac{m_q}{m_b} [3(\mathbb{1} + \tilde{\Sigma}) + \Sigma_3 + E] \\
C_{\tilde{H}} = -\frac{3}{4} [(3\mathbb{1} + \Sigma_3) + \frac{m_s}{m_b} (3\Sigma + \tilde{E})] \\
C_{\mathcal{H}} = -\frac{7}{8} \frac{m_q}{m_b} [3(\mathbb{1} + \tilde{\Sigma}) + \Sigma_3 + E] \\
C_{\tilde{\mathcal{H}}} = -\frac{7}{8} (3\mathbb{1} + \Sigma_3) - \frac{7}{8} \frac{m_s}{m_b} (3\Sigma + \tilde{E}) \\
D_H = -\frac{3}{4} (\Sigma_3 + \frac{m_s}{m_b} \tilde{E}) \\
D_{\tilde{H}} = -\frac{3}{4} \frac{m_q}{m_b} (\Sigma_3 + E) \\
D_{\mathcal{H}} = \frac{1}{8} (11\mathbb{1} - 7\Sigma_3) + \frac{m_s}{8m_b} (11\Sigma - 7\tilde{E}) \\
D_{\tilde{\mathcal{H}}} = \frac{m_q}{8m_b} [11(\mathbb{1} + \tilde{\Sigma}) - 7(\Sigma_3 + E)]
\end{cases}
\begin{cases}
\Sigma_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \\
E = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \\
\tilde{E} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \\
\Delta = \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}
\end{cases}$$

$$\tilde{\mathcal{M}}^{qq} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-12 \frac{m_q}{m_b} \Xi & \frac{1}{12} (\Xi + \frac{m_s}{m_b} \Phi) & 0 & 0 & 0 & 0 & -\frac{1}{4} (\Xi + \frac{m_s}{m_b} \Phi) & 0 & 0 & 0 \\
24 \frac{m_q}{m_b} \tilde{\Xi} & \frac{1}{6} (\Xi' + \frac{m_s}{m_b} \Phi') & 0 & 0 & 0 & 0 & -\frac{1}{2} (\Xi' + \frac{m_s}{m_b} \Phi') & 0 & 0 & 0 \\
-\frac{11}{8} \frac{m_q}{m_b} \Xi & \frac{7}{144} (\Xi + \frac{m_s}{m_b} \Phi) & 0 & 0 & 0 & -\frac{15}{8} \frac{m_q}{m_b} \Xi & -\frac{7}{48} (\Xi + \frac{m_s}{m_b} \Phi) & 0 & 0 & 0 \\
\frac{11}{4} \frac{m_q}{m_b} \tilde{\Xi} & \frac{7}{72} (\Xi' + \frac{m_s}{m_b} \Phi') & 0 & 0 & 0 & \frac{15}{4} \frac{m_q}{m_b} \tilde{\Xi} & -\frac{7}{24} (\Xi' + \frac{m_s}{m_b} \Phi') & 0 & 0 & 0 \\
0 & 0 & -\frac{3}{64} \frac{m_q}{m_b} \mathbb{1} & 0 & 0 & 0 & 0 & \frac{9}{64} \frac{m_q}{m_b} \mathbb{1} & 0 & 0
\end{bmatrix}
\begin{matrix}
\leftarrow E \\
\cdots \\
\leftarrow \mathcal{H}^T \\
\leftarrow \mathcal{Q}^D \\
\leftarrow \tilde{\mathcal{Q}}^D \\
\leftarrow \mathcal{Q}^S \\
\leftarrow \tilde{\mathcal{Q}}^S \\
\leftarrow \mathcal{Q}^T
\end{matrix}$$

$$\times \begin{bmatrix} \mathbb{1} \\ -\tilde{V}_{\text{Fierz}}^q \end{bmatrix}_{q=b,s}, \tilde{\Xi} \equiv \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix} \quad (22)$$

The definition and meaning of the notation $\begin{bmatrix} \mathbb{1} \\ -\tilde{V}_{\text{Fierz}}^q \end{bmatrix}_{q=b,s}$ should be obvious by now.

(iii) $(\bar{b}s)$ -gauge couplings:

$$\begin{aligned}
\frac{d(C^g)}{d \ln \alpha_S} = & -\frac{C_2(3)}{\beta_0} \left\{ (C^g) \left[\frac{1}{4} \text{diag}(5\mathbb{1}, 14\mathbb{1}, -19\mathbb{1}, -19\mathbb{1}, -19\mathbb{1}, -10\mathbb{1}, -10\mathbb{1}, -10\mathbb{1}, -1, -1, -1, -1, -1) - \tilde{\mathcal{M}}^{gg} \right] \right. \\
& \left. - (C^g) \tilde{\mathcal{M}}^{gg} \right\} + O(\alpha_S). \quad (23)
\end{aligned}$$

The matrices $\tilde{\mathcal{M}}^{gg,qq}$ proceed from our calculation of the divergent contributions,

$$\tilde{\mathcal{M}}^{gg} = \begin{bmatrix}
 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 -4Q_d \mathbb{1} & \frac{11}{4} \mathbb{1} & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 4\mathbb{1} & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 4\mathbb{1} & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 -\frac{1}{4}\Omega & 0 & -\frac{2}{3}Q_d \mathbb{1} & -\frac{1}{3}Q_d \sigma_3 & 0 & -\frac{55}{24} \mathbb{1} & \frac{11}{48} \sigma_3 \\
 \frac{1}{2}\tilde{\Omega} & 0 & -\frac{4}{3}Q_d \sigma_3 & -\frac{2}{3}Q_d \mathbb{1} & 0 & \frac{11}{12} \sigma_3 & -\frac{55}{24} \mathbb{1} \\
 \left(1 - \frac{m_s^2}{m_b^2}\right) \mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & -\frac{3}{8}\Omega & 0 & 0 & 0 & -Q_d \mathbb{1} & -\frac{1}{2}Q_d \sigma_3 \\
 0 & \frac{3}{4}\tilde{\Omega} & 0 & 0 & 0 & -2Q_d \sigma_3 & -Q_d \mathbb{1} \\
 0 & -\frac{7}{32}\Omega & 0 & 0 & 0 & -\frac{7}{12}Q_d \mathbb{1} & -\frac{7}{24}Q_d \sigma_3 \\
 0 & \frac{7}{16}\tilde{\Omega} & 0 & 0 & 0 & -\frac{7}{6}Q_d \sigma_3 & -\frac{7}{12}Q_d \mathbb{1} \\
 0 & -\frac{9}{8}\left(1 + \frac{m_s^2}{m_b^2}\right) \mathbb{1} & 0 & 0 & 0 & -\frac{9}{2}Q_d \mathbb{1} & \frac{9}{4}Q_d \sigma_3
 \end{bmatrix}$$

$$\begin{bmatrix}
 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 \frac{1}{4}\mathbb{1} & 0 & 0 & 0 & 0 & 0 \\
 0 & \frac{17}{2}\mathbb{1} & 0 & -1 & -\frac{1}{2}\sigma_3 & \frac{9}{2}\mathbb{1} \\
 0 & 0 & \frac{17}{2}\mathbb{1} & -2\sigma_3 & -1 & -9\sigma_3 \\
 0 & \frac{21}{16}\mathbb{1} & -\frac{3}{32}\sigma_3 & \frac{379}{24}\mathbb{1} & -\frac{41}{48}\sigma_3 & \frac{11}{8}\mathbb{1} \\
 0 & -\frac{3}{8}\sigma_3 & \frac{21}{16}\mathbb{1} & -\frac{41}{12}\sigma_3 & \frac{379}{24}\mathbb{1} & -\frac{11}{4}\sigma_3 \\
 0 & \frac{3}{2}\mathbb{1} & -\frac{3}{4}\sigma_3 & \frac{45}{2}\mathbb{1} & -\frac{45}{4}\sigma_3 & -\frac{9}{4}\mathbb{1}
 \end{bmatrix}$$

(24)

$$\begin{cases} \Omega \equiv \left(1 + \frac{m_s^2}{m_b^2}\right) \mathbb{1} - \frac{2}{3} \frac{m_s}{m_b} \sigma_1, \\ \tilde{\Omega} \equiv \left(1 + \frac{m_s^2}{m_b^2}\right) \sigma_3 - \frac{2}{3} \frac{m_s}{m_b} \epsilon, \end{cases} \quad \sigma_1 \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_3 \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \epsilon \equiv \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix},$$

$$\tilde{\mathcal{M}}^{qq} = [\mathbb{1}; -\tilde{V}_{Fierz}^q]_{q=b,s} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 18Q_q \frac{m_q}{m_b} \mathbb{1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 & \dots \\ 0 & 0 & 0 \\ 6Q_q \frac{m_q}{m_b} \mathbb{1} & 6\frac{m_q}{m_b} \mathbb{1} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (25)$$

III. SOLVING THE RGEs

Having presented in the previous section what is meant to be the main result of this paper, i.e., the RGEs for all operators of mass dimension ≤ 7 intervening in $b \rightarrow s$ transitions, we shall now propose a partial solution for these RGEs partial in the sense that we will not provide a general solution for operators of dimension-six but only recover the anomalous-dimension matrix for those operators that are usually considered. On the contrary, for dimension-seven operators, we will present the full result. This section shall also provide us with the opportunity to discuss the limits of our approach and how our extended analysis may be combined with the far more advanced one of dimension-six vector operators.

A. Dimension-six operators

First of all, we wish to check whether we can recover the usual leading-order anomalous-dimension matrix for dimension-six vector operators. We thus consider the basis proposed in Eq. (2.15) of Ref. [12]. These can be viewed as specific linear combinations of ours (including also a rescaling), so that we should be able to read the corresponding anomalous-dimension matrix $\gamma_C^{[1-6]}$ from our

results. For the time being, we forget about the dimension-seven entries.

For the four-quark operators O_{1-6} of [12], we obtain

$$\gamma_C^{[1-6]} = \frac{g_S^2}{8\pi^2} \begin{bmatrix} -1 & 3 & 0 & 0 & 0 & 0 \\ 3 & -1 & -1/9 & 1/3 & -1/9 & 1/3 \\ 0 & 0 & -11/9 & 11/3 & -2/9 & 2/3 \\ 0 & 0 & 22/9 & 2/3 & -5/9 & 5/3 \\ 0 & 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & -5/9 & 5/3 & -5/9 & -19/3 \end{bmatrix}, \quad (26)$$

which coincides with the first 6×6 sub-block of Eq. (3.4) of Ref. [12].

Similarly, we consider the so-called electroweak four-quark operators Q_{7-10} of Eq. (VII.2) of Ref. [3] and recover the matrix elements presented in Table XIV of that reference.

Let us turn to the magnetic and chromomagnetic operators. In our approach, they satisfy the RGE, with $(C_G) \equiv (C_E, C_Q)^{L,R}$,

$$\begin{aligned} \frac{d(C_G)}{d \ln \alpha_S} &= -\frac{C_2(3)}{\beta_0} \left\{ (C_G) \begin{bmatrix} \frac{5}{4} & 0 \\ 4Q_d & \frac{3}{4} \end{bmatrix} - (C^q) \tilde{\mathcal{M}}^{qq} \right\} \\ &\simeq -\frac{C_2(3)}{\beta_0} \left\{ (C_G) \begin{bmatrix} \frac{5}{4} & 0 \\ -\frac{4}{3} & \frac{3}{4} \end{bmatrix} + (C_S^b, C_T^b) \begin{bmatrix} \frac{1}{16} \Delta & -\frac{3}{16} \Delta \\ 7\mathbb{1} & -3\mathbb{1} \end{bmatrix} \right\}, \end{aligned} \quad (27)$$

where we have neglected $\frac{m_q}{m_b} \ll 1$ for $q \neq b$ and replaced $\tilde{V}_{Fierz}^b$ explicitly. The contributions from the four-quark operators that we obtain here have no equivalent in the usual approach: the corresponding operators are simply not considered in the classical case, so that it is pointless for us to keep track of them here.¹² Therefore,

¹²They are symbolically replaced by $(\dots)$ in Eq. (28). Note that Δ was defined in the previous subsection and appears through $\tilde{V}_{Fierz}^q$; see Eqs. (21) and (25).

$$\begin{cases} C_E(\mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{-\frac{5C_2(3)}{4\beta_0}} C_E(\mu_0) + \frac{8}{3} \left[\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{-\frac{3C_2(3)}{4\beta_0}} - \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{-\frac{5C_2(3)}{4\beta_0}} \right] C_Q(\mu_0) + (\dots), \\ C_Q(\mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{-\frac{3C_2(3)}{4\beta_0}} C_Q(\mu_0) + (\dots). \end{cases} \quad (28)$$

To recover the usual scaling of the operators $O_{7,8}$ [12], one has to multiply these coefficients by $\frac{m_b}{\alpha_s}$, which leads to the additional factor $\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{-\frac{11C_2(3)}{4\beta_0}}$. This is, again, consistent with Eq. (3.4) of Ref. [12], for the sub-block 7 – 8.

However, one notices at once an apparent discrepancy with Eq. (3.4) of Ref. [12]: the operators O_{1-6} do not lead to divergences in the $O_{7,8}$ directions. This is not surprising though, in the sense that the corresponding off-diagonal elements appear at the two-loop QCD level, while we considered only the one-loop divergences. One may wonder why such two-loop effects in Ref. [12] are competitive with the diagonal one-loop elements of the O_{7-8} sub-block and, if so, whether it endangers the validity of our one-loop analysis. The explanation is to be found in the relative order of the SM matching for the four-quark and magnetic operators [considering our normalization in Eqs. (5) and (6)]: if both were of the same order, then the two-loop mixing would only generate a subleading effect of relative importance $O(\alpha_s)$, which, for consistency, could be neglected. However, if one refers to the magnitude of the matching elements that we outlined at the end of Sec. II A—which is relevant for, e.g., the SM—one observes that the matching coefficients of the magnetic operators are already of order α_s (translating the fact that it arises at the loop level even though the α_s factor itself is only an artefact of the relative normalization of the operators), so that the two-loop mixing effect, related to the $O(\alpha_s^0)$ four-quark matching, becomes competitive; in other words, the naive hierarchy has been destabilized by the SM matching. More pragmatically, this situation in the SM is related to the fact that the closure of the charm-quark loop does not entail any α_s suppression. In our naive approach, we would naturally miss such an effect. Nevertheless, one also observes from the estimates at the end of Sec. II A that this imbalance among matching conditions only concerns the dimension-six four-quark operators O_{1-6} ¹³, at least in an SM-like model: the corresponding matrix elements are therefore already known and may be included straightforwardly within our analysis. This potentially misleading aspect of our naive approach should, however, be kept in mind in order to combine our findings consistently with the usual dimension-six analysis, directed at realistic high-energy models.

Let us finally consider the semileptonic operators $(C^l) = (C_S^l, C_V^l, C_T^l)$. They satisfy the RGE

$$\frac{d(C^l)}{d \ln \alpha_s} = \frac{C_2(3)}{\beta_0} \left\{ (C^l) \frac{1}{4} \begin{bmatrix} 23\mathbb{1} & 0 & 0 \\ 0 & 11\mathbb{1} & 0 \\ 0 & 0 & 7\mathbb{1} \end{bmatrix} + (C^q) \tilde{\mathcal{M}}_{ql} \right\}, \quad (29)$$

leading to

$$\begin{cases} C_S^l(\mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{23C_2(3)}{4\beta_0}} C_S^l(\mu_0), \\ C_V^l(\mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{11C_2(3)}{4\beta_0}} C_V^l(\mu_0) + (\dots), \\ C_T^l(\mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{7C_2(3)}{4\beta_0}} C_T^l(\mu_0). \end{cases} \quad (30)$$

(...) stands for the inhomogeneous terms due to four-quark operators, which we discuss later. If one factors out α_s^{-1} (for S^l) or $\alpha_s^{-1} m_b$ (for V^l, T^l) in the definition of the operators, the scaling of the coefficients is modified by the factors $\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{-\frac{23C_2(3)}{4\beta_0}}$ and $\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{-\frac{11C_2(3)}{4\beta_0}}$, respectively, leading to the commonly known nonrunning of scalar and vector coefficients; see, e.g., Ref. [31]. Let us now consider the inhomogeneous terms in the classical case of $O_9 \equiv (\bar{b}\gamma^\mu P_L s)(\bar{l}\gamma_\mu l)$ and $O_{10} \equiv (\bar{b}\gamma^\mu P_L s)(\bar{l}\gamma_\mu \gamma_5 l)$. Then we find that the off-diagonal elements of the anomalous-dimension matrix (which translates the mixing with the operators O_{1-6}) are given by $\gamma_C^{i7} = \frac{g_s^2}{8\pi^2} [-\frac{4}{3}, -\frac{4}{9}, -\frac{2}{9}, \frac{10}{9}, -\frac{2}{3}, -\frac{2}{9}]$, $\gamma_C^{i8} = 0$, $i = 1, \dots, 6$. Up to a factor 2 accounting for the relative normalization of the operators, these matrix elements coincide with Eq. (VIII.11) of Ref. [3].

We have thus verified that—except for the mixing of four-quark vector operators with the magnetic and chromomagnetic operators, which is a two-loop effect—our calculation consistently covers all the leading-order elements of the anomalous-dimension matrix for those operators that have been considered in the literature.

Before turning to the solution for the dimension-seven running, it is worthwhile to discuss how dimension-seven corrections to these dimension-six RGEs should be implemented. First, we note that we are talking here about corrections of order $\frac{m_b}{M_Z}$ at most, so that it would make little sense to include them if the dimension-six coefficients were considered at leading order only: dimension-seven effects will make sense numerically only if dimension-six contributions are known up to $O(\alpha_s)$ or even $O(\alpha_s^2)$ (depending on the matching conditions). A second remark involves the absence

¹³Only O_2 is affected at the level of the matching conditions. However, given that O_2 affects the whole O_{1-6} basis via renormalization, the whole subset has to be considered.

of dimension-six effects in the RGEs of dimension-seven operators (in other words, the anomalous-dimension matrix is block-triangular); this naturally proceeds from the fact that the dimension-six basis is stable and does not require dimension-seven counterterms to cancel its divergences (this can be derived from simple power counting). The dimension-seven RGEs can therefore be considered separately while their mixing effect is injected directly as a small inhomogeneous term in the dimension-six RGEs.

B. Dimension-seven RGEs

We present the solution for the RGEs of dimension-seven operators. We stress again that these are independent

from the dimension-six running, which allows us to solve them separately. The block-triangular shape appearing in $\gamma_C^{\text{dim}-7}$ —the restriction of the anomalous-dimension matrix to dimension-seven operators—allows for a splitting into sub-block RGEs with inhomogeneous terms; this observation makes an explicit solution tractable analytically, which we perform for the sake of completeness. Note that the methodology, consisting in splitting the anomalous-dimension matrix according to its block-triangular shape and resulting in inhomogeneous linear equations, is a recurring feature in EFTs and has been used, e.g., in Ref. [32] (although in a different context; see Sec. 3.4.2 of that reference).

1. $(\bar{b}s)$ -gauge operators

Gluonic operators— $(C_Q) \equiv (C_{Q^D}, C_{\bar{Q}^D}, C_{Q^S}, C_{\bar{Q}^S}, C_{Q^T})^{L,R}$:

The RGE is a simple homogeneous first-order differential equation,

$$\begin{aligned} \frac{d(C_Q)}{d \ln \alpha_s} &= \frac{C_2(3)}{\beta_0} (C_Q) \begin{bmatrix} \frac{35}{4} \mathbb{1} & 0 & -1 & -\frac{1}{2} \sigma_3 & \frac{9}{2} \mathbb{1} \\ 0 & \frac{35}{4} \mathbb{1} & -2 \sigma_3 & -1 & -9 \sigma_3 \\ \frac{21}{16} \mathbb{1} & -\frac{3}{32} \sigma_3 & \frac{385}{24} \mathbb{1} & -\frac{41}{48} \sigma_3 & \frac{11}{8} \mathbb{1} \\ -\frac{3}{8} \sigma_3 & \frac{21}{16} \mathbb{1} & -\frac{41}{12} \sigma_3 & \frac{385}{24} \mathbb{1} & -\frac{11}{4} \sigma_3 \\ \frac{3}{2} \mathbb{1} & -\frac{3}{4} \sigma_3 & \frac{45}{2} \mathbb{1} & -\frac{45}{4} \sigma_3 & -2 \mathbb{1} \end{bmatrix} \\ &\equiv \frac{C_2(3)}{\beta_0} (C_Q) \mathcal{A}_{QQ} \\ &\Rightarrow (C_Q)(\mu) = (C_Q)(\mu_0) \exp \left[\frac{C_2(3)}{\beta_0} \mathcal{A}_{QQ} \ln \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right] = (C_Q)(\mu_0) \mathcal{O}_Q \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\frac{C_2(3)}{\beta_0} \mathcal{D}_Q} \mathcal{O}_Q^{-1}, \end{aligned} \quad (31)$$

with $\mathcal{A}_{QQ} \equiv \mathcal{O}_Q \mathcal{D}_Q \mathcal{O}_Q^{-1}$, where $\mathcal{D}_Q$ is a diagonal matrix: $\mathcal{D}_Q = \text{diag}[-\frac{19}{4} \mathbb{1}, \frac{1}{8} (117 - \sqrt{3073}) \mathbb{1}, \frac{1}{24} (277 - \sqrt{3193}) \mathbb{1}, \frac{1}{24} (277 + \sqrt{3193}) \mathbb{1}, \frac{1}{8} (117 + \sqrt{3073}) \mathbb{1}]$. The details of this diagonalization can be found in Appendix C.

Hybrid operators— $(C_H) \equiv (C_{H^S}, C_{\bar{H}^S}, C_{H^T})^{L,R}$:

$$\begin{aligned} \frac{d(C_H)}{d \ln \alpha_s} &= \frac{C_2(3)}{\beta_0} \left\{ (C_H) \begin{bmatrix} \frac{5}{24} \mathbb{1} & \frac{11}{48} \sigma_3 & 0 \\ \frac{11}{12} \sigma_3 & \frac{5}{24} \mathbb{1} & 0 \\ 0 & 0 & \frac{11}{4} \mathbb{1} \end{bmatrix} \right\} - Q_d(C_Q) \begin{bmatrix} 1 & \frac{1}{2} \sigma_3 & 0 \\ 2 \sigma_3 & 1 & 0 \\ \frac{7}{12} \mathbb{1} & \frac{7}{24} \sigma_3 & 0 \\ \frac{7}{2} \sigma_3 & \frac{7}{12} \mathbb{1} & 0 \\ \frac{9}{2} \mathbb{1} & -\frac{9}{4} \sigma_3 & 0 \end{bmatrix} \\ &\equiv \frac{C_2(3)}{\beta_0} \{ (C_H) \mathcal{A}_{HH} - (C_Q) \mathcal{A}_{QH} \} \\ &\Rightarrow (C_H)(\mu) = \left\{ (C_H)(\mu_0) - \frac{C_2(3)}{\beta_0} \int_0^{\ln(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)})} (C_Q)(\mu') \mathcal{A}_{QH} \exp \left[-\frac{C_2(3)}{\beta_0} \mathcal{A}_{HH} \ln \frac{\alpha_s(\mu')}{\alpha_s(\mu_0)} \right] d \ln \left(\frac{\alpha_s(\mu')}{\alpha_s(\mu_0)} \right) \right\} \\ &\quad \times \exp \left[\frac{C_2(3)}{\beta_0} \mathcal{A}_{HH} \ln \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right]. \end{aligned} \quad (32)$$

$$\Rightarrow (C_H)(\mu) = \left\{ (C_H)(\mu_0) - \frac{C_2(3)}{\beta_0} \int_0^{\ln(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)})} (C_Q)(\mu') \mathcal{A}_{QH} \exp \left[-\frac{C_2(3)}{\beta_0} \mathcal{A}_{HH} \ln \frac{\alpha_s(\mu')}{\alpha_s(\mu_0)} \right] d \ln \left(\frac{\alpha_s(\mu')}{\alpha_s(\mu_0)} \right) \right\} \times \exp \left[\frac{C_2(3)}{\beta_0} \mathcal{A}_{HH} \ln \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right]. \quad (33)$$

The exponentiation of $\mathcal{A}_{\mathcal{H}\mathcal{H}}$ can be achieved without difficulty through its diagonalization: $\mathcal{A}_{\mathcal{H}\mathcal{H}} = \mathcal{O}_{\mathcal{H}} D_{\mathcal{H}} \mathcal{O}_{\mathcal{H}}^{-1}$, with $\mathcal{D}_H \equiv \text{diag}[-\frac{1}{4}\mathbb{1}, \frac{2}{3}\mathbb{1}, \frac{11}{4}\mathbb{1}]$ (refer to Appendix C).

Photonic operators— $(C_{\mathcal{E}}) \equiv (C_{\mathcal{E}^S}, C_{\mathcal{E}^{\bar{S}}}, C_{\mathcal{E}^T})^{L,R}$:

$$\frac{d(C_{\mathcal{E}})}{d \ln \alpha_S} = \frac{C_2(3)}{\beta_0} \left\{ (C_{\mathcal{E}}) \frac{1}{4} \begin{bmatrix} 35\mathbb{1} & 0 & 0 \\ 0 & 35\mathbb{1} & 0 \\ 0 & 0 & 19\mathbb{1} \end{bmatrix} - (C_{\mathcal{H}}) Q_d \begin{bmatrix} \frac{2}{3}\mathbb{1} & \frac{1}{3}\sigma_3 & 0 \\ \frac{4}{3}\sigma_3 & \frac{2}{3}\mathbb{1} & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\} \quad (34)$$

$$\Rightarrow (C_{\mathcal{E}})(\mu) = (C_{\mathcal{E}})(\mu_0) \begin{bmatrix} \left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)^{\frac{35C_2(3)}{4\beta_0}} \mathbb{1} & 0 & 0 \\ 0 & \left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)^{\frac{35C_2(3)}{4\beta_0}} \mathbb{1} & 0 \\ 0 & 0 & \left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)^{\frac{19C_2(3)}{4\beta_0}} \mathbb{1} \end{bmatrix} - \frac{C_2(3)Q_d}{\beta_0} \left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)^{\frac{35C_2(3)}{4\beta_0}} \int_0^{\ln \left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)} (C_{\mathcal{H}})(\mu') \left(\frac{\alpha_S(\mu')}{\alpha_S(\mu_0)}\right)^{-\frac{35C_2(3)}{4\beta_0}} d \ln \left(\frac{\alpha_S(\mu')}{\alpha_S(\mu_0)}\right) \begin{bmatrix} \frac{2}{3}\mathbb{1} & \frac{1}{3}\sigma_3 & 0 \\ \frac{4}{3}\sigma_3 & \frac{2}{3}\mathbb{1} & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (35)$$

Interestingly, some of these operators have a negative anomalous dimension, meaning that they will grow at low energy, and thus become important. Note however that this negative dimension may simply be an artifact of artificial factors α_S^{-1} in the normalization of the operators: the matching conditions in a definite model are therefore likely to correct this feature by introducing suppression factors. Otherwise, such growing directions in the dimension-seven anomalous-dimension matrix could lead to a significant perturbation of the dimension-six results. While the computation of the anomalous dimension is particularly simple, e.g., in the case of the photonic operators (leaving little room for false moves), the prospect of having enhanced dimension-seven contributions at low energy should be considered with caution.

2. Dimension-seven $(\bar{b}s)(\bar{q}q)$ operators

$$(C_H^q) \equiv (C_{H^q}^q, C_{\bar{H}^q}^q, C_{\mathcal{H}^q}^q, C_{\bar{\mathcal{H}}^q}^q)^{L,RL,R}.$$

$$\begin{aligned} \frac{d(C_H^q)}{d \ln \alpha_S} &= -\frac{C_2(3)}{\beta_0} \left\{ (C_H^q) - (C_H^{q'}) \frac{1}{4} [1; -\tilde{V}_{\text{Fierz}}^{q'}]_{q'=b,s} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{3}(1 + \tilde{\Sigma}) & 0 & -(1 + \tilde{\Sigma}) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -\tilde{V}_{\text{Fierz}}^q \end{bmatrix}_{q=b,s} \right\} \\ &\equiv -\frac{C_2(3)}{\beta_0} (C_H^q) [1 + \mathcal{A}_{qq}] \\ &\Rightarrow (C_H^q)(\mu) = (C_H^q)(\mu_0) \exp \left[-\frac{C_2(3)}{\beta_0} (1 + \mathcal{A}_{qq}) \ln \frac{\alpha_S(\mu')}{\alpha_S(\mu_0)} \right]. \end{aligned} \quad (36)$$

$\mathcal{A}_{qq}$ can be explicitly diagonalized: $\mathcal{A}_{qq} = \mathcal{O}_q \text{diag}(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{8}{3}\mathcal{D}_{24}) \mathcal{O}_q^{-1}$ (refer to Appendix C).

3. Dimension-seven $(\bar{b}s)(\bar{l}l)$ operators

$$(C_H^l) \equiv (C_{H^l}^l, C_{\bar{H}^l}^l):$$

$$\frac{d(C_H^l)}{d \ln \alpha_s} = \frac{C_2(3)}{4\beta_0} (C_H^l) \begin{bmatrix} 19\mathbb{1} & 0 \\ 0 & 23\mathbb{1} \end{bmatrix} - \frac{2Q_l Q_q}{\beta_0} (C_H^q) [\mathbb{1}; -\tilde{V}_{\text{Fierz}}^q]_{q=b,s} \begin{bmatrix} \mathbb{1} + \tilde{\Sigma} & 0 \\ 0 & 0 \\ \frac{1}{3}(\mathbb{1} + \tilde{\Sigma}) & 0 \\ 0 & 0 \end{bmatrix} \quad (37)$$

$$\Rightarrow (C_H^l)(\mu) = (C_H^l)(\mu_0) \begin{bmatrix} \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{19C_2(3)}{4\beta_0}} \mathbb{1} & 0 \\ 0 & \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{23C_2(3)}{4\beta_0}} \mathbb{1} \end{bmatrix} - \frac{2Q_l Q_q}{\beta_0} \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{19C_2(3)}{4\beta_0}} \int_0^{\ln \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)} (C_H^q)(\mu') \left(\frac{\alpha_s(\mu')}{\alpha_s(\mu_0)}\right)^{-\frac{19C_2(3)}{4\beta_0}} d \ln \left(\frac{\alpha_s(\mu')}{\alpha_s(\mu_0)}\right) \times [\mathbb{1}; -\tilde{V}_{\text{Fierz}}^q]_{q=b,s} \begin{bmatrix} \mathbb{1} + \tilde{\Sigma} & 0 \\ 0 & 0 \\ \frac{1}{3}(\mathbb{1} + \tilde{\Sigma}) & 0 \\ 0 & 0 \end{bmatrix}. \quad (38)$$

Note that the positive eigenvalues (negative anomalous dimensions) change sign once the artificial factor α_s^{-1} is factored away from the operators.

This concludes the solution of all dimension-seven RGEs. We will now consider a simple example featuring these dimension-seven effects.

IV. A SIMPLE APPLICATION

A. Setup

Let us assume that leading new physics effects would arise in the possible (though unlikely) form of the following dimension-six SM operator [28]:

$$\mathcal{L}_{\text{NP}} = -\frac{iK_{sb}}{M_Z^2} H^\dagger \frac{g' B_{\mu\nu} + 2g W_{\mu\nu}^a \tau^a}{\sqrt{g^2 + g'^2}} \bar{s} \sigma^{\mu\nu} P_L \begin{pmatrix} t \\ b \end{pmatrix} + \text{H.c.}, \quad (39)$$

where g' and g are the $U(1)_Y$ and $SU(2)_L$ coupling constants, $B_{\mu\nu}$ and $W_{\mu\nu}^a \tau^a$ are the corresponding field-strength tensors [the τ^a correspond to the $SU(2)_L$ generators], while H is the Higgs doublet (with negative hypercharge). K_{sb} is a complex coefficient encoding new physics effects. At low energy, it results in a modified Z - b - s coupling,

$$\frac{iK_{sb}^* v}{M_Z^2} [\bar{b} \sigma^{\mu\nu} P_R s] (\partial_\mu Z_\nu - \partial_\nu Z_\mu) + \text{H.c.}, \quad (40)$$

where $v = (2\sqrt{2}G_F)^{-1/2}$ is the electroweak vacuum expectation value. Such a term could generate flavor

effects at the Z -pole, resulting in a perturbation of the precision measurements (such as $Z \rightarrow \bar{b}s$, $\bar{s}b$ being mistaken experimentally for a $Z \rightarrow \bar{b}b$ decay).

Considering the modified Z coupling, we compute the associated matching conditions, at tree level, for the operators of the $b \rightarrow s$ transition,

$$\begin{cases} \delta^{\text{NP}} C_{Vf}^{LL}(M_Z) = -4\sqrt{2}(I_3^f - Q^f s_W^2) \frac{m_s K_{sb}^*}{M_Z^3} \simeq 0, \\ \delta^{\text{NP}} C_{Vf}^{LR}(M_Z) = 4\sqrt{2} Q^f s_W^2 \frac{m_s K_{sb}^*}{M_Z^3} \simeq 0, \\ \delta^{\text{NP}} C_{Vf}^{RR}(M_Z) = -4\sqrt{2} Q^f s_W^2 \frac{m_b(M_Z) K_{sb}^*}{M_Z^3}, \\ \delta^{\text{NP}} C_{Vf}^{RL}(M_Z) = 4\sqrt{2}(I_3^f - Q^f s_W^2) \frac{m_b(M_Z) K_{sb}^*}{M_Z^3}, \\ \delta^{\text{NP}} C_{Hf}^{LL}(M_Z) = 0, \\ \delta^{\text{NP}} C_{Hf}^{LR}(M_Z) = 0, \\ \delta^{\text{NP}} C_{Hf}^{RR}(M_Z) = 4\sqrt{2} Q^f s_W^2 \frac{K_{sb}^*}{M_Z^3}, \\ \delta^{\text{NP}} C_{Hf}^{RL}(M_Z) = -4\sqrt{2}(I_3^f - Q^f s_W^2) \frac{K_{sb}^*}{M_Z^3}, \end{cases} \quad (41)$$

where we here use the following normalization of the operators (f stands for any of the low-energy quarks and leptons):

$$\begin{aligned} (V^f)^{L,RL,R} &= (\bar{b} \gamma^\mu P_{L,R} s) (\bar{f} \gamma_\mu P_{L,R} f), \\ (H^f)^{L,RL,R} &= [\bar{b} i(\vec{D} - \vec{D})^\mu P_{L,R} s] (\bar{f} \gamma_\mu P_{L,R} f). \end{aligned} \quad (42)$$

We note at this point that the classical framework of dimension-six vector-type operators would not allow for

a consistent description of these new physics effects: even though the $(V^f)^{L,RL,R}$ operators receive a contribution, the associated effect is of the same order as that of the dimension-seven $(H^f)^{L,RL,R}$ operators. One should thus rely on our extended analysis.

B. $\text{Br}(B_s \rightarrow l^+ l^-)$

Now let us consider the decay $B_s \rightarrow l^+ l^-$. This rate is one of the traditional search channels for new physics. Evidence for its observation (in the case $l = \mu$) was reported a few months ago at LHCb [1], showing good agreement with the SM and hence constraining new physics effects more tightly. For a recent review, see Ref. [2]. The well-known SM matching provides [3] (the function Y is also defined in Ref. [3])

$$\begin{aligned} C_{V^l}^{LL}(M_Z) &\simeq -\frac{\sqrt{2}G_F\alpha}{\pi s_W^2} V_{ts}^* V_{tb} Y \left(\frac{m_l^2}{M_W^2} \right) \\ &\simeq -6.2 \times 10^{-9} \equiv C_{V^l}^{\text{SM}}. \end{aligned} \quad (43)$$

We (safely) neglect dimension-seven effects associated with the SM and assume that only new physics

associated with K_{sb} may lead to a significant deviation.

Then, we consider the following matrix elements (in terms of the B_s -meson decay constant f_B , and its four-momentum P_μ):

$$\begin{aligned} \langle 0 | \bar{b} \gamma_5 s | B_s \rangle &= -i f_B \frac{M_B^2}{m_b + m_s}, \\ \langle 0 | \bar{b} s | B_s \rangle &= 0, \\ \langle 0 | \bar{b} \gamma_\mu \gamma_5 s | B_s \rangle &= i f_B P_\mu, \\ \langle 0 | \bar{b} \gamma_\mu s | B_s \rangle &= 0, \\ \langle 0 | \bar{b} l (\vec{D} - \vec{\bar{D}})_\mu \gamma_5 s | B_s \rangle &= i f_B (m_b - m_s) P_\mu, \\ \langle 0 | \bar{b} l (\vec{D} - \vec{\bar{D}})_\mu s | B_s \rangle &= 0. \end{aligned} \quad (44)$$

Here we have followed the conventions of Ref. [33] and derived the result for the matrix elements $\langle 0 | \bar{b} l (\vec{D} - \vec{\bar{D}})_\mu \mathbb{1} / \gamma_5 s | B_s \rangle$. One then derives (where we consider only the operators V^l and H^l , which are relevant in this particular example)

$$\text{Br}(B_s \rightarrow l^+ l^-) \simeq \frac{f_B^2 M_B m_l^2}{32\pi\Gamma_B} \sqrt{1 - 4 \frac{m_l^2}{M_B^2}} |C_{V^l}^{LR} - C_{V^l}^{LL} + C_{V^l}^{RL} - C_{V^l}^{RR} + m_b (C_{H^l}^{LR} - C_{H^l}^{LL} + C_{H^l}^{RL} - C_{H^l}^{RR})|^2 (\mu_b). \quad (45)$$

Note that for the combinations $C_{V^l}^{L,RL} - C_{V^l}^{L,RR}$ and $C_{H^l}^{L,RL} - C_{H^l}^{L,RR}$, the mixing with the four-quark operators cancels in the RGE [Eq. (16), due to the $\mathbb{1} + \tilde{\Sigma}$ in $\tilde{\mathcal{M}}^{q,l}$], so that we obtain simple scalings for both of these quantities: $(\frac{\alpha_S(\mu_b)}{\alpha_S(M_Z)})^0 = 1$ for $C_{V^l}^{L,RL} - C_{V^l}^{L,RR}$ and $(\frac{\alpha_S(\mu_b)}{\alpha_S(M_Z)})^{-\frac{C_2(3)}{\beta_0}} = (\frac{\alpha_S(\mu_b)}{\alpha_S(M_Z)})^{-\frac{4}{23}}$ for $C_{H^l}^{L,RL} - C_{H^l}^{L,RR}$. Note however the off-diagonal term in $\tilde{\mathcal{M}}^{ll}$ that mixes the V^l with the H^l operator. Hence, the RGE for the vector coefficients reads

$$\begin{aligned} \frac{d}{d \ln \alpha_S} [C_{V^l}^{L,RL} - C_{V^l}^{L,RR}] &= -\frac{2C_2(3)}{\beta_0} m_b [C_{H^l}^{L,RL} - C_{H^l}^{L,RR}] \Rightarrow [C_{V^l}^{L,RL} - C_{V^l}^{L,RR}](\mu_b) \\ &= [C_{V^l}^{L,RL} - C_{V^l}^{L,RR}](M_Z) - \frac{2C_2(3)}{\beta_0} m_b(M_Z) [C_{H^l}^{L,RL} - C_{H^l}^{L,RR}](M_Z) \int_0^{\ln(\frac{\alpha_S(\mu_b)}{\alpha_S(M_Z)})} e^{\frac{2C_2(3)}{\beta_0} x} dx \\ &= [C_{V^l}^{L,RL} - C_{V^l}^{L,RR}](M_Z) - m_b(M_Z) [C_{H^l}^{L,RL} - C_{H^l}^{L,RR}](M_Z) \left[\left(\frac{\alpha_S(\mu_b)}{\alpha_S(M_Z)} \right)^{\frac{2C_2(3)}{\beta_0}} - 1 \right]. \end{aligned} \quad (46)$$

It is also more convenient to work with a low-energy b mass: $m_b = m_b(M_Z) (\frac{\alpha_S(\mu_b)}{\alpha_S(M_Z)})^{\frac{3C_2(3)}{\beta_0}}$. Note that this replacement does not concern the explicit factor m_b multiplying the H^l coefficients in Eq. (45). Replacing the coefficients at the M_Z scale by the matching conditions, we then observe a complete cancellation of the new physics contributions,

$$\text{Br}(B_s \rightarrow l^+ l^-) \simeq \frac{f_B^2 M_B m_l^2}{32\pi\Gamma_B} \sqrt{1 - 4 \frac{m_l^2}{M_B^2}} |C_{V^l}^{\text{SM}}|^2. \quad (47)$$

Therefore, the scenario under investigation receives no constraint from $\text{Br}(B_s \rightarrow l^+ l^-)$. Although this is what the

naive tree-level calculation would have predicted, it is a nontrivial result to observe that this feature is preserved by the resummation of the leading logarithms through the RGEs: inconsistently neglecting the H contributions or the V - H mixing, for instance, would have generated limits of the order $|K_{sb}| \lesssim 10^{-3}$. Note however that the cancellation of the new physics effects is tightly related to the form of the matching conditions of Eq. (41), and that a perturbation of Eq. (41)—e.g., through the implementation of additional dimension-six operators—would again lead to relevant limits.

C. $\text{Br}(B \rightarrow K\nu\bar{\nu})$

Many other $b \rightarrow s$ transitions are of course relevant to constrain K_{sb} . Let us consider another simple example: the

decay $B \rightarrow K\nu\bar{\nu}$ is bounded by the limit $\text{Br}(B^- \rightarrow K^- + \text{Inv}) < 13 \times 10^{-6}$ (90% C.L.) from *BABAR* [34]. The SM provides $\text{Br}(B^- \rightarrow K^- \nu\bar{\nu}) \approx (4.4^{+1.3+0.8+0.0}_{-1.1-0.7-0.7}) \times 10^{-6}$; see Ref. [35] for a summary. The relevant matching coefficient for the SM can be found in Ref. [3] (the function X is also defined there),

$$C_{V\nu}^{LL}(M_Z) \simeq \frac{\sqrt{2}G_F\alpha}{\pi s_W^2} V_{ts}^* V_{tb} X\left(\frac{m_t^2}{M_W^2}\right) \simeq 1 \times 10^{-8} \equiv C_{V\nu}^{\text{SM}}. \quad (48)$$

Concerning the decay constants, we again follow the conventions of Ref. [33] and derive the matrix element corresponding to the H -type operators,

$$\begin{aligned} \langle K(P_K) | \bar{s} \gamma_\mu b | B(P_B) \rangle &= [P_B + P_K]_\mu f_+(s) + \frac{M_B^2 - M_K^2}{s} [P_B - P_K]_\mu (f_0(s) - f_+(s)), \quad s \equiv (P_B - P_K)^2, \\ \langle K(P_K) | \bar{s} \frac{i}{2} \sigma_{\mu\nu} (P_B - P_K)^\nu b | B(P_B) \rangle &= i \left\{ [P_B + P_K]_\mu \frac{s}{M_B + M_K} - [P_B - P_K]_\mu (M_B - M_K) \right\} f_T(s), \\ \langle K(P_K) | \bar{s} i (\vec{D} - \vec{\bar{D}})_\mu b | B(P_B) \rangle &\simeq [P_B + P_K]_\mu \left\{ m_b f_+(s) - \frac{s f_T(s)}{M_B + M_K} \right\} \\ &\quad + [P_B - P_K]_\mu \left\{ m_b \frac{M_B^2 - M_K^2}{s} (f_0(s) - f_+(s)) + (M_B - M_K) f_T(s) \right\}. \end{aligned} \quad (49)$$

For the form factors, we closely follow the discussion in Ref. [35] and employ the parametrization of Ref. [36] and the approximate relation $f_T(s)/f_+(s) \simeq \frac{M_B + M_K}{M_B}$.

The estimation of the branching ratio is now straightforward ($N_\nu = 3$ is the number of neutrino flavors),

$$\begin{aligned} \text{Br}(B \rightarrow K\nu\bar{\nu}) &= \frac{N_\nu/3}{512\pi^3 M_B^3 \Gamma_B} \int_0^{(M_B - M_K)^2} ds [s^2 - 2s(M_B^2 + M_K^2) + (M_B^2 - M_K^2)^2]^{3/2} \\ &\quad \times \left| [C_{V\nu}^{LL} + C_{V\nu}^{RL} + m_b(C_{H\nu}^{LL} + C_{H\nu}^{RL})] - \frac{s f_T(s)}{M_B + M_K} (C_{H\nu}^{LL} + C_{H\nu}^{RL}) \right|^2 (\mu_b), \end{aligned} \quad (50)$$

where we will use $M_B = 5.27925(17)$ GeV, $\tau_B = (1.641 \pm 0.008) \times 10^{-12}$ s, and $M_K = 0.493677(16)$ GeV [37].

The running of the coefficients is again very simple, as the neutrality of the neutrinos ensures that there is no mixing with the four-quark operators. Similarly to Eq. (46), we have

$$\begin{aligned} [C_{V\nu}^{LL} + C_{V\nu}^{RL}](\mu_b) &= [C_{V\nu}^{LL} + C_{V\nu}^{RL}](M_Z) - m_b(M_Z) [C_{H\nu}^{LL} + C_{H\nu}^{RL}](M_Z) \left[\left(\frac{\alpha_S(\mu_b)}{\alpha_S(M_Z)} \right)^{\frac{2C_2(3)}{\beta_0}} - 1 \right], \\ [C_{H\nu}^{LL} + C_{H\nu}^{RL}](\mu_b) &= \left(\frac{\alpha_S(\mu_b)}{\alpha_S(M_Z)} \right)^{\frac{C_2(3)}{\beta_0}} [C_{H\nu}^{LL} + C_{H\nu}^{RL}](M_Z). \end{aligned} \quad (51)$$

Recalling our explicit matching conditions in Eq. (41), we observe that new physics effects persist here in the part multiplying $f_T(s)$. Numerically, we find K_{sb} (which we assume to be real) in the range $[-4, 1.5] \times 10^{-3}$. The naive estimate $K_{sb} \sim 10^{-3} \sim \frac{M_Z^2}{\Lambda_{NP}^2}$ returns $\Lambda_{NP} \sim \text{TeV}$; this scale

can be lowered further if one assumes that the new physics operators are loop suppressed, that they follow minimal flavor violation, etc. The bound on new physics appearing in this fashion is therefore relatively loose. Note however that several other observables in the $b \rightarrow s$ sector, e.g.,

$\bar{B} \rightarrow X_s l^+ l^-$, should be considered before drawing any conclusion concerning the viability of $K_{sb} \sim 10^{-3}$. Since our focus in this section was merely to consider a concrete yet simple case where the inclusion of the dimension-seven RGEs was relevant, we shall not pursue this analysis further.

Let us briefly summarize our achievements. We have established the most general basis of (on-shell) operators for the EFT describing $b \rightarrow s$ transitions up to mass dimension seven. After computing the associated ultraviolet QCD divergences at leading order, we derived the corresponding RGEs. A comparison with the existing studies concerning dimension-six operators proved satisfactory. We have also solved all the RGEs describing the evolution of pure dimension-seven operators; interestingly, we observed that some directions exhibited negative anomalous dimensions, which could lead to an enhancement effect at low energy. Note however that this property depends on the normalization of the operators so that the matching conditions in a definite high-energy model are likely to regulate it. Finally, we used this analysis to constrain—using the measurement of the decay $B_s^0 \rightarrow \mu^+ \mu^-$ at LHCb and the limit set by BABAR on $B \rightarrow K \nu \bar{\nu}$ —a very naive extension of the SM resulting

from the addition of a dimension-six/SM operator. More generally, let us recall that the inclusion of dimension-seven effects has little relevance in, e.g., the SM, where such operators receive extra suppression due to the very constrained pattern of flavor violation. Beyond the SM, allowing new physics to only project onto operators of higher mass dimension would be an elegant way of circumventing the strong limits on nonstandard flavor violation; however, such models would have to be specifically designed for this purpose.

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APPENDIX A: FIERZ IDENTITIES

We derive/recover the following Fierz identities¹⁴:

(i) Scalar identities:

$$\begin{cases} (\bar{\psi}' P_L \psi) (\bar{\chi}' P_L \chi) = \frac{1}{2} (\bar{\psi}' P_L \chi) (\bar{\chi}' P_L \psi) - \frac{1}{32} (\bar{\psi}' \sigma^{\mu\nu} P_L \chi) (\bar{\chi}' \sigma_{\mu\nu} P_L \psi), \\ (\bar{\psi}' P_R \psi) (\bar{\chi}' P_R \chi) = \frac{1}{2} (\bar{\psi}' P_R \chi) (\bar{\chi}' P_R \psi) - \frac{1}{32} (\bar{\psi}' \sigma^{\mu\nu} P_R \chi) (\bar{\chi}' \sigma_{\mu\nu} P_R \psi), \\ (\bar{\psi}' P_L \psi) (\bar{\chi}' P_R \chi) = \frac{1}{2} (\bar{\psi}' \gamma^\mu P_R \chi) (\bar{\chi}' \gamma_\mu P_L \psi), \\ (\bar{\psi}' P_R \psi) (\bar{\chi}' P_L \chi) = \frac{1}{2} (\bar{\psi}' \gamma^\mu P_L \chi) (\bar{\chi}' \gamma_\mu P_R \psi). \end{cases} \quad (\text{A1})$$

(ii) Vector identities:

$$\begin{cases} (\bar{\psi}' \gamma^\mu P_L \psi) (\bar{\chi}' \gamma_\mu P_L \chi) = -(\bar{\psi}' \gamma^\mu P_L \chi) (\bar{\chi}' \gamma_\mu P_L \psi), \\ (\bar{\psi}' \gamma^\mu P_R \psi) (\bar{\chi}' \gamma_\mu P_R \chi) = -(\bar{\psi}' \gamma^\mu P_R \chi) (\bar{\chi}' \gamma_\mu P_R \psi), \\ (\bar{\psi}' \gamma^\mu P_L \psi) (\bar{\chi}' \gamma_\mu P_R \chi) = 2(\bar{\psi}' P_R \chi) (\bar{\chi}' P_L \psi), \\ (\bar{\psi}' \gamma^\mu P_R \psi) (\bar{\chi}' \gamma_\mu P_L \chi) = 2(\bar{\psi}' P_L \chi) (\bar{\chi}' P_R \psi). \end{cases} \quad (\text{A2})$$

(iii) Tensor identities:

$$\begin{cases} (\bar{\psi}' \sigma^{\mu\nu} P_L \psi) (\bar{\chi}' \sigma_{\mu\nu} P_L \chi) = -24(\bar{\psi}' P_L \chi) (\bar{\chi}' P_L \psi) - \frac{1}{2} (\bar{\psi}' \sigma^{\mu\nu} P_L \chi) (\bar{\chi}' \sigma_{\mu\nu} P_L \psi), \\ (\bar{\psi}' \sigma^{\mu\nu} P_R \psi) (\bar{\chi}' \sigma_{\mu\nu} P_R \chi) = -24(\bar{\psi}' P_R \chi) (\bar{\chi}' P_R \psi) - \frac{1}{2} (\bar{\psi}' \sigma^{\mu\nu} P_R \chi) (\bar{\chi}' \sigma_{\mu\nu} P_R \psi). \end{cases} \quad (\text{A3})$$

¹⁴Note that here we only consider the identities from the point of view of the Clifford algebra and do not include the $-$ sign resulting from the anticommutation of fermion fields.

(iv) Hybrid identities:

$$\left\{ \begin{array}{l} (\bar{\psi}' P_L \not{D}^\mu \psi) (\bar{\chi}' \gamma_\mu P_L \chi) = \frac{1}{2} (\bar{\psi}' P_L \chi) (\bar{\chi}' P_R \not{D} \psi) - \frac{1}{4} (\bar{\psi}' \sigma^{\mu\nu} P_L \chi) (\bar{\chi}' \gamma_\nu P_L \not{D}_\mu \psi), \\ (\bar{\psi}' P_R \not{D}^\mu \psi) (\bar{\chi}' \gamma_\mu P_R \chi) = \frac{1}{2} (\bar{\psi}' P_R \chi) (\bar{\chi}' P_L \not{D} \psi) - \frac{1}{4} (\bar{\psi}' \sigma^{\mu\nu} P_R \chi) (\bar{\chi}' \gamma_\nu P_R \not{D}_\mu \psi), \\ (\bar{\psi}' P_L \not{D}^\mu \psi) (\bar{\chi}' \gamma_\mu P_R \chi) = (\bar{\psi}' \gamma^\mu P_R \chi) (\bar{\chi}' P_L \not{D}_\mu \psi) - \frac{1}{2} (\bar{\psi}' \gamma^\mu P_R \chi) (\bar{\chi}' \gamma_\mu P_R \not{D} \psi), \\ (\bar{\psi}' P_R \not{D}^\mu \psi) (\bar{\chi}' \gamma_\mu P_L \chi) = (\bar{\psi}' \gamma^\mu P_L \chi) (\bar{\chi}' P_R \not{D}_\mu \psi) - \frac{1}{2} (\bar{\psi}' \gamma^\mu P_L \chi) (\bar{\chi}' \gamma_\mu P_L \not{D} \psi). \end{array} \right. \quad (\text{A4})$$

One can straightforwardly generalize these results to the case where the covariant derivative acts on the first spinor instead of the second. Moreover, with a little bit of algebra,

$$\begin{aligned} & (\bar{\psi}' \sigma^{\mu\nu} P_{L,R} \chi) (\bar{\chi}' \gamma_\nu P_{L,R} \not{D}_\mu \psi) \\ &= -[\bar{\psi}' \not{D} (\vec{D} - \vec{D})_\mu P_{L,R} \chi] (\bar{\chi}' \gamma^\mu P_{L,R} \psi) - [\bar{\psi}' \not{D} (\vec{D} - \vec{D})_\mu \gamma_5 P_{L,R} \chi] (\bar{\chi}' \gamma^\mu \gamma_5 P_{L,R} \psi) \\ &\quad - [\bar{\psi}' (\not{D} \gamma^\mu P_{L,R} - \gamma^\mu P_{R,L} \not{D}) \chi] (\bar{\chi}' \gamma_\mu P_{L,R} \psi) - [\bar{\psi}' (\not{D} \gamma^\mu \gamma_5 P_{L,R} + \gamma^\mu \gamma_5 P_{R,L} \not{D}) \chi] (\bar{\chi}' \gamma_\mu \gamma_5 P_{L,R} \psi) \\ &\quad - \frac{1}{4} (\bar{\psi}' \sigma^{\mu\nu} P_{L,R} \chi) [\bar{\chi}' (\vec{D} \sigma_{\mu\nu} P_{L,R} + \sigma_{\mu\nu} P_{R,L} \vec{D}) \psi] - \frac{1}{4} i \partial_\rho [(\bar{\psi}' \sigma_{\mu\nu} P_{L,R} \chi) (\bar{\chi}' \gamma^\rho \gamma^\nu) \gamma^\mu P_{L,R} \psi]. \end{aligned} \quad (\text{A5})$$

Consequently, for $q = b, s$, $\mathcal{O}_{L,RL,R}^q = \mathcal{S}, \mathcal{V}, \mathcal{T}, \mathcal{H}, \tilde{\mathcal{H}}_{L,RL,R}^q$ can be expressed in terms of $\mathcal{O}_{L,RL,R}^q = \mathcal{S}, \mathcal{V}, \mathcal{T}, \mathcal{H}, \tilde{\mathcal{H}}_{L,RL,R}^q$ as $\mathcal{O}_{L,RL,R}^q = -\tilde{\mathcal{V}}_{\text{Fierz}}^q \mathcal{O}_{L,RL,R}^q$ (the sign originates from the necessity of anticommuting fermion fields), and vice versa. Chiralities are ordered as LL, LR, RR, RL .

$$\tilde{\mathcal{V}}_{\text{Fierz}}^b = \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{32} & 0 & & & & & \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & & & & & \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{32} & 0 & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & & & & & \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & & & & & & & \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & & & & & & \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & & & & & & & \\ -24 & 0 & 0 & 0 & & & & & -\frac{1}{2} & 0 & & & & & \\ 0 & 0 & -24 & 0 & & & & & 0 & -\frac{1}{2} & & & & & \\ -\frac{1}{2} & 0 & 0 & \frac{m_s}{m_b} & -1 & -\frac{1}{2} & 0 & 0 & -\frac{1}{32} & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 & 0 & 0 & -\frac{1}{2} & -\frac{m_s}{m_b} & 0 & -\frac{1}{32} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{m_s}{m_b} & -\frac{1}{2} & 0 & 0 & 0 & -1 & -\frac{1}{2} & 0 & -\frac{1}{32} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{2} & 1 & -\frac{m_s}{m_b} & 0 & 0 & -\frac{1}{2} & 0 & -\frac{1}{32} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 & 0 & -1 & 0 & 0 & -\frac{m_s}{2m_b} & -\frac{1}{32} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{m_s}{2m_b} & 0 & -1 & -\frac{1}{2} & 0 & 0 & 0 & -\frac{m_s}{32m_b} & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 1 & 0 & -\frac{m_s}{2m_b} & -1 & 0 & 0 & -\frac{1}{32} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -\frac{m_s}{2m_b} & 0 & 0 & 1 & 0 & 0 & -1 & -\frac{1}{2} & -\frac{m_s}{32m_b} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (\text{A6})$$

$$\tilde{V}_{\text{Fierz}}^s = \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{32} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{32} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -24 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & -24 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ -\frac{m_s}{2m_b} & \frac{m_s}{m_b} & 0 & 0 & -1 & 0 & 0 & -\frac{m_s}{2m_b} & -\frac{m_s}{32m_b} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{m_s}{2m_b} & 0 & 0 & 1 & 0 & 0 & -\frac{m_s}{m_b} & -\frac{m_s}{2m_b} & -\frac{m_s}{32m_b} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{m_s}{2m_b} & \frac{m_s}{m_b} & 0 & -\frac{m_s}{2m_b} & -1 & 0 & 0 & -\frac{m_s}{32m_b} & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{m_s}{2m_b} & 0 & -\frac{m_s}{m_b} & -\frac{m_s}{2m_b} & 0 & 0 & 0 & -\frac{m_s}{32m_b} & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -\frac{1}{2} & 0 & 0 & \frac{m_s}{m_b} & -\frac{m_s}{m_b} & -\frac{m_s}{2m_b} & 0 & 0 & -\frac{1}{32} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -\frac{m_s}{2m_b} & \frac{m_s}{m_b} & -\frac{m_s}{m_b} & 0 & 0 & -\frac{1}{2} & 0 & -\frac{m_s}{32m_b} & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{m_s}{m_b} & -\frac{1}{2} & 0 & 0 & 0 & -\frac{m_s}{m_b} & -\frac{m_s}{2m_b} & 0 & -\frac{1}{32} & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{m_s}{2m_b} & \frac{m_s}{m_b} & 0 & 0 & 0 & -\frac{1}{2} & -\frac{m_s}{m_b} & 0 & -\frac{m_s}{32m_b} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (\text{A7})$$

APPENDIX B: ANOMALOUS-DIMENSION MATRIX

Considering the fact that RGEs in EFTs already represent a significantly equipped industry, we have decided to collect here our results for the anomalous-dimension matrix γ_C , so that one may easily implement this matrix without having to refer to our discussion in Sec. II C 2 and thereafter, which is oriented towards a more progressive, (if exhaustive) solution. Note however that, due to the size of the matrix, we are bound to present it block by block. Remember that our operators are ordered as in Eqs. (4), (5), and (6) and that the chiralities follow the ordering LL , LR , RR , RL , or L , R (depending on the number of chirality indices the operator carries). The definition of the blocks in chirality space is recalled at the very end of this appendix. Recalling Eq. (15),

$$\gamma_C = \begin{bmatrix} \gamma_C^{ll} & 0 & 0 \\ \gamma_C^{ql} & \gamma_C^{qq} & \gamma_C^{qg} \\ \gamma_C^{gl} & \gamma_C^{gq} & \gamma_C^{gg} \end{bmatrix}. \quad (\text{B1})$$

- (i) *Sub-block γ_C^{ll}* : We remind the reader that operators with different lepton flavors do not mix (at least as long as massless neutrinos are concerned), so that the sub-block γ_C^{ll} is diagonal in lepton flavor space,

$$\gamma_C^{ll} = \begin{bmatrix} \tilde{\gamma}_C^{ee} & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{\gamma}_C^{\mu\mu} & 0 & 0 & 0 & 0 \\ 0 & 0 & \tilde{\gamma}_C^{\tau\tau} & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{\gamma}_C^{\nu_e\nu_e} & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{\gamma}_C^{\nu_\mu\nu_\mu} & 0 \\ 0 & 0 & 0 & 0 & 0 & \tilde{\gamma}_C^{\nu_\tau\nu_\tau} \end{bmatrix}. \quad (\text{B2})$$

From Eqs. (16) and (17), we read, for any lepton flavor l (but note that the basis of operators is shortened in the neutrino case, due to the absence of low-energy right-handed neutrinos),

$$\tilde{\gamma}_C^{ll} = \frac{2\alpha_s C_2(3)}{4\pi} \times \begin{bmatrix} -\frac{23}{4} \mathbb{1} & 0 & 0 & 0 & 0 \\ 0 & -\frac{11}{4} \mathbb{1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{7}{4} & 0 & 0 \\ 0 & 2\left(1 + \frac{m_s}{m_b} \Sigma\right) & 0 & -\frac{19}{4} & 0 \\ 0 & 0 & 0 & 0 & -\frac{23}{4} \mathbb{1} \end{bmatrix}. \quad (\text{B3})$$

(ii) *Sub-block* γ_C^{ql} :

$$\gamma_C^{ql} = \begin{bmatrix} \tilde{\gamma}_C^{ue} & \tilde{\gamma}_C^{u\mu} & \tilde{\gamma}_C^{u\tau} & \tilde{\gamma}_C^{u\nu_e} & \tilde{\gamma}_C^{u\nu_\mu} & \tilde{\gamma}_C^{u\nu_\tau} \\ \tilde{\gamma}_C^{de} & \tilde{\gamma}_C^{d\mu} & \tilde{\gamma}_C^{d\tau} & \tilde{\gamma}_C^{d\nu_e} & \tilde{\gamma}_C^{d\nu_\mu} & \tilde{\gamma}_C^{d\nu_\tau} \\ \tilde{\gamma}_C^{se} & \tilde{\gamma}_C^{s\mu} & \tilde{\gamma}_C^{s\tau} & \tilde{\gamma}_C^{s\nu_e} & \tilde{\gamma}_C^{s\nu_\mu} & \tilde{\gamma}_C^{s\nu_\tau} \\ \tilde{\gamma}_C^{ce} & \tilde{\gamma}_C^{c\mu} & \tilde{\gamma}_C^{c\tau} & \tilde{\gamma}_C^{c\nu_e} & \tilde{\gamma}_C^{c\nu_\mu} & \tilde{\gamma}_C^{c\nu_\tau} \\ \tilde{\gamma}_C^{be} & \tilde{\gamma}_C^{b\mu} & \tilde{\gamma}_C^{b\tau} & \tilde{\gamma}_C^{b\nu_e} & \tilde{\gamma}_C^{b\nu_\mu} & \tilde{\gamma}_C^{b\nu_\tau} \end{bmatrix}. \quad (\text{B4})$$

From Eqs. (16) and (19), we read, for any lepton flavor l and $q = u, d, c$,

$$\tilde{\gamma}_C^{ql} = -\frac{2\alpha_s C_2(3)}{4\pi} \frac{3}{2} Q_l Q_q \times \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -(1+\tilde{\Sigma}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(1+\tilde{\Sigma}) & 0 \\ 0 & \frac{m_q}{m_b}(1+\tilde{\Sigma}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{3}(1+\tilde{\Sigma}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{3}(1+\tilde{\Sigma}) & 0 \\ 0 & \frac{m_q}{3m_b}(1+\tilde{\Sigma}) & 0 & 0 & 0 \end{bmatrix}. \quad (\text{B5})$$

For $q = b, s$, we have

$$\tilde{\gamma}_C^{ql} = \frac{2\alpha_s C_2(3)}{4\pi} \frac{3}{2} Q_l Q_q \times \left\{ - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -(1+\tilde{\Sigma}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(1+\tilde{\Sigma}) & 0 \\ 0 & \frac{m_q}{m_b}(1+\tilde{\Sigma}) & 0 & 0 & 0 \end{bmatrix} + \tilde{V}_{\text{Fierz}}^q \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{3}(1+\tilde{\Sigma}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{3}(1+\tilde{\Sigma}) & 0 \\ 0 & \frac{m_q}{3m_b}(1+\tilde{\Sigma}) & 0 & 0 & 0 \end{bmatrix} \right\}. \quad (\text{B6})$$

(iii) *Sub-block* γ_C^{gl} :

$$\gamma_C^{gl} = [\tilde{\gamma}_C^{ge} \quad \tilde{\gamma}_C^{g\mu} \quad \tilde{\gamma}_C^{g\tau} \quad \tilde{\gamma}_C^{g\nu_e} \quad \tilde{\gamma}_C^{g\nu_\mu} \quad \tilde{\gamma}_C^{g\nu_\tau}]. \quad (\text{B7})$$

From Eqs. (16) and (18), we read, for any lepton flavor l ,

$$\tilde{\gamma}_C^{gl} = \frac{2\alpha_s C_2(3)}{4\pi} \frac{1}{3} Q_l \times \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \Xi + \frac{m_s}{m_b} \Phi & 0 & 0 & 0 \\ 0 & 2\left[\Xi' + \frac{m_s}{m_b} \Phi'\right] & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (\text{B8})$$

(iv) *Sub-block* γ_C^{qq} :

$$\gamma_C^{ql} = \begin{bmatrix} \tilde{\gamma}_C^{uu} & \tilde{\gamma}_C^{ud} & \tilde{\gamma}_C^{us} & \tilde{\gamma}_C^{uc} & \tilde{\gamma}_C^{ub} \\ \tilde{\gamma}_C^{du} & \tilde{\gamma}_C^{dd} & \tilde{\gamma}_C^{ds} & \tilde{\gamma}_C^{dc} & \tilde{\gamma}_C^{db} \\ \tilde{\gamma}_C^{su} & \tilde{\gamma}_C^{sd} & \tilde{\gamma}_C^{ss} & \tilde{\gamma}_C^{sc} & \tilde{\gamma}_C^{sb} \\ \tilde{\gamma}_C^{cu} & \tilde{\gamma}_C^{cd} & \tilde{\gamma}_C^{cs} & \tilde{\gamma}_C^{cc} & \tilde{\gamma}_C^{cb} \\ \tilde{\gamma}_C^{bu} & \tilde{\gamma}_C^{bd} & \tilde{\gamma}_C^{bs} & \tilde{\gamma}_C^{bc} & \tilde{\gamma}_C^{bb} \end{bmatrix}. \quad (\text{B9})$$

For two quark flavors q and q' , we read from Eqs. (20) and (21) the following:

(i) If q and $q' \neq b, s$,

$$\tilde{\gamma}_C^{q'q} = \frac{2\alpha_s C_2(3)}{4\pi} \{ \delta^{q'q} \text{diag} \times (51, 51, 51, 21, 21, 51, 51, 51, 21, 21) - \delta^{q'q} \tilde{\mathcal{M}}_{\text{diag}}^{q'q} - \tilde{\mathcal{M}}_{\text{univ}}^{q'q} \}. \quad (\text{B10})$$

(ii) If q and $q' = b$ or s ,

$$\tilde{\gamma}_C^{q'q} = \frac{2\alpha_s C_2(3)}{4\pi} \left\{ \delta^{q'q} \text{diag}(51, 51, 51, 21, 21) - \delta^{q'q} [1, 0] \tilde{\mathcal{M}}_{\text{diag}}^{q'q} \begin{bmatrix} 1 \\ 0 \end{bmatrix} - [1, -\tilde{V}_{\text{Fierz}}^{q'}] \tilde{\mathcal{M}}_{\text{univ}}^{q'q} \begin{bmatrix} 1 \\ -\tilde{V}_{\text{Fierz}}^q \end{bmatrix} \right\}. \quad (\text{B11})$$

(iii) If $q \neq b, s$ and $q' = b$ or s ,

$$\tilde{\gamma}_C^{q'q} = -\frac{2\alpha_s C_2(3)}{4\pi} [\mathbb{1}, -\tilde{V}_{\text{Fierz}}^{q'}] \tilde{\mathcal{M}}_{\text{univ}}^{q'q}. \quad (\text{B12})$$

(iv) If $q = b$ or s and $q' \neq b, s$,

$$\tilde{\gamma}_C^{q'q} = -\frac{2\alpha_s C_2(3)}{4\pi} \tilde{\mathcal{M}}_{\text{univ}}^{q'q} \left[-\tilde{V}_{\text{Fierz}}^q \right], \quad (\text{B13})$$

with

$$\tilde{\mathcal{M}}_{\text{diag}}^{q'q} = \begin{bmatrix} 8\mathbb{1} & 0 & \frac{1}{32}\Delta & 0 & 0 & 0 & 0 & -\frac{3}{32}\Delta & 0 & 0 \\ 0 & 2\mathbb{1} + \frac{3}{4}\Sigma_3 & 0 & 0 & 0 & 0 & -\frac{9}{4}\Sigma_3 & 0 & 0 & 0 \\ 24\Delta^T & 0 & 0 & 0 & 0 & -72\Delta^T & 0 & 0 & 0 & 0 \\ A_H & B_H & 0 & \mathbb{1} & 0 & C_H & D_H & 0 & 0 & 0 \\ A_{\tilde{H}} & B_{\tilde{H}} & 0 & 0 & \mathbb{1} & C_{\tilde{H}} & D_{\tilde{H}} & 0 & 0 & 0 \\ \frac{9}{4}\mathbb{1} & 0 & -\frac{3}{64}\Delta & 0 & 0 & \frac{5}{4}\mathbb{1} & 0 & -\frac{7}{64}\Delta & 0 & 0 \\ 0 & -\frac{9}{8}(\mathbb{1} + \Sigma_3) & 0 & 0 & 0 & 0 & \frac{1}{8}(43\mathbb{1} - 21\Sigma_3) & 0 & 0 & 0 \\ -36\Delta^T & 0 & -\frac{9}{4}\mathbb{1} & 0 & 0 & -84\Delta^T & 0 & \frac{27}{4}\mathbb{1} & 0 & 0 \\ A_{\mathcal{H}} & B_{\mathcal{H}} & 0 & 0 & 0 & C_{\mathcal{H}} & D_{\mathcal{H}} & 0 & \mathbb{1} & 0 \\ A_{\tilde{\mathcal{H}}} & B_{\tilde{\mathcal{H}}} & 0 & 0 & 0 & C_{\tilde{\mathcal{H}}} & D_{\tilde{\mathcal{H}}} & 0 & 0 & \mathbb{1} \end{bmatrix},$$

$$\tilde{\mathcal{M}}_{\text{univ}}^{q'q} = \frac{1}{4} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3}(\mathbb{1} + \tilde{\Sigma}) & 0 & 0 & 0 & 0 & -(\mathbb{1} + \tilde{\Sigma}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3}(\mathbb{1} + \tilde{\Sigma}) & 0 & 0 & 0 & 0 & -(\mathbb{1} + \tilde{\Sigma}) & 0 \\ 0 & -\frac{m_q}{3m_b}(\mathbb{1} + \tilde{\Sigma}) & 0 & 0 & 0 & 0 & \frac{m_q}{m_b}(\mathbb{1} + \tilde{\Sigma}) & 0 & 0 & 0 \end{bmatrix}.$$

(v) *Sub-block* γ_C^{qq} :

$$\gamma_C^{qq} = \begin{bmatrix} \tilde{\gamma}_C^{qu} & \tilde{\gamma}_C^{qd} & \tilde{\gamma}_C^{qs} & \tilde{\gamma}_C^{gc} & \tilde{\gamma}_C^{gb} \end{bmatrix}. \quad (\text{B14})$$

For a quark flavor $q = u, d, s$, we read from Eqs. (20) and (22)

$$\tilde{\gamma}_C^{qq} = -\frac{2\alpha_S C_2(3)}{4\pi} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -12\frac{m_q}{m_b}\Xi & \frac{1}{12}(\Xi + \frac{m_s}{m_b}\Phi) & 0 & 0 & 0 & 0 & -\frac{1}{4}(\Xi + \frac{m_s}{m_b}\Phi) & 0 & 0 & 0 \\ 24\frac{m_q}{m_b}\tilde{\Xi} & \frac{1}{6}(\Xi' + \frac{m_s}{m_b}\Phi') & 0 & 0 & 0 & 0 & -\frac{1}{2}(\Xi' + \frac{m_s}{m_b}\Phi') & 0 & 0 & 0 \\ -\frac{11}{8}\frac{m_q}{m_b}\Xi & \frac{7}{144}(\Xi + \frac{m_s}{m_b}\Phi) & 0 & 0 & 0 & -\frac{15}{8}\frac{m_q}{m_b}\Xi & -\frac{7}{48}(\Xi + \frac{m_s}{m_b}\Phi) & 0 & 0 & 0 \\ \frac{11}{4}\frac{m_q}{m_b}\tilde{\Xi} & \frac{7}{72}(\Xi' + \frac{m_s}{m_b}\Phi') & 0 & 0 & 0 & \frac{15}{4}\frac{m_q}{m_b}\tilde{\Xi} & -\frac{7}{24}(\Xi' + \frac{m_s}{m_b}\Phi') & 0 & 0 & 0 \\ 0 & 0 & -\frac{3}{64}\frac{m_q}{m_b}\mathbb{1} & 0 & 0 & 0 & 0 & \frac{9}{64}\frac{m_q}{m_b}\mathbb{1} & 0 & 0 \end{bmatrix}. \quad (\text{B15})$$

For $q = b, s$, we have

$$\tilde{\gamma}_C^{qq} = \frac{2\alpha_S C_2(3)}{4\pi} \left\{ - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -12\frac{m_q}{m_b}\Xi & \frac{1}{12}(\Xi + \frac{m_s}{m_b}\Phi) & 0 & 0 & 0 \\ 24\frac{m_q}{m_b}\tilde{\Xi} & \frac{1}{6}(\Xi' + \frac{m_s}{m_b}\Phi') & 0 & 0 & 0 \\ -\frac{11}{8}\frac{m_q}{m_b}\Xi & \frac{7}{144}(\Xi + \frac{m_s}{m_b}\Phi) & 0 & 0 & 0 \\ \frac{11}{4}\frac{m_q}{m_b}\tilde{\Xi} & \frac{7}{72}(\Xi' + \frac{m_s}{m_b}\Phi') & 0 & 0 & 0 \\ 0 & 0 & -\frac{3}{64}\frac{m_q}{m_b}\mathbb{1} & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{4}(\Xi + \frac{m_s}{m_b}\Phi) & 0 & 0 & 0 \\ 0 & -\frac{1}{2}(\Xi' + \frac{m_s}{m_b}\Phi') & 0 & 0 & 0 \\ -\frac{15}{8}\frac{m_q}{m_b}\Xi & -\frac{7}{48}(\Xi + \frac{m_s}{m_b}\Phi) & 0 & 0 & 0 \\ \frac{15}{4}\frac{m_q}{m_b}\tilde{\Xi} & -\frac{7}{24}(\Xi' + \frac{m_s}{m_b}\Phi') & 0 & 0 & 0 \\ 0 & 0 & \frac{9}{64}\frac{m_q}{m_b}\mathbb{1} & 0 & 0 \end{bmatrix} \tilde{V}_{\text{Fierz}}^q \right\}. \quad (\text{B16})$$

(vi) *Sub-block* γ_C^{qq} :

$$\gamma_C^{qq} = \begin{bmatrix} \tilde{\gamma}_C^{ug} \\ \tilde{\gamma}_C^{dg} \\ \tilde{\gamma}_C^{sg} \\ \tilde{\gamma}_C^{cg} \\ \tilde{\gamma}_C^{bg} \end{bmatrix}. \quad (\text{B17})$$

$$\tilde{\gamma}_C^{qq} = -\frac{2\alpha_S C_2(3)}{4\pi} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 18Q_q \frac{m_q}{m_b} \mathbb{1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 & \dots \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 6Q_q \frac{m_q}{m_b} \mathbb{1} & 6\frac{m_q}{m_b} \mathbb{1} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (\text{B18})$$

For a quark flavor $q = u, d, s$, we read from Eqs. (23)–(25)

For $q = b, s$, we have

$$\tilde{\gamma}_C^{qq} = \frac{2\alpha_S C_2(3)}{4\pi} \left\{ - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 18Q_q \frac{m_q}{m_b} \mathbb{1} & 0 & 0 & \dots \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \tilde{V}_{\text{Fierz}}^q \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 6Q_q \frac{m_q}{m_b} \mathbb{1} & 6\frac{m_q}{m_b} \mathbb{1} & 0 & \dots \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\}. \quad (\text{B19})$$

(vii) *Sub-block γ_C^{gg} :*

$$\gamma_C^{gg} = \frac{2\alpha_S C_2(3)}{4\Pi} \begin{bmatrix} \frac{5}{4} \mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4Q_d \mathbb{1} & \frac{3}{4} \mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{35}{4} \mathbb{1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{35}{4} \mathbb{1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{19}{4} \mathbb{1} & 0 & 0 & 0 \\ \frac{1}{4} \Omega & 0 & \frac{2}{3} Q_d \mathbb{1} & \frac{1}{3} Q_d \sigma_3 & 0 & -\frac{5}{24} \mathbb{1} & -\frac{11}{48} \sigma_3 & 0 \\ -\frac{1}{2} \tilde{\Omega} & 0 & \frac{4}{3} Q_d \sigma_3 & \frac{2}{3} Q_d \mathbb{1} & 0 & -\frac{11}{12} \sigma_3 & -\frac{5}{24} \mathbb{1} & 0 \\ -\left(1 - \frac{m_s^2}{m_b^2}\right) \mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{11}{4} \mathbb{1} \\ 0 & \frac{3}{8} \Omega & 0 & 0 & 0 & Q_d \mathbb{1} & \frac{1}{2} Q_d \sigma_3 & 0 \\ 0 & -\frac{3}{4} \tilde{\Omega} & 0 & 0 & 0 & 2Q_d \sigma_3 & Q_d \mathbb{1} & 0 \\ 0 & \frac{7}{32} \Omega & 0 & 0 & 0 & \frac{7}{12} Q_d \mathbb{1} & \frac{7}{24} Q_d \sigma_3 & 0 \\ 0 & -\frac{7}{16} \tilde{\Omega} & 0 & 0 & 0 & \frac{7}{6} Q_d \sigma_3 & \frac{7}{12} Q_d \mathbb{1} & 0 \\ 0 & \frac{9}{8} \left(1 + \frac{m_s^2}{m_b^2}\right) \mathbb{1} & 0 & 0 & 0 & \frac{9}{2} Q_d \mathbb{1} & -\frac{9}{4} Q_d \sigma_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{35}{4} \mathbb{1} & 0 & 1 & \frac{1}{2} \sigma_3 & -\frac{9}{2} \mathbb{1} & 0 & 0 & 0 \\ 0 & -\frac{35}{4} \mathbb{1} & 2\sigma_3 & 1 & 9\sigma_3 & 0 & 0 & 0 \\ -\frac{21}{16} \mathbb{1} & \frac{3}{32} \sigma_3 & -\frac{385}{24} \mathbb{1} & \frac{41}{48} \sigma_3 & -\frac{11}{8} \mathbb{1} & 0 & 0 & 0 \\ \frac{3}{8} \sigma_3 & -\frac{21}{16} \mathbb{1} & \frac{41}{12} \sigma_3 & -\frac{385}{24} \mathbb{1} & \frac{11}{4} \sigma_3 & 0 & 0 & 0 \\ -\frac{3}{2} \mathbb{1} & \frac{3}{4} \sigma_3 & -\frac{45}{2} \sigma_3 & \frac{45}{4} \sigma_3 & 21 & 0 & 0 & 0 \end{bmatrix} \quad (\text{B20})$$

We finally collect the definition of the various relevant sub-blocks in chirality space (with the exception of the identity $\mathbb{1}$ and the null matrix 0):

$$\begin{aligned}
\Sigma &\equiv \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}; & \tilde{\Sigma} &\equiv \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}; & \begin{cases} \Xi &\equiv \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{pmatrix}; \\ \Phi &\equiv \begin{pmatrix} 1 & 1 & 0 & 0 \end{pmatrix} \end{cases} & \begin{cases} \Xi' &\equiv \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 1 \\ -1 & -1 & 0 & 0 \end{pmatrix}; \\ \Phi' &\equiv \begin{pmatrix} 0 & 0 & 1 & 1 \\ -1 & -1 & 0 & 0 \end{pmatrix} \end{cases} \\
\tilde{\Xi} &\equiv \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix}; & \sigma_1 &\equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; & \sigma_3 &\equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; & \epsilon &\equiv \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}; \\
\Sigma_3 &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}; & E &= \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}; & \tilde{E} &= \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}; & \Delta &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \\
\begin{cases} A_H = \frac{m_q}{4m_b} [3(\mathbb{1} + \tilde{\Sigma}) + \Sigma_3 + E] \\ A_{\tilde{H}} = \frac{1}{4} [3\mathbb{1} + \Sigma_3 + \frac{m_s}{m_b} (3\Sigma + \tilde{E})] \\ A_{\mathcal{H}} = -\frac{3}{8} \frac{m_q}{m_b} [3(\mathbb{1} + \tilde{\Sigma}) + \Sigma_3 + E] \\ A_{\tilde{\mathcal{H}}} = -\frac{3}{8} (3\mathbb{1} + \Sigma_3) - \frac{3}{8} \frac{m_s}{m_b} (3\Sigma + \tilde{E}) \end{cases} & \begin{cases} B_H = \frac{1}{4} [-8\mathbb{1} + \Sigma_3 + \frac{m_s}{m_b} (-8\Sigma + \tilde{E})] \\ B_{\tilde{H}} = \frac{m_q}{4m_b} [-8(\mathbb{1} + \tilde{\Sigma}) + \Sigma_3 + E] \\ B_{\mathcal{H}} = -\frac{3}{8} (3\mathbb{1} + \Sigma_3) - \frac{3}{8} \frac{m_s}{m_b} (3\Sigma + \tilde{E}) \\ B_{\tilde{\mathcal{H}}} = -\frac{3}{8} \frac{m_q}{m_b} [3(\mathbb{1} + \tilde{\Sigma}) + \Sigma_3 + E] \end{cases} \\
\begin{cases} C_H = -\frac{3}{4} \frac{m_q}{m_b} [3(\mathbb{1} + \tilde{\Sigma}) + \Sigma_3 + E] \\ C_{\tilde{H}} = -\frac{3}{4} [(3\mathbb{1} + \Sigma_3) + \frac{m_s}{m_b} (3\Sigma + \tilde{E})] \\ C_{\mathcal{H}} = -\frac{7}{8} \frac{m_q}{m_b} [3(\mathbb{1} + \tilde{\Sigma}) + \Sigma_3 + E] \\ C_{\tilde{\mathcal{H}}} = -\frac{7}{8} (3\mathbb{1} + \Sigma_3) - \frac{7}{8} \frac{m_s}{m_b} (3\Sigma + \tilde{E}) \end{cases} & \begin{cases} D_H = -\frac{3}{4} (\Sigma_3 + \frac{m_s}{m_b} \tilde{E}) \\ D_{\tilde{H}} = -\frac{3}{4} \frac{m_q}{m_b} (\Sigma_3 + E) \\ D_{\mathcal{H}} = \frac{1}{8} (11\mathbb{1} - 7\Sigma_3) + \frac{m_s}{8m_b} (11\Sigma - 7\tilde{E}) \\ D_{\tilde{\mathcal{H}}} = \frac{m_q}{8m_b} [11(\mathbb{1} + \tilde{\Sigma}) - 7(\Sigma_3 + E)] \end{cases} \\
\Omega &\equiv \left(1 + \frac{m_s^2}{m_b^2}\right) \mathbb{1} - \frac{2}{3} \frac{m_s}{m_b} \sigma_1, & \tilde{\Omega} &\equiv \left(1 + \frac{m_s^2}{m_b^2}\right) \sigma_3 - \frac{2}{3} \frac{m_s}{m_b} \epsilon.
\end{aligned} \tag{B21}$$

APPENDIX C: DIAGONALIZATION OF THE ANOMALOUS-DIMENSION MATRIX FOR DIMENSION-SEVEN OPERATORS

1. Gluonic operators

We consider the matrix $\mathcal{A}_{\mathcal{Q}\mathcal{Q}}$ defined in Eq. (31),

$$\mathcal{A}_{\mathcal{Q}\mathcal{Q}} = \begin{bmatrix} \frac{35}{4} \mathbb{1} & 0 & -1 & -\frac{1}{2} \sigma_3 & \frac{9}{2} \mathbb{1} \\ 0 & \frac{35}{4} \mathbb{1} & -2\sigma_3 & -1 & -9\sigma_3 \\ \frac{21}{16} \mathbb{1} & -\frac{3}{32} \sigma_3 & \frac{385}{24} \mathbb{1} & -\frac{41}{48} \sigma_3 & \frac{11}{8} \mathbb{1} \\ -\frac{3}{8} \sigma_3 & \frac{21}{16} \mathbb{1} & -\frac{41}{12} \sigma_3 & \frac{385}{24} \mathbb{1} & -\frac{11}{4} \sigma_3 \\ \frac{3}{2} \mathbb{1} & -\frac{3}{4} \sigma_3 & \frac{45}{2} \mathbb{1} & -\frac{45}{4} \sigma_3 & -2\mathbb{1} \end{bmatrix} = \mathcal{O}_{\mathcal{Q}} D_{\mathcal{Q}} \mathcal{O}_{\mathcal{Q}}^{-1}. \tag{C1}$$

Its diagonalization happens to be tractable,

$$\mathcal{D}_Q = \text{diag} \left[-\frac{19}{4} \mathbb{1}, \frac{1}{8} (117 - \sqrt{3073}) \mathbb{1}, \frac{1}{24} (277 - \sqrt{3193}) \mathbb{1}, \frac{1}{24} (277 + \sqrt{3193}) \mathbb{1}, \frac{1}{8} (117 + \sqrt{3073}) \mathbb{1} \right],$$

$$\mathcal{O}_Q = \begin{bmatrix} 1 & -\frac{1}{12} (47 + \sqrt{3073}) & -\frac{1}{54} (67 + \sqrt{3193}) & \frac{1}{2} & \frac{1}{24} (-47 + \sqrt{3073}) \\ -2 & \frac{1}{6} (47 + \sqrt{3073}) & -\frac{1}{27} (67 + \sqrt{3193}) & 1 & \frac{1}{12} (47 - \sqrt{3073}) \\ \frac{7}{60} & 1 & \frac{1}{2} & -\frac{1}{96} (67 + \sqrt{3193}) & \frac{1}{2} \\ -\frac{7}{30} & -2 & 1 & -\frac{1}{48} (67 + \sqrt{3193}) & -1 \\ -3 & 2 & 0 & 0 & 1 \end{bmatrix}, \quad (\text{C2})$$

so that the exponentiation is fully determined.

2. Hybrid $(\bar{b}s)$ -gauge operators

We consider the matrix $\mathcal{A}_{\mathcal{H}\mathcal{H}}$ defined in Eq. (32),

$$\mathcal{A}_{\mathcal{H}\mathcal{H}} = \begin{bmatrix} \frac{5}{24} \mathbb{1} & \frac{11}{48} \sigma_3 & 0 \\ \frac{11}{12} \sigma_3 & \frac{5}{24} \mathbb{1} & 0 \\ 0 & 0 & \frac{11}{4} \mathbb{1} \end{bmatrix} = \mathcal{O}_{\mathcal{H}} D_{\mathcal{H}} \mathcal{O}_{\mathcal{H}}^{-1}. \quad (\text{C3})$$

Its diagonalization is straightforward,

$$\mathcal{O}_{\mathcal{H}} \equiv \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbb{1} & \frac{1}{2} \sigma_3 & 0 \\ -2 \sigma_3 & \mathbb{1} & 0 \\ 0 & 0 & \sqrt{2} \mathbb{1} \end{bmatrix}, \quad D_{\mathcal{H}} \equiv \text{diag} \left[-\frac{1}{4} \mathbb{1}, \frac{2}{3} \mathbb{1}, \frac{11}{4} \mathbb{1} \right], \quad (\text{C4})$$

allowing for a simple exponentiation,

$$\exp \left[\frac{C_2(3)}{\beta_0} \mathcal{A}_{\mathcal{H}\mathcal{H}} \ln \frac{\alpha_S(\mu)}{\alpha_S(\mu_0)} \right] = \left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)} \right)^{\frac{5C_2(3)}{24\beta_0}} \begin{bmatrix} \cosh \vartheta(\mu) \mathbb{1} & \frac{1}{2} \sinh \vartheta(\mu) \sigma_3 & 0 \\ -2 \sinh \vartheta(\mu) \sigma_3 & \cosh \vartheta(\mu) \mathbb{1} & 0 \\ 0 & 0 & \left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)} \right)^{\frac{61C_2(3)}{24\beta_0}} \mathbb{1} \end{bmatrix},$$

$$\vartheta(\mu) \equiv \frac{11C_2(3)}{24\beta_0} \ln \frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}. \quad (\text{C5})$$

3. $(\bar{b}s)(\bar{q}q)$ sector

We consider the matrix $\mathcal{A}_{qq}$ defined in Eq. (36),

$$\mathcal{A}_{qq} = \frac{1}{4} [\mathbb{1}; -\tilde{V}_{\text{Fierz}}^{q'}]_{q'=b,s} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{1}{3} (\mathbb{1} + \tilde{\Sigma}) & 0 & \mathbb{1} + \tilde{\Sigma} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbb{1} \\ -\tilde{V}_{\text{Fierz}}^{q'} \end{bmatrix}_{q=b,s}$$

$$= \mathcal{O}_q \text{diag} \left(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{8}{3} \mathcal{D}_{24} \right) \mathcal{O}_q^{-1}. \quad (\text{C6})$$

Let us introduce the following blocks:

$$\begin{aligned}
 \mathcal{D}_{24} &\equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; & \mathcal{R} &\equiv \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}; \\
 \mathcal{U}^s &\equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & -4 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{pmatrix}; & \mathcal{U}^b &\equiv \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & -4 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & -4 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{pmatrix}; \quad (C7)
 \end{aligned}$$

$$\mathcal{O}_q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \mathcal{R} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \mathcal{R} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \mathcal{R} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathcal{U}^s & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathcal{U}^b & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathcal{U}^s & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathcal{U}^b \end{bmatrix}$$

$$\times \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathcal{D}_{24} & 0 & \mathcal{D}_{24} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\mathcal{D}_{24} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \mathcal{D}_{24} & 0 & \mathcal{D}_{24} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\mathcal{D}_{24} & 0 & 0 & 0 & 1 - \mathcal{D}_{24} & 0 & \mathcal{D}_{24} & 0 & \mathcal{D}_{24} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathcal{D}_{24} & 0 & 1 - \frac{16}{7}\mathcal{D}_{24} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \mathcal{D}_{24} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathcal{D}_{24} & 0 & -\frac{9}{7}\mathcal{D}_{24} & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (\text{C8})$$

$\mathcal{O}_q$ turns out to diagonalize $\mathcal{A}_{qq}$ satisfactorily.

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