Production of one or two vector mesons in peripheral high-energy collisions of heavy ions

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We study the production of spin-one mesons in high-energy heavy-ion collisions with peripheral kinematics in the framework of quantum electrodynamics (QED). The cross sections of the production of a single vector meson and of two different ones are presented. The explicit dependence on the virtuality of the intermediate vector meson is obtained within a quark model. The effect of Reggeization of the intermediate vector-meson state in the case of the production of two vector mesons is taken into account.

I. INTRODUCTION

The CERN Large Hadron Collider (LHC) provides the opportunity to study experimentally the production of scalar, pseudoscalar, and vector mesons in peripheral collision of heavy ions. Peripheral kinematics implies the detection of particles produced in directions close to the axis of the colliding beams [1]. The main feature of these processes is the nondecreasing total and differential cross sections. The invariant mass of the created particles is assumed to be small in the fragmentation and central regions in comparison with the total energy in the center of mass of the colliding beams $\sqrt{s} = 2E$. The application of the known theoretical approaches, such as the Nambu-Iona-Lasinio model as well as chiral perturbation theory, seems to be legitimate. We shall consider, in pure quantum electrodynamics (QED), the processes of the production of a single vector meson and of two vector mesons separated by a rapidity gap. It is known that the main contributions to the amplitudes of peripheral processes arise from the interaction mechanism of ions, mediated by the exchange of spin-one particles, such as virtual photons, vector mesons, and gluons. For sufficiently large electric charges of the ions, virtual-photon exchanges will eventually play the dominant role. Actually, the effect of the replacement $\alpha \to Z\alpha$ for the case of charged heavy ions, e.g., for Pb-Pb collisions, exceeds the corresponding QCD contribution for typical regions of momentum transfer, where $\alpha_s \sim 0.1$–0.2, which seems to be essential in the experimental setup. We shall consider processes of the creation of one and two vector mesons, such as orthopositronium or the ρ, ω, and J/ψ

mesons.

$$Y_1(Z_1, P_1) + Y_2(Z_2, P_2) \to V(e, r) + Y_1(Z_1, P_1') + Y_2(Z_2, P_2'),$$
$$Y_1(Z_1, P_1) + Y_2(Z_2, P_2) \to V(e_1, r_1) + V(e_2, r_2)$$
$$+ Y_1(Z_1, P_1') + Y_2(Z_2, P_2'). \quad (1)$$

Here, P_i, P_i' are the four-momenta of the incoming and scattered ions, and e, e_i and r, r_i are the polarization four-vectors and four-momenta and of the created vector mesons, which obey the transversality conditions $e(r)r = e_i(r_i)r_i = 0$. To describe the peripheral kinematics, it is convenient to introduce the light-cone four-momenta p_i as linear combinations of the incoming-ion four-momenta P_i:

$$p_1 = P_1 - \lambda P_2, \quad p_2 = P_2 - \eta P_1, \quad p_1^2 = p_2^2 = 0,$$
$$P_1^2 = m_1^2, \quad P_2^2 = m_2^2, \quad 2P_1P_2 \approx 2p_1p_2 = s \gg m_i^2, \quad (2)$$

where m_i are the masses of the ions. In the calculation of the differential cross section, the effects of the off-mass-shell-ness of the exchanged photons and vector meson must be taken into account. Our approach is based on taking the constituent quarks to be QED fermions. The additional factors for the QED amplitudes associated with the color and charge of the quarks will be discussed later.

The details of the wave function of the bound state were discussed in Ref. [2]. For our approach, only one structure R is relevant. The virtual-photon polarization effects in the process $\gamma^\star gg \to \psi$ were considered in Ref. [3]. The analysis of the inclusive annihilation of heavy quarkonium beyond the Born approximation of QCD was presented in Ref. [4]. In our work, we shall obtain the differential cross sections for the creation of one or two vector mesons considered as bound states of the relevant quarks.

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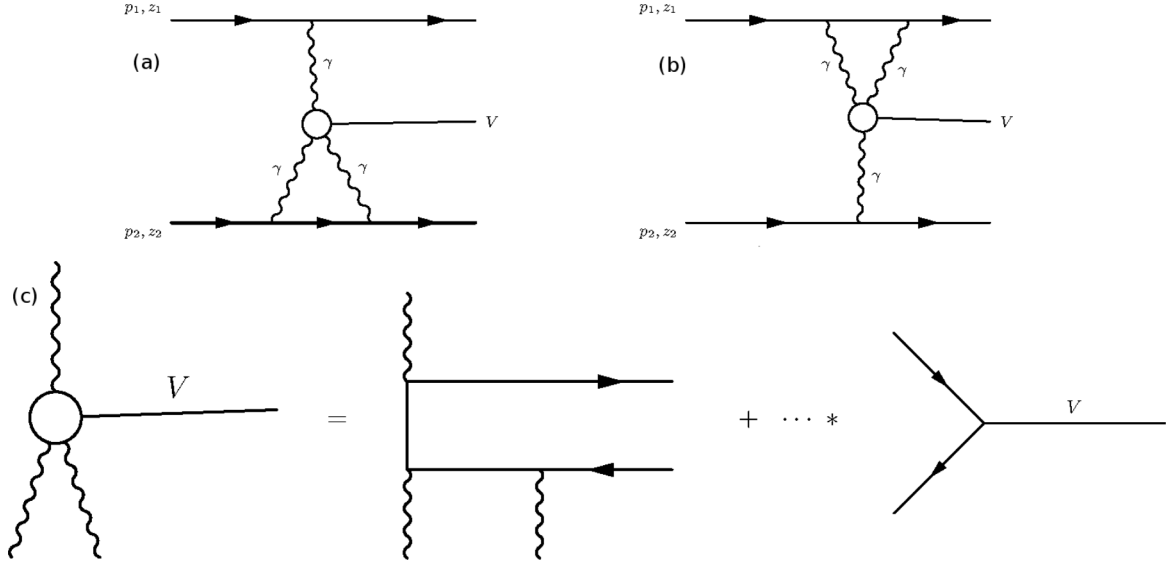[†]kniehl@desy.de
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FIG. 1. Feynman diagrams pertinent to single vector-meson production in peripheral heavy-ion collisions.

II. MATRIX ELEMENTS OF THE $2 \to 3$ PROCESSES

To lowest order in perturbation theory, there are two sets of Feynman diagrams involving three virtual-photon exchanges (see Fig. 1). The contribution of each of them to the total cross section has the form $\sigma \sim \sigma_0(aL^2 + bL + c)$, where $L = \ln(s/M^2)$ is the "large" logarithm and M is the mass of the created vector meson. The interference of the relevant amplitudes only contributes terms devoid of the "large" logarithm. Below, we restrict ourselves to the consideration of just one of the amplitudes, which corresponds to exchanges of one virtual photon with one ion $Y_1(Z_1, P_1)$ and of two virtual photons with the other ion $Y_2(Z_2, P_2)$. Using the prescriptions proposed in Ref. [1] for evaluating the matrix element of the peripheral process of single vector-meson production, we obtain

$$
M^{Y_1 Y_2 \to Y_1 Y_2 V} = \frac{(4\pi \alpha Z_1)(4\pi \alpha Z_2)^2}{q_1^2} \left(\frac{2}{s}\right)^3 s N_1
$$
$$
\times \int \frac{d^4 q}{(2\pi)^4 q^2 q_3^2} s^2 N_2 s F C^V, \quad (3)
$$

where the factor C^V accounts for the color and charge of the quarks and N_1, N_2, and F are given by the following expressions:

$$
N_1 = \frac{1}{s}\bar{u}(P_1')\hat{p}_2 u(P_1),
$$
$$
N_2 = \frac{1}{2s^2}\bar{u}(P_2')\left[\hat{p}_1 \frac{\hat{p}_2 - \hat{q} + m_2}{(p_2 - q)^2 - m_2^2}\hat{p}_1 \right.
$$
$$
\left. + \hat{p}_1 \frac{\hat{p}_2 - \hat{q}_3 + m_2}{(p_2 - q_3)^2 - m_2^2}\hat{p}_1\right]u(P_2), \quad (4)
$$
$$
F = \frac{1}{s}\frac{M\mathcal{A}}{2}\frac{1}{4}\operatorname{Tr} O^{\mu\nu\lambda}(\hat{p} + M)\hat{e}\,p_{1\mu}p_{2\nu}p_{2\lambda}, \quad (5)
$$

where $q_3 = q_2 - q$. The coupling constant $\mathcal{A}$, which we shall specify below, measures the strength of the interaction of the vector meson with incoming photons. The quantity $\bar{u}(q)O^{\mu\nu\lambda}v(q_+)$ is the matrix element of the subprocess $3\gamma \to q\bar{q}$ depicted in Fig. 1. So, we have

$$
O^{\mu\nu\lambda}p_{1\mu}p_{2\nu}p_{2\lambda} = \hat{p}_1\frac{\hat{q}_- - \hat{q}_1 + m}{D_1}\left[\frac{1}{D_3}\hat{p}_2(-\hat{q}_+ + \hat{q}_3 + m)\hat{p}_2 + \frac{1}{D_2}\hat{p}_2(-\hat{q}_+ + \hat{q}_2 + m)\hat{p}_2\right]
$$
$$
+ \left[\frac{1}{D_2}\hat{p}_2(\hat{q}_- - \hat{q} + m)\hat{p}_2 + \frac{1}{D_3}\hat{p}_2(\hat{q}_- - \hat{q}_3 + m)\hat{p}_2\right]\frac{-\hat{q}_+ + \hat{q}_1 + m}{D_1}\hat{p}_1
$$
$$
+ \frac{1}{D_2 D_3}[\hat{p}_2(\hat{q}_- - \hat{q} + m)\hat{p}_1(-\hat{q}_+ + \hat{q}_3 + m)\hat{p}_2 + \hat{p}_2(\hat{q}_- - \hat{q}_3 + m)\hat{p}_1(-\hat{q}_+ + \hat{q}_2 + m)\hat{p}_2], \quad (6)
$$

where $q_\pm$ are the four-momenta of the created quarks and the denominators are given by the expressions $D_1 = (q_- - q_1)^2 - m^2 + i0$, $D_2 = (-q_- + q)^2 - m^2 + i0$,

and $D_3 = (-q_+ + q_3)^2 - m^2 + i0$. Following the rules for the construction of the matrix element of the quark-antiquark bound state [5], we must put $q_+ = q_- = p/2$ in this

expression. Let us now introduce the Sudakov parametrization of the loop momenta and the momenta of the quarks and virtual photons:

$$q_1 = \beta_1 p_1 + q_{1\perp}, \quad q_3 = \alpha_3 p_2 + \beta_3 p_1 + q_{3\perp},$$
$$q = \alpha p_2 + \beta p_1 + q_\perp, \tag{7}$$
$$q_\pm = \alpha_\pm p_2 + \beta_\pm p_1 + q_{\pm\perp}, \quad d^4q = \frac{s}{2} d\alpha d\beta d^2\vec{q}.$$

From four-momentum conservation and the on-mass-shell conditions of the quarks, we have

$$\beta_+ = \beta_- = \frac{1}{2}\beta_1, \quad \alpha + \alpha_3 = \alpha_2,$$
$$\vec{q}_1 + \vec{q}_2 = \vec{q}_1 + \vec{q} + \vec{q}_3 = \vec{p}, \quad s\alpha_\pm = \frac{1}{2\beta_1}(\vec{p}^2 + 4m^2). \tag{8}$$

The expressions for the denominators can now be rewritten as

$$D_1 = -\vec{q}_1^2 + \vec{p}\vec{q}_1 - \frac{1}{2}(\vec{p}^2 + 4m^2)$$
$$= -\frac{1}{2}(\vec{q}_1^2 + \vec{q}_2^2 + M^2) = -\frac{1}{2}R,$$
$$D_2 = -\vec{q}^2 + \vec{p}\vec{q} - \frac{1}{2}s\beta_1\alpha + i0, \tag{9}$$
$$D_3 = -\vec{q}_3^2 + \vec{p}\vec{q}_3 - \frac{1}{2}s(\alpha - \alpha_2)\beta_1 + i0.$$

Performing the integration over the component β_2 of the loop momentum, we obtain for N_2

$$s \int d\beta N_2 = \frac{1}{s}\bar{u}(P_2')\hat{p}_1 u(P_2) \int_{-\infty}^{\infty} sd\beta \left[\frac{1}{s\beta + a + i0}\right.$$
$$\left. + \frac{1}{-s\beta + b + i0}\right] = (-2\pi i)\frac{1}{s}\bar{u}(P_2')\hat{u}(P_2)$$
$$= -2\pi i N_2'. \tag{10}$$

Summing the squared moduli of N_1 and N_2' over the spin states, we obtain

$$\sum |N_1|^2 = 2, \quad \sum |N_2'|^2 = 2. \tag{11}$$

In a similar way, the integral over the Sudakov variable α gives us

$$\int_{-\infty}^{\infty} sd\alpha \left(\frac{1}{D_2} + \frac{1}{D_3}\right) = (-2\pi i)\frac{2}{\beta_1}. \tag{12}$$

