Systematic studies of the Richardson–Lucy deconvolution algorithm applied to VHE gamma data

S. Heinz^a, I. Jung^{a,*}, C. Stegmann^{b,c,d}

^a Department of Physics, University of Erlangen-Nuremberg, Erwin-Rommel-Str. 1, Erlangen D-91058, Germany

^b DESY, Platanenallee 6, D-15738 Zeuthen, Germany

^c Institut für Physik und Astronomie, Universität Potsdam, Karl-Liebknecht-Str. 24/25, D-14476 Potsdam-Golm, Germany

ARTICLE INFO

Article history:

Received 13 February 2012

Received in revised form 4 May 2012

Accepted 10 May 2012

Available online 2 June 2012

Keywords:

Very high energy gamma-ray astronomy
Deconvolution

ABSTRACT

The Richardson–Lucy deconvolution algorithm was applied to astronomical images in the very high-energy regime with photon energies above 100 GeV. Through a systematic study with respect to source significance, background level and source morphology we were able to derive optimal deconvolution parameters. The results presented show that deconvolution makes it possible to study structural details well below the angular resolution of the very high-energy γ -ray experiment.

© 2012 Elsevier B.V. All rights reserved.

1. Introduction

Owing to the currently operating Cherenkov telescope systems, the number of detected very high-energy γ -ray (VHE; >100 GeV) sources has increased in the last years to more than 100 and has revealed a variety of different source classes [1,2]. Most of the sources are extended sources with a detailed structure. For example in the case of supernova remnants, ring-like morphologies have been observed [3–6]. This may be a hint to the γ -ray emission processes taking place in the sources, but up to now these are still subject of debate. Correlation studies between different wavebands and the detailed investigation of source morphologies are suitable approaches to pin-point the emission region and to shed light on the underlying processes responsible for the γ -ray emission. Detailed correlation studies are hampered by differences in angular resolution between VHE γ -ray and X-ray instruments. ASCA, for example, has an angular resolution of $2.9'$ half power diameter [7] which is better by a factor of two compared to VHE γ -ray instruments.

In order to unveil the true morphology of VHE γ -ray sources, we studied the application of the Richardson–Lucy deconvolution algorithm (RLA) [8,9] to VHE γ -ray images. The Richardson–Lucy algorithm is commonly used in astronomy. Its advantage is the conservation of the global flux. Disadvantages are noise amplification and unstable solutions. In the VHE regime, the noise is usually

of the same order as the signal strength, requiring a detailed understanding of the behavior of the algorithm in order to extract improved images of VHE γ -ray sources. In this paper, detailed systematic studies on deconvolving VHE γ -ray data based on the RLA are presented. First, the algorithm is described followed by detailed simulation studies of this one. Finally, the algorithm is applied to simulated VHE γ -ray and X-ray data and it is shown how the correlation coefficient depends on the region size for which the coefficient was calculated before and after deconvolution. The paper closes with conclusions and expectations with respect to the improvements to be gained by application to already detected VHE γ -ray sources.

2. Deconvