

Subleading- N_c improved parton showers

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We present an algorithm for improving subsequent parton shower emissions by full $SU(3)$ colour correlations in the framework of a dipole-type shower. As a proof of concept, we present results from the first implementation of such an algorithm for a final state shower.

1 Introduction and motivation

Parton showers and event generators are indispensable tools for predicting and understanding collider results [1, 2, 3]. Considering their importance for interpreting LHC results, it is essential to have a good understanding of their approximations and limitations. These simulations have up to now all been based on QCD as an $SU(N_c)$ gauge theory in the limit of large N_c . For $N_c = 3$, this approximation seems to work remarkably well despite the fact that $1/N_c = 1/3$, in the general case, and $1/N_c^2 = 1/9$ in most cases are not truly small parameters. Otherwise, significant deviations from parton shower predictions as compared to experimentally measured observables would have already hinted towards a severe underestimate of colour suppressed terms.

Including colour suppressed terms in parton shower simulations has so far been an unexplored continent, and investigating effects caused by colour correlations beyond the large- N_c limit is mandatory in the age of ever improving simulations, particularly when including higher-order QCD corrections. Especially when considering the matching of parton showers to NLO QCD corrections, subleading- N_c improved parton showers provide a valuable input in making these matchings more precise such that the matching conditions are indeed satisfied exactly, and not only modulo colour suppressed terms.¹

We here present an approach to subleading colour contributions [5] which is simple in the sense that it fits very well into the framework of existing Monte Carlo event generators. We note that this is not the end of the story, as for the general case an evolution at amplitude level would have to be considered. Our approach of colour matrix element corrections is a first step towards quantifying the size of the expected effects.

2 From dipole factorization to dipole showering

Dipole factorization, [6], states that the behaviour of QCD tree-level matrix elements squared in any singly unresolved limit involving two partons i, j (*i.e.* whenever i and j become collinear

¹In this context an independent approach, considering only one emission, has been presented in [4].

or one of them soft), can be cast into the form

$$|\mathcal{M}_{n+1}(\dots, p_i, \dots, p_j, \dots, p_k, \dots)|^2 \approx \sum_{k \neq i, j} \frac{1}{2p_i \cdot p_j} \langle \mathcal{M}_n(\dots, p_{\tilde{i}\tilde{j}}, \dots, p_{\tilde{k}}, \dots) | \mathbf{V}_{ij,k}(p_i, p_j, p_k) | \mathcal{M}_n(\dots, p_{\tilde{i}\tilde{j}}, \dots, p_{\tilde{k}}, \dots) \rangle, \quad (1)$$

where $|\mathcal{M}_n\rangle$ – which is a vector in the space of helicity and colour configurations – denotes the amplitude for an n -parton final state. Here an emitter $\tilde{i}\tilde{j}$ undergoes splitting to two partons i and j in the presence of a spectator $\tilde{k}$ which absorbs the longitudinal recoil of the splitting, $\tilde{k} \rightarrow k$. This factorization formula, which is well established to provide a subtraction scheme for NLO calculations, can actually be used to derive a dipole-type shower algorithm [7]. Results of an implementation have been reported in [8], and similar approaches have been considered in [9, 10]. In these cases, the colour correlations present in $\mathbf{V}_{ij,k}$ are approximated in the large- N_c limit, while keeping the colour factor for gluon emission off quarks, $C_F = \mathbf{T}_{q_i}^2$, exact. In turn, chains of colour connected dipoles evolve through subsequent emissions generating more dipoles in a chain or leading to a breakup of the chain in case a gluon splits into a $q\bar{q}$ pair until eventually the transverse momentum of potential emissions is below an infrared cutoff in the region of one GeV.