As a result for the contribution of these terms to the matrix element, we have

$$\frac{8(4\pi\alpha Z_1)(4\pi\alpha Z_2)^2}{\vec{q}_1^2} N_1 N_2' \frac{(2\pi i)^2}{4(2\pi)^4} \frac{1}{D_1} \frac{M_1\mathcal{A}}{2} \int \frac{d^2\vec{q}}{\pi\vec{q}^2\vec{q}_3^2} \frac{1}{4} \mathrm{Tr}\, Q, \tag{13}$$

with the trace

$$\frac{1}{4}\mathrm{Tr}\, Q = \frac{1}{4}\mathrm{Tr}\left[\hat{p}_2\left(-\frac{1}{2}\hat{p} + \hat{q}_1 + m\right)\hat{p}_1\right.$$
$$\left. - \hat{p}_1\left(\frac{1}{2}\hat{p} - \hat{q}_1 + m\right)\hat{p}_2\right](\hat{p} + M)\hat{e}$$
$$= \frac{1}{4}\mathrm{Tr}[\hat{p}_2\hat{q}_1\hat{p}_1 + \hat{p}_1\hat{q}_1\hat{p}_2]M\hat{e} = sM(\vec{q}\vec{e}). \tag{14}$$

The contribution of the last two terms may be found using the relation

$$\int_{-\infty}^{\infty} sd\alpha \frac{1}{D_2 D_3} = (-2\pi i)\frac{2}{\beta_1}\frac{1}{D}, \tag{15}$$

where $D = R - 4\vec{q}\vec{q}_3$. The calculation of the relevant trace leads to

$$\frac{1}{4}\mathrm{Tr}\, \hat{p}_2[(\hat{q}_{-\perp} - \hat{q}_{2\perp} + m)(-\hat{q}_{+\perp} + \hat{q}_{3\perp} - m)$$
$$+ (\hat{q}_{-\perp} - \hat{q}_{3\perp} + m)(-\hat{q}_{+\perp} - \hat{q}_{2\perp} - m)](\hat{p} + M)\hat{e}$$
$$= -2m\frac{1}{4}\mathrm{Tr}\, \hat{p}_2\hat{q}_1\hat{p}\hat{e} = -s\frac{M}{2}\beta_1(\vec{q}_1\vec{e}). \tag{16}$$

Here, the expression contained within the square brackets is $2m(\hat{q}_3 + \hat{q}_2 - \hat{q}_+ - \hat{q}_-) = -2m\hat{q}_1$, the replacement $(\hat{p} + M) \to \hat{p}$ is due to the relation $pe = 0$, and $p_2 p = \frac{1}{2}s\beta_1$. Furthermore, we have

$$\frac{1}{D} - \frac{1}{R} = \frac{4\vec{q}(\vec{q}_2 - \vec{q})}{RD}. \tag{17}$$

We thus obtain

$$\int d\alpha F = \frac{(2\pi i)8(\vec{q}\vec{q}_3)(\vec{e}\vec{q}_1)}{RD} \frac{M\mathcal{A}}{2}. \tag{18}$$

It is worth noting that the right-hand side of this equation vanishes in the the limit where the transverse momenta of each of the three virtual photons vanish, which is a consequence of gauge invariance.

The final result for matrix element corresponding to the Feynman diagram in Fig. 1(a) is

$$M^{Y_1 Y_2 \to Y_1 Y_2 V} = M^2 s \frac{2^7 \pi^2 Z_1 Z_2^2 \alpha^3}{\vec{q}_1^2} \mathcal{A}N_1 N_2'(\vec{q}_1\vec{e})\frac{1}{R} J(\vec{q}_2^2, R)C^V, \tag{19}$$

with

$$J(\vec{q}_2^2, R) = \int \frac{d^2\vec{q}}{\pi\vec{q}^2\vec{q}_3^2} \frac{(\vec{q}\vec{q}_3)}{R - 4(\vec{q}\vec{q}_3)}. \tag{20}$$

As shown in Appendix A, the function J has the form

$$J(\vec{q}_2^2, R) = \frac{1}{2\vec{q}_2^2 - R}\left[2\ln\frac{R}{2\vec{q}_2^2} - \ln\frac{R - \vec{q}_2^2}{\vec{q}_2^2}\right]$$
$$= \frac{1}{\vec{q}_1^2 + M^2}\mathcal{L}(x_1), \tag{21}$$

where

$$\mathcal{L}(x) = \frac{1}{x - 1}\ln\frac{(1 + x)^2}{4x}, \quad x_{1,2} = \frac{\vec{q}_{2,1}^2}{\vec{q}_{1,2}^2 + M^2}. \tag{22}$$

III. DIFFERENTIAL CROSS SECTION OF SINGLE VECTOR-MESON PRODUCTION

The Feynman diagrams contributing to the process of single vector-meson production are shown in Fig. 1. The phase space

volume has the form

$$d\Gamma_3 = \frac{(2\pi)^4}{(2\pi)^9} \frac{d^3 P'_1}{2E'_1} \frac{d^3 P'_2}{2E'_2} \frac{d^3 p}{2p^0} \delta^4(P_1 + P_2 - P'_1 - P'_2 - p). \quad (23)$$

We introduce the factor $d^4 q_1 \delta^4(p_1 - q_1 - p'_1) d^4 q_2 \delta^4(p_2 - q_2 - p'_2)$ in terms of Sudakov variables and use the standard Sudakov parametrization of Eq. (7) [1]. Allowing for the vector meson to be unstable, the three-particle phase space volume then becomes

$$d\Gamma_3 = \frac{1}{4s} \frac{(2\pi)^4}{(2\pi)^9} \pi^2 \frac{d^2\vec{q}_1 d^2\vec{q}_2}{\pi^2} \frac{d\beta_1}{\beta_1} dp^2 R(p^2), \quad (24)$$

where $p^2 = s\alpha_2\beta_1 - (\vec{q}_1 + \vec{q}_2)^2$ and we have replaced the delta function by a Breit-Wigner resonance,

$$\delta(p^2 - M^2) \rightarrow R(p^2) = \frac{1}{\pi} \frac{\Gamma M}{(p^2 - M^2)^2 + M^2\Gamma^2}, \quad (25)$$

where $M \approx 2m_q$ and Γ are the mass and the total decay width of the vector-meson resonance and m_q is the mass of the bound quarks. The quantity $p^2 = (P_1 + P_2 - P'_1 - P'_2)^2$ may be associated with the square of the missing mass in the process of single vector-meson production.

With the matrix element of Eq. (19), we now have

$$M^{Y_1 Y_2 \rightarrow Y_1 Y_2 V} = s \frac{2^7 \pi^2 M^2 \alpha^3 Z_1 Z_2 N_1 N_2}{R} \mathcal{A} Z, \quad (26)$$

where

$$Z = \left[\frac{Z_2(\vec{q}_1 \vec{e})}{\vec{q}_1^2(\vec{q}_1^2 + M^2)} \mathcal{L}(x_1) + \frac{Z_1(\vec{q}_2 \vec{e})}{\vec{q}_2^2(\vec{q}_2^2 + M^2)} \mathcal{L}(x_2) \right] C^V. \quad (27)$$

The relevant differential cross section is

$$\frac{d\sigma^{(1)}}{dp^2} = \frac{2^6 \pi (Z_1 Z_2)^2 \alpha^6 \mathcal{A}^2 M^4}{R^2} Z^2 \frac{d^2\vec{q}_1 d^2\vec{q}_2}{\pi^2} \frac{d\beta_1}{\beta_1} R(p^2). \quad (28)$$

The interference term in Z^2 is canceled by the average over the azimuthal angle. For the case of extremely small transverse momentum $|\vec{q}_1|$ of ion Y_1, a modification of this formula is necessary, which consists in replacing

$$\frac{1}{(\vec{q}_1^2)^2} \frac{d\beta_1}{\beta_1} \rightarrow \frac{1 - \beta_1}{(\vec{q}_1^2 + m_1^2 \beta_1^2)^2} \frac{d\beta_1}{\beta_1}, \quad (29)$$

and a similar replacement for small values of $|\vec{q}_2|$. This leads to the so-called Weizsäcker-Williams enhancement factor in the total cross section:

$$\frac{d\sigma^{(1)}}{dp^2} = \sigma_0 R(p^2) \left[Z_2^2 (L_1^2 - 5L_1) + Z_1^2 (L_2^2 - 5L_2) \right.$$
$$\left. + c(Z_1^2 + Z_2^2) + O\left(\frac{m_1^2}{s}, \frac{m_2^2}{s} \right) \right], \quad (30)$$

where

$$\sigma_0 = \frac{32\pi (Z_1 Z_2 \alpha^3)^2 \mathcal{A}^2}{M^2} (C^V)^2 (1 - \ln 2),$$

$$L_{1,2} = \ln \frac{s}{m_{1,2}^2},$$

$$c = 2 \int_0^1 \frac{dx}{x} \left[-\frac{5 + x}{2(1 - x)^3} + \frac{5}{2} + \left(\frac{1 + 2x}{(1 - x)^4} - 1 \right) \ln \frac{1}{x} \right]$$

$$\approx 10.4565. \quad (31)$$

In Sec. V, we present typical values of $\sigma^{(1)}$ for orthopositronium and the ρ, ω, and J/ψ mesons.

IV. DIFFERENTIAL CROSS SECTION OF DOUBLE VECTOR-MESON PRODUCTION

In the case of two vector mesons in the final state, we must consider the two mechanisms due to the sets of Feynman diagrams indicated in Figs. 2(a) and 2(b).

The phase space volume of the four-particle final state is

$$d\Gamma_4 = \frac{(2\pi)^4}{(2\pi)^{12}} \pi^3 \frac{d\beta_1}{\beta_1} \frac{d\beta'}{\beta'} \frac{d^2\vec{q}_1 d^2\vec{q}' d^2\vec{q}_2}{\pi^3}$$
$$\times dr_1^2 dr_2^2 R(r_1^2) R(r_2^2) \frac{1}{8s}, \quad (32)$$

where $r_1^2 = s\alpha'\beta_1 - (\vec{q}_1 - \vec{q}')^2$ and $r_2^2 = s\alpha_2\beta' - (\vec{q}_2 + \vec{q}')^2$. Also here, we assume that $m_{1,2}^2/s \ll \beta' \ll \beta_1 \ll 1$. Let us introduce another auxiliary four-vector: $q' = \alpha' p_2 + \beta' p_1 + q'_\perp$.

The matrix element of the process of double vector-meson production mediated by a virtual vector meson [see Fig. 2(a)] has the form

$$M_a^{Y_1 Y_2 \rightarrow Y_1 Y_2 V_1 V_2} = s \frac{(Z_1 Z_2 \alpha^2)^2 N_1 N_2 \mathcal{A}_1 \mathcal{A}_2 (M_{V_1} M_{V_2})^2 g^2 \pi^2 2^9}{\vec{q}^2 + M_V^2}$$
$$\times T(\vec{q}\, \vec{e}_1)(\vec{q}\, \vec{e}_2) C_2^{V_1} C_2^{V_2}, \quad (33)$$

where T is given by the expression

$$T = \frac{J(q_1^2, R_1) J(q_2^2, R_2)}{R_1 R_2} + \frac{J(q_1^2, \bar{R}_1) J(q_2^2, \bar{R}_2)}{\bar{R}_1 \bar{R}_2}, \quad (34)$$

with $R_1 = q^2 + q_1^2 + M_1^2$, $R_2 = q^2 + q_2^2 + M_2^2$, $\bar{R}_1 = q^2 + q_1^2 + M_2^2$, and $\bar{R}_2 = q^2 + q_2^2 + M_1^2$. Here, M_V is the mass of the virtual vector meson, g is its coupling constant, and $M_{1,2}$ are the masses of the created vector mesons.

The cross section of the process of double vector-meson production is

$$\frac{d\sigma_a^{(2)}}{dr_1^2 dr_2^2} = s \frac{2^6 \alpha^8 (M_1 M_2 Z_1 Z_2)^4}{(\vec{q}^2 + M_V^2)^2} g^4 (\mathcal{A}_1 \mathcal{A}_2)^2 R(r_1^2) R(r_2^2) |T|^2 (\vec{q}\vec{e}_1)^2 (\vec{q}\vec{e}_2)^2 (C_2^{V_1} C_2^{V_2})^2 \frac{d^2\vec{q}_1 d^2\vec{q}_2 d^2\vec{q}}{\pi^3} \frac{d\beta_1}{\beta_1} \frac{d\beta}{\beta}. \quad (35)$$

After integration over $d^2\vec{q}$, we obtain

$$\frac{d\sigma_a^{(2)}}{dr_1^2 dr_2^2} = \frac{8 Z_1^4 Z_2^4 g^4 \alpha^8}{\pi} (1 + 2\cos^2\theta_{12}) \frac{d\beta_1}{\beta_1} \frac{d\beta}{\beta} (\mathcal{A}_1 \mathcal{A}_2)^2 \left(\frac{M_{V_1}^2 M_{V_2}^2}{M_V^5} \right)^2 R(r_1^2) R(r_2^2) (C_2^{V_1} C_2^{V_2})^2 \frac{d\vec{q}_1^2 d\vec{q}_2^2}{M_V^4} P\left(\frac{\vec{q}_1^2}{M_V^2}, \frac{\vec{q}_2^2}{M_V^2} \right), \quad (36)$$
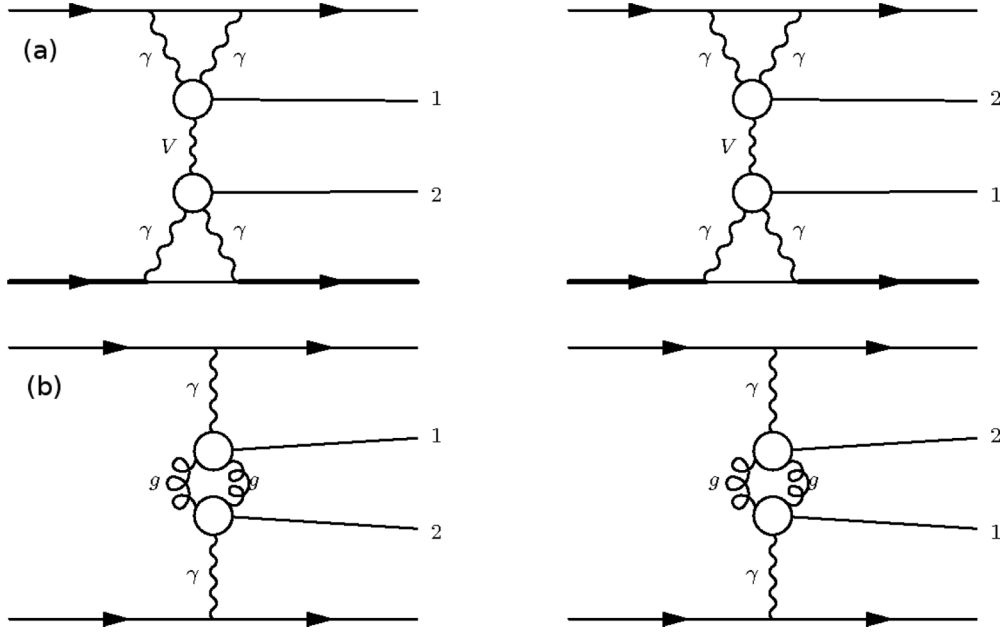
FIG. 2. Feynman diagrams pertinent to two vector-meson production in peripheral heavy-ion collisions via intermediate (a) vector-meson and (b) two-gluon states.

where $\cos\theta_{12} = \cos\widehat{\vec{e}_1\vec{e}_2}$ and

$$P\left(\frac{\vec{q}_1^2}{M_V^2}, \frac{\vec{q}_2^2}{M_V^2}\right) = \int_0^\infty \frac{d\vec{q}^2}{\left(\vec{q}^2 + M_V^2\right)^2} \frac{(\vec{q}^2)^2}{M_V^2} \left|TM_V^8\right|^2 \tag{37}$$

is evaluated in Appendix B yielding the numerical values presented in Table I.

For the case of two-gluon exchange [see Fig. 2(b)], the matrix element has the form

$$M_b^{Y_1Y_2 \to Y_1Y_2V_1V_2} = is\frac{2^{12}\pi^3\left(Z_1Z_2\alpha^2\alpha_s^2\right)\left(M_{V_1}M_{V_2}\right)^2N_1N_2}{\vec{q}_1^2\vec{q}_2^2}\mathcal{A}_1\mathcal{A}_2QC_1^{V_1}C_1^{V_2}, \tag{38}$$

where

$$Q = \frac{I(R_1, R_2)}{R_1R_2}(\vec{e}_1\vec{q}_1)(\vec{e}_2\vec{q}_2) + \frac{I(\bar{R}_1, \bar{R}_2)}{\bar{R}_1\bar{R}_2}(\vec{e}_1\vec{q}_2)(\vec{e}_2\vec{q}_1). \tag{39}$$

The quantity $I(R_1, R_2)$ is given in Appendix A. For the cross section, we may write

$$\frac{d\sigma_b^{(2)}}{dr_1^2dr_2^2} = \frac{2^{12}\pi\left(Z_1Z_2\alpha^2\alpha_s^2\right)^2(M_{V_1}M_{V_2})^4}{\left(\vec{q}_1^2\vec{q}_2^2\right)^2}R(r_1^2)R(r_2^2)Q^2\frac{d\beta_1}{\beta_1}\frac{d\beta}{\beta}(\mathcal{A}_1\mathcal{A}_2)^2\frac{d^2\vec{q}_1d^2\vec{q}_2d^2\vec{q}}{\pi^3}\left(C_2^{V_1}C_2^{V_2}\right)^2. \tag{40}$$

Performing the integration over $d^2\vec{q}$, we obtain

$$\frac{d\sigma_b^{(2)}}{dr_1^2dr_2^2} = \frac{2^{10}\pi\left(Z_1Z_2\alpha^2\alpha_s^2\right)^2dx_1dx_2}{x_1x_2M_{V_1}M_{V_2}}R(r_1^2)R(r_2^2)\frac{d\beta_1}{\beta_1}\frac{d\beta}{\beta}(\mathcal{A}_1\mathcal{A}_2)^2\left(C_2^{V_1}C_2^{V_2}\right)^2[\Phi + \bar{\Phi} + 2\cos^2\theta_{12}G]W, \tag{41}$$

where the functions Φ, $\bar{\Phi}$, and G are evaluated in Appendix B yielding the numerical values presented in Table II,

$$W = \frac{d\vec{q}_1^2}{\vec{q}_1^2}\frac{d\vec{q}_2^2}{\vec{q}_2^2} \tag{42}$$

is the Weizsäcker-Williams enhancement factor, and

$$x_{1,2} = \frac{\vec{q}_{1,2}^2}{M_{V_1}M_{V_2}}. \tag{43}$$

In the region of small values of $\vec{q}_{1,2}^2$, we must replace $\vec{q}_1^2 \to \vec{q}_1^2 + \beta_1^2m_1^2$, $\vec{q}_2^2 \to \vec{q}_2^2 + \alpha_2^2m_1^2$.

We note that the interference term of the two amplitudes is absent, so that $|M_a^{(2)} + M_b^{(2)}|^2 = |M_a^{(2)}|^2 + |M_b^{(2)}|^2$.

V. DISCUSSION AND CONCLUSIONS

We studied the production of one or two vector mesons in peripheral heavy-ion collisions at high energies. In the case

TABLE I. Values of the function $P(x_1, x_2; 1, 1)$, defined in Eq. (B1), for different values of x_1 and x_2.

$x_1\backslash x_2$	0.1	0.5	1	1.5	2	3	4	5
0.1	0.076	0.0047	0.00049	0.000082	0.00004	0.000046	0.000045	0.000038
0.5	0.0047	0.00036	0.000049	0.00001	3.66×10^{-6}	2.51×10^{-6}	2.48×10^{-6}	2.23×10^{-6}
1	0.00049	0.000049	9.0049×10^{-6}	2.34×10^{-6}	7.85×10^{-7}	2.52×10^{-7}	2.136×10^{-7}	2.03×10^{-7}
1.5	0.000082	0.00001	2.34×10^{-6}	7.39×10^{-7}	2.78×10^{-7}	6.46×10^{-8}	3.357×10^{-8}	2.85×10^{-8}
2	0.00004	3.66×10^{-6}	7.85×10^{-7}	2.78×10^{-7}	1.29×10^{-7}	4.59×10^{-8}	2.52×10^{-8}	1.76×10^{-8}
3	0.000046	2.51×10^{-6}	2.52×10^{-7}	6.46×10^{-8}	4.59×10^{-8}	4.02×10^{-8}	3.21×10^{-8}	2.45×10^{-8}
4	0.000045	2.48×10^{-6}	2.14×10^{-7}	3.36×10^{-8}	2.52×10^{-8}	3.21×10^{-8}	2.91×10^{-8}	2.35×10^{-8}
5	0.000038	2.23×10^{-6}	2.029×10^{-7}	2.85×10^{-8}	1.76×10^{-8}	2.45×10^{-8}	2.35×10^{-8}	1.96×10^{-8}