3 Colour matrix element corrections

The dipole factorization formula eq. 1 implies a factorization at the level of cross sections; here, the differential cross section for $n+1$ partons factorizes into the cross section for producing n partons times a radiation density as the sum over all dipole configurations $\tilde{i}\tilde{j}, \tilde{k}$, which undergo radiation, $\tilde{i}\tilde{j}, \tilde{k} \rightarrow i, j, k$:

$$dP_{ij,k}(p_\perp^2, z) = V_{ij,k}(p_\perp^2, z) \frac{d\phi_{n+1}(p_\perp^2, z)}{d\phi_n} \times \frac{-1}{\mathbf{T}_{\tilde{i}\tilde{j}}^2} \frac{\langle \mathcal{M}_n | \mathbf{T}_{\tilde{i}\tilde{j}} \cdot \mathbf{T}_k | \mathcal{M}_n \rangle}{|\mathcal{M}_n|^2} \quad (2)$$

Here, we have used the spin-averaged version of the dipole kernels including the product of colour charges encoding the colour correlations, $\mathbf{V}_{ij,k} = -V_{ij,k} \mathbf{T}_{\tilde{i}\tilde{j}} \cdot \mathbf{T}_k / \mathbf{T}_{\tilde{i}\tilde{j}}^2$, and $d\phi_k$ denotes the k -parton phase space. In the large- N_c limit, this formula yields the basis for the dipole shower considered so far: $-\mathbf{T}_{\tilde{i}\tilde{j}} \cdot \mathbf{T}_k / \mathbf{T}_{\tilde{i}\tilde{j}}^2 \rightarrow \delta(\tilde{i}\tilde{j}, k \text{ colour connected}) / (1 + \delta_{\tilde{i}\tilde{j}})$, where $\delta_{\tilde{i}\tilde{j}} = 1(0)$ for $\tilde{i}\tilde{j} = g(q/\bar{q})$. To obtain an algorithm which instead keeps the full colour correlations, we do not consider this approximation but keep the second factor in eq. 2 exactly. Owing to the similarity of matrix element corrections present in parton showers so far, we refer to this improvement as ‘colour matrix element corrections’.

Eq. 2 describes how a single emission incorporates colour correlations. Indeed, for the first emission off the hard subprocess, $|\mathcal{M}_n\rangle$ is known, though it has to be recalculated after each subsequent emission to define the colour matrix element correction for the next emission. Instead of directly calculating the next amplitude, which would only be possible if we had derived splitting amplitudes in the singly unresolved limits, we observe that

$$|\mathcal{M}_n|^2 = \mathcal{M}_n^\dagger S_n \mathcal{M}_n = \text{Tr} (S_n \times \mathcal{M}_n \mathcal{M}_n^\dagger) \quad (3)$$

and

$$\langle \mathcal{M}_n | \mathbf{T}_{\tilde{i}\tilde{j}} \cdot \mathbf{T}_{\tilde{k}} | \mathcal{M}_n \rangle = \text{Tr} (S_{n+1} \times T_{\tilde{k},n} \mathcal{M}_n \mathcal{M}_n^\dagger T_{\tilde{i}\tilde{j},n}^\dagger), \quad (4)$$

where we have chosen a definite basis $\{|\alpha\rangle\}$ for the colour space, $|\mathcal{M}_n\rangle = \sum_{\alpha=1}^{d_n} c_{n,\alpha} |\alpha\rangle \leftrightarrow \mathcal{M}_n = (c_{n,1}, \dots, c_{n,d_n})^T$ and introduced the scalar product matrix $S_n = \{\langle\alpha_n|\beta_n\rangle\}$ as well as matrix representations of the colour charge operators for n partons, $\mathbf{T}_i \rightarrow T_{i,n}$.

These representations then imply that we can work with an amplitude matrix M_n as the fundamental object,

$$M_{n+1} = - \sum_{i \neq j} \sum_{k \neq i,j} \frac{4\pi\alpha_s}{p_i \cdot p_j} \frac{V_{ij,k}(p_i, p_j, p_k)}{\mathbf{T}_{ij}^2} T_{k,n} M_n T_{ij,n}^\dagger, \quad (5)$$

where the initial matrix for the hard subprocess is given by $M_{n_{\text{hard}}} = \mathcal{M}_{n_{\text{hard}}} \mathcal{M}_{n_{\text{hard}}}^\dagger$.

4 Technicalities

Having outlined the principle of the algorithm for including colour correlations for subsequent parton shower emissions, two major technical issues have to be addressed: On the one hand, an general treatment of the colour basis for an arbitrary number of partons is required. On the other hand, sampling from the probability (Sudakov-type) density driving the next parton shower emission has to be generalized to the case of non-positive splitting rates as typically encountered for $1/N_c$ suppressed contributions.

For the first task, we have implemented a C++ library `ColorFull` [11] implementing the trace bases of colour space, [12]. This library is interfaced to the `Matchbox` framework presented in [8]. The colour matrix element corrections calculated in this part of the simulations, are inserted as correction weights into an existing dipole shower implementation, which uses the `ExSample` library [13] to sample Sudakov-type densities derived from the absolute value of the colour-corrected splitting rates. For the second task, we then employ the interleaved competition/veto algorithm outlined in [14] to arrive at events distributed according to the desired density (note that the sum of all splitting rates approximates a squared matrix element and is thus positive).

5 Results

As a proof-of-concept, we present results from the subleading- N_c improved parton shower for final state radiation, more precisely considering $e^+e^- \rightarrow q\bar{q}$ at LEP1 energies. We compare three different approximations: ‘full’ colour correlations, the ‘shower’ approximation where the $\mathbf{T}_i \cdot \mathbf{T}_j / \mathbf{T}_i^2$ are taken in the large- N_c limit, and a ‘strict’ large- N_c approximation, where also $C_F \approx C_A/2$. Interestingly, the ‘shower’ approximation does not exactly reproduce the shower implementation; from four partons onwards, this evolution is sensitive to the emission history, though matches the shower implementation if one sequence of dipoles has dominated. Indeed, the differences between the ‘shower’ approximation and the default shower implementation are at the per-mille level as one would expect from strong ordering in the emission history.

In the results presented here, we include up to six improved shower emissions. $g \rightarrow q\bar{q}$ splittings are neglected, as there are no associated colour correlations; we also do not include hadronization. For event shapes and jet rates we find small subleading- N_c effects when considering the shower approximation; for tailored observables, probing the dynamics of soft radiation with respect to a hard subsystem of the event, larger effects are seen. The strict approximation shows larger deviations. This fact can mainly be attributed to the change in the Sudakov exponent for gluon emission off a quark. A few sample results are shown in figure 1.

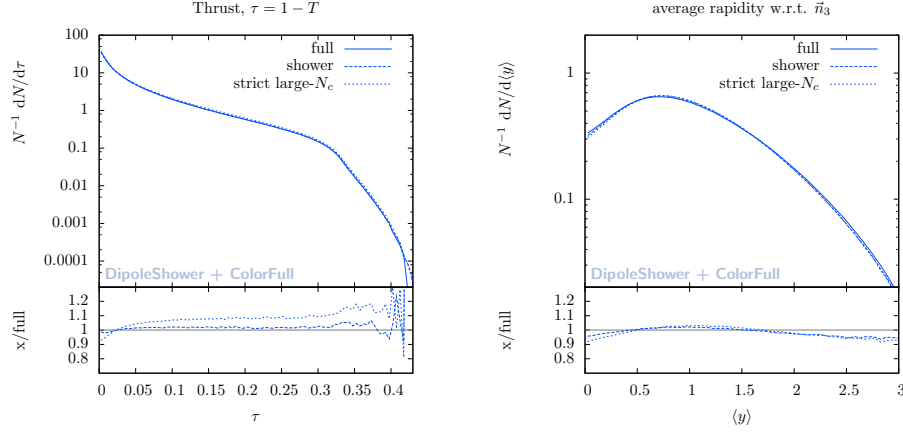


Figure 1: The thrust distribution (left) and the average rapidity w.r.t the thrust axis defined by the three hardest partons using the different approximations.

6 Conclusions

We have presented the first implementation of a subleading- N_c improved parton showers. For $e^+e^- \rightarrow$ jets small effects are seen except for very special observables. The technical issues associated with the implementation, particularly the treatment of the colour basis and the presence of negative splitting kernels will serve as input for related and future work; we also anticipate that larger effects can be seen in hadron collisions, *e.g.* $pp \rightarrow$ jets.

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