of $Z\alpha > \alpha_s$, a simplified version of a general theory [4] can be used to lowest order of QED and QCD that is based on the subprocesses $\gamma^\star \gamma^\star g \to V$ and $\gamma^\star g g \to V$. In our study of double vector-meson production, a vector meson can also appear virtually as an intermediate state. In this case, it is important to replace it by the relevant vector Reggeon state, with the Regge trajectory

$$\alpha_V((\vec{q}')^2) = \alpha_V(0) + \alpha_V'(0)(\vec{q}')^2 \approx \alpha_V(0) \approx 1/2. \quad (44)$$

This results in an additional factor

$$\left(\frac{p^2}{s_0}\right)^{2(\alpha_V(0)-1)} \sim \frac{s_0}{p^2}, \quad (45)$$

where $s_0 \sim 1$ GeV and p^2 is the missing-mass square, in the cross section $d\sigma_a^{(2)}$.

When constructing the invariant mass square of the decay products of one vector meson, $p^2 = (P_1 + P_2 - P_1' - P_2')^2 = s\beta_1\alpha_2 - (\vec{q}_1 + \vec{q}_2)^2$, we take into account that the part $s\beta_1\alpha_2$ is the combination $(\sum E_i)^2 - (\sum p_{iz})^2$, with E_i and p_{iz} being the energies and z components of the three-momenta of the decay products, while the part $-(\vec{q}_1 + \vec{q}_2)^2$ is the contribution from the transverse components $-(\sum p_{i\perp})^2$. Here, it is understood that the z direction is taken along the beam axis in the center-of-mass frame. Such is the case for double jet production with $r_1^2 = (P_1 - P_1' - q')^2 = s\alpha'\beta_1 - (\vec{q}_1 - \vec{q}')^2$ and $r_2^2 = (P_2 - P_2' + q')^2 = s\alpha_2\beta' - (\vec{q}_2 + \vec{q}')^2$.

The coupling constant $\mathcal{A}$ of the meson-photon interaction in the case of single vector-meson production, appearing in Eq. (5), is given by $\mathcal{A} = \alpha^{3/2}/(2\sqrt{\pi})$ for orthopositronium and by $\mathcal{A}_i = 2f_{V_i}/M_{V_i}$, with $f_\rho = f_\omega = 0.21$ GeV and $f_\psi = 0.38$ GeV, for the ω, ρ, and J/ψ mesons, respectively (see Ref. [6] for details).

In the case of single vector-meson production, the color and charge factors are $C^V = 1$ for orthopositronium, $C^V = 3\sum_{u,d} Q_q^3 = \frac{7}{3}$ for the ρ and ω mesons, and $C^V = \frac{8}{3}$ for the J/ψ meson. In the case of double vector-meson production [7] through the mechanism shown in Fig. 2(a), we have $C_2^{V_1} = 3\sum Q_q^2 = \frac{5}{3}$ for the ρ and ω mesons, and $C_2^{V_1} = \frac{4}{3}$ for the J/ψ meson. In the case of the mechanism shown in Fig. 2(b), we have $C_1^{V_1} = 3\sum Q_q = 1$ for the ρ and ω mesons, and $C_1^{V_1} = 3\frac{2}{3} = 2$ for the J/ψ meson.

Note that the mechanism involving single γ^* exchange [see Fig. 2(b)] yields a L^4 enhancement, whereas double γ^* exchange [see Fig. 2(a)] only produces a L^2 enhancement. For pp collisions at the LHC, the "large" logarithm is as large as $L = \ln \frac{s}{m^2} \approx 7$.

We do not consider gluon exchange between heavy ions and vector mesons to avoid channels with ion excitation.

Our study was performed in terms of the momenta of the particles and the fractions β_1 and α_2 of energy transferred from the ions via the virtual photons to the created vector mesons. In the case of the production of a single vector meson, its on-mass-shell condition implies that $s\alpha_2\beta_1 > M_V^2$ with $\alpha_2, \beta_1 < 1$, and its rapidity is

$$\int \frac{d\beta_1}{\beta_1} = \ln \frac{E_V + p_{Vz}}{E_V - p_{Vz}}. \quad (46)$$

As for the production of two vector mesons, there is a rapidity gap between them. The impact factor ρ is inversely related to the modulus $|q|$ of the two-dimensional transverse momentum q. In the Weizsäcker-Williams approximation, we have for the

TABLE II. Values of the function $\Phi(x_1, x_2; 1, 1) = \bar{\Phi}(x_1, x_2; 1, 1) = G(x_1, x_2; 1, 1)$, defined in Eq. (B4), for different values of x_1 and x_2.

$x_1\backslash x_2$	0.1	0.5	1	1.5	2	3	4	5
0.1	1.67	1.006	0.623	0.428	0.314	0.191	0.129	0.094
0.5	1.006	0.6142	0.386	0.268	0.198	0.123	0.08399	0.0615
1	0.623	0.386	0.246	0.173	0.129	0.0808	0.0559	0.0414
1.5	0.428	0.268	0.173	0.1221	0.0918	0.0583	0.04078	0.0304
2	0.314	0.198	0.129	0.0918	0.0695	0.0446	0.0314	0.0235
3	0.191	0.123	0.0808	0.0583	0.0446	0.029	0.02073	0.0157
4	0.1297	0.08399	0.0559	0.04078	0.0314	0.02073	0.149	0.01142
5	0.094	0.0615	0.0414	0.0304	0.0235	0.0157	0.01142	0.0088

distributions

$$\int_0^{q^2_{\max}} \frac{d\vec{q}^2}{\vec{q}^2 + q^2_{\min}} = \int_{R^2_{ion}}^{\rho^2_{\max}} \frac{d\rho^2}{\rho^2}, \qquad (47)$$

where $|q_{\min}| \approx m_{ion} M_V^2/s$ is determined by the characteristic values of the momentum transfer to the ion, and R_{ion} and m_{ion} are the radius and mass of the ion, respectively. For $Z = 80$ and $A = 200$, we have $R_{ion} \approx 5$ fm. Also assuming $M_V = 3$ GeV and $\sqrt{s} = 7$ TeV, we have $|q_{\min}| \approx 0.04$ MeV. The maximum allowed value of the momentum transfer $|q_{\max}|$ is determined by the condition that the ions do not reach exited states. Nuclear resonances have masses of typically 30–100 MeV. Thus, the momentum transfer varies in the range 0.04 MeV $< |q| <$ 100 MeV. The corresponding range of variation of the impact parameter is $R_{ion} < \rho < \rho_{\max}$, where $\rho_{\max} = h/|q_{\min}| \approx 5$ pm.

We now estimate the cross section of single vector-meson production according to Eq. (30). Using the approximation $\int dp^2 R(p^2) = 1$, we estimate $\sigma^{(1)}$ for $Z_1 = Z_2 = 80$ and $L_1 = L_2 = 7$ to be 0.8 pb for orthopositronium, $\sigma_\rho = 3.2$ pb for ρ and σ mesons, and 4 pb for J/ψ mesons.

Using similar approximations in the case of double vector-meson production, we find

$$\frac{\sigma_a^{(2)}}{\sigma^\rho} \sim (Z\alpha)^2 \approx 0.34, \quad \frac{\sigma_b^{(2)}}{\sigma^\rho} \sim \frac{\alpha_s^4 L^2}{(Z\alpha)^2} \sim 1. \quad (48)$$

APPENDIX A

In this section, we shall explain how to calculate the two-dimensional Euclidean integrals appearing in Eqs. (19) and (39),

$$J(\vec{q}_2^2, R) = \int \frac{d^2\vec{k}}{\pi} \frac{\vec{k}(\vec{q}_2 - \vec{k})}{\vec{k}^2(\vec{q}_2 - \vec{k})^2 D},$$

$$I(R_1, R_2) = 4 \int \frac{d^2\vec{k}}{\pi} \frac{(\vec{k}(\vec{q} - \vec{k}))^2}{\vec{k}^2(\vec{q} - \vec{k})^2 D_1 D_2}, \qquad (A1)$$

where $D = R - 4\vec{k}(\vec{q} - \vec{k})$, $R = \vec{q}_1^2 + \vec{q}_2^2 + M^2$, $D_i = R_i - 4\vec{k}(\vec{q} - \vec{k})$, and $R_i = q_i^2 + q^2 + M_i^2$. For the first one, we have

$$4J = \int \frac{d^2\vec{k}}{\pi} \frac{R - D}{\vec{k}^2(\vec{q}_2 - \vec{k})^2 D} = \lim_{\lambda \to 0}[RJ_1 - J_0], \quad (A2)$$

with

$$J_1 = \int \frac{d^2\vec{k}}{\pi} \frac{1}{(\vec{k}^2 + \lambda^2)((\vec{q}_2 - \vec{k})^2 + \lambda^2)D},$$

$$J_0 = \int \frac{d^2\vec{k}}{\pi} \frac{1}{(\vec{k}^2 + \lambda^2)((\vec{q}_2 - \vec{k})^2 + \lambda^2)}. \qquad (A3)$$

Using Feynman's trick of joining the denominators,

$$\frac{1}{ab} = \int_0^1 dx \frac{1}{(xa + \bar{x}b)^2}, \qquad (A4)$$

with $\bar{x} = 1 - x$, we obtain for J_0:

$$J_0 = \int_0^1 dx \int \frac{d^2\vec{k}}{\pi} \frac{1}{\left[(\vec{k} - \vec{q}_2 x)^2 + \vec{q}_2^2 x\bar{x} + \lambda^2\right]^2}$$

$$= \int_0^1 \frac{dx}{\vec{q}_2^2 x\bar{x} + \lambda^2} = \frac{2}{\vec{q}_2^2} \ln \frac{\vec{q}_2^2}{\lambda^2}. \qquad (A5)$$

For J_1, we have

$$4J_1 = \int_0^1 dx \int_0^1 \frac{ydy}{\left[\vec{q}_2^2 x\bar{x} y^2 + \frac{\bar{y}}{4}(R - \bar{y}\vec{q}_2^2) + y\lambda^2\right]^2}, \quad (A6)$$

where $\bar{y} = 1 - y$. Introducing the variable $t = 1 - 2x$, we may cast this into the form:

$$J_1 = 4 \int_0^1 ydy \int_0^1 \frac{dt}{(A - Bt^2)^2}, \qquad (A7)$$

where

$$A = \vec{q}_2^2 y^2 + \bar{y}(R - \vec{q}_2^2 \bar{y}) + 4y\lambda^2, \quad B = \vec{q}_2^2 y^2. \quad (A8)$$

Performing the integration over t, we obtain

$$J_1 = \int_0^1 \frac{ydy}{A^{3/2}B^{1/2}} \left[\frac{2(AB)^{1/2}}{A - B} + \ln \frac{A^{1/2} + B^{1/2}}{A^{1/2} - B^{1/2}} \right]$$

$$= (I_1 + I_2)\left(\frac{1}{\vec{q}^2}\right)^2, \qquad (A9)$$

where

$$I_1 = 2 \int_0^1 \frac{ydy}{[\bar{y}(\rho - \bar{y}) + 4y\sigma]T}, \quad I_2 = \int_0^1 \frac{dy}{T^{3/2}} \ln \frac{T^{1/2} + y}{T^{1/2} - y}, \quad (A10)$$

with

$$T = y^2 + \bar{y}(\rho - \bar{y}), \quad \rho = \frac{R}{\vec{q}^2}, \quad \sigma = \frac{\lambda^2}{\vec{q}^2}. \quad (A11)$$

The first integral I_1 contains an infrared singularity. Introducing the small parameters σ and ϵ, with $\sigma \ll \epsilon \ll 1$, we rewrite it as

$$I_1 = 2 \int_0^{1-\epsilon} \frac{ydy}{\bar{y}(\rho - \bar{y})[\rho - 1 + y(2 - \rho)]} + 2 \int_{1-\epsilon}^1 \frac{dy}{\bar{y}\rho + 4\sigma}$$

$$= \frac{2}{\rho} \ln \frac{\rho}{4\sigma} - \frac{2}{\rho(\rho - 1)}[(\rho - 1)\ln(\rho - 1) + \ln \rho]. \quad (A12)$$

For the second integral I_2, which is infrared finite, the substitutions $T = t^2$, $y = \frac{t^2 - a}{b}$, $a = \rho - 1$, and $a + b = 1$

yield

$$I_2 = \frac{2}{b}\int_{\sqrt{a}}^1 \frac{dt}{t^2}\ln\frac{tb+t^2-a}{tb-(t^2-a)} = \frac{2}{b}\int_{\sqrt{a}}^1 \frac{dt(1-t)}{t}\left[\frac{b+2t}{tb+t^2-a} - \frac{b-2t}{tb-t^2+a}\right]$$

$$= 4\int_{\sqrt{a}}^1 dt\frac{t^2+a}{t(1+t)(t^2-a^2)} = \frac{2}{b}[\ln a - 2\ln 2 + \frac{a+1}{a}\ln(a+1)]. \tag{A13}$$

The total answer for J_1 is

$$J_1(\rho) = 2\left(\frac{1}{\vec{q}^2}\right)^2\left[\frac{1}{\rho}\ln\frac{1}{\sigma} + \frac{2}{\rho(2-\rho)}(2\ln\rho - 2\ln 2 - \ln(\rho-1))\right]. \tag{A14}$$

For the sum $J = RJ_1 - J_0$, we obtain

$$J(R, \vec{q}_2^2) = \frac{1}{2\vec{q}_2^2 - R}\ln\frac{R^2}{4(R-\vec{q}_2^2)\vec{q}_2^2} = \frac{1}{\vec{q}_1^2 + M^2}\frac{1}{x-1}\ln\frac{(x+1)^2}{4x}, \tag{A15}$$

where $x = \vec{q}_2^2/(\vec{q}_1^2 + M^2)$.

For the integral I, we have

$$4I = \int \frac{d^2\vec{k}}{\pi}\frac{1}{\vec{k}^2(\vec{q}-\vec{k})^2}\frac{(R_1-D_1)(R_2-D_2)}{D_1 D_2} = J_0 + \frac{R_2^2}{R_1-R_2}J_1(R_2) - \frac{R_1^2}{R_1-R_2}J_1(R_1), \tag{A16}$$

where J_1 given in Eq. (A9) and $R_{1,2} = \vec{q}_{1,2}^2 + \vec{q}^2 + M_{1,2}^2$. The infrared singularity is canceled using

$$I = \frac{1}{R_1-R_2}\left\{\frac{R_2}{2\vec{q}^2-R_2}\ln\frac{R_2^2}{4\vec{q}^2(R_2-\vec{q}^2)} - \frac{R_1}{2\vec{q}^2-R_1}\ln\frac{R_1^2}{4\vec{q}^2(R_1-\vec{q}^2)}\right\}. \tag{A17}$$

For the case of $R_1 = R_2 = R = \vec{q}_1^2 + \vec{q}^2 + M^2$, we have

$$I(R, R) = -\frac{\partial}{\partial R}\int \frac{d^2\vec{k}}{\pi}\frac{\vec{k}(\vec{q}-\vec{k})(R-D)}{\vec{k}^2(\vec{q}-\vec{k})^2 D} = -4\frac{\partial}{\partial R}[RJ(R, \vec{q}^2)] = 4\left[\frac{1}{R-\vec{q}_1^2} - \frac{2\vec{q}_1^2}{2\vec{q}_1^2-R}\ln\frac{R^2}{4\vec{q}_1^2(R^2-\vec{q}_1^2)}\right]. \tag{A18}$$

For the cross section of single vector-meson production, integrated over $\vec{q}_1^2$ and $\vec{q}_2^2$, we have

$$\frac{d\sigma^{(1)}}{dp^2 R(p^2)} = \frac{16\pi(Z_1 Z_2\alpha^3)^2 A^2}{M^2}\int_0^\infty \frac{dx}{(x+1)^2(x-1)^2}\ln^2\left(\frac{(x+1)^2}{4x}\right)\left\{Z_1^2\int_{\frac{m_1^2}{s}}^1 \frac{d\beta_1}{\beta_1}(1-\beta_1)\int_0^\infty \frac{dt\cdot t}{(1+t)^3(t+\beta_1^2\rho_1^2)}\right.$$

$$\left. + \left(\beta_1 \to \alpha_2, Z_1 \leftrightarrow Z_2, \rho_1 = \frac{m_1^2}{M^2} \to \rho_2 = \frac{m_2^2}{M^2}\right)\right\}. \tag{A19}$$

Using

$$\int_0^\infty \frac{dx}{(x^2-1)^2}\ln^2\left(\frac{(x+1)^2}{4x}\right) = 2(1-\ln 2), \tag{A20}$$

we obtain the expression given in Eq. (31).

APPENDIX B

In this section, we shall explain how to evaluate the integrals appearing in Eqs. (36) and (41) relevant for the mechanisms based on vector-meson [see Fig. 2(a)] and two-gluon [see Fig. 2(b)] exchange, respectively.

In the first case, we have

$$P(x_1, x_2; \rho_1, \rho_2) = \int_0^\infty \frac{dx\cdot x^2}{(x+1)^2}\tau^2, \tag{B1}$$

with

$$\tau = \frac{i(x_1, r_1)i(x_2, r_2)}{r_1 r_2} + \frac{i(x_1, \bar{r}_1)i(x_2, \bar{r}_2)}{\bar{r}_1\bar{r}_2}, \tag{B2}$$

where

$$i(x, r) = \frac{1}{2x - r} \ln \frac{r^2}{4x(r - x)},$$
(B3)

and $r_1 = x + x_1 + \rho_1$, $r_2 = x + x_2 + \rho_2$, $\bar{r}_1 = x + x_1 + \rho_2$, $\bar{r}_2 = x + x_2 + \rho_1$, $\rho_1 = M_{V_1}^2/M_V^2$, and $\rho_2 = M_{V_2}^2/M_V^2$. Numerical values of $P(x_1, x_2; 1, 1)$, appropriate for the important case $\rho_1 = \rho_2 = 1$, are listed in Table I.

On the other hand, we have

$$\Phi(x_1, x_2; \rho_1, \rho_2) = \int_0^\infty dx \left(\frac{j(r_1, r_2)}{r_1 r_2} \right)^2, \quad \bar{\Phi}(x_1, x_2; \rho_1, \rho_2) = \int_0^\infty dx \left(\frac{j(\bar{r}_1, \bar{r}_2)}{\bar{r}_1 \bar{r}_2} \right)^2,$$

$$G(x_1, x_2; \rho_1, \rho_2) = \int_0^\infty dx \frac{j(r_1, r_2) j(\bar{r}_1, \bar{r}_2)}{r_1 r_2 \bar{r}_1 \bar{r}_2}$$
(B4)

where

$$j(r_1, r_2) = \frac{1}{r_1 - r_2} \left[\frac{r_2}{2x - r_2} \ln \frac{r_2^2}{4x(r_2 - x)} - \frac{r_1}{2x - r_1} \ln \frac{r_1^2}{4x(r_1 - x)} \right].$$
(B5)

In the case $r_1 = r_2$, we have [see Eq. (A18)]

$$j(r_1, r_1) = 4 - \frac{8x}{(x - 1)^2} \ln \left(\frac{(x + 1)^2}{4x} \right), \quad x = \frac{\vec{q}_1^2}{\vec{q}^2 + M^2}.$$
(B6)

For $\rho_1 = \rho_2$, we have $\Phi(x_1, x_2; \rho_1, \rho_1) = \bar{\Phi}(x_1, x_2; \rho_1, \rho_1) = G(x_1, x_2; \rho_1, \rho_1)$. Numerical values for the important choice $\rho_1 = \rho_2 = 1$ are listed in Table II.

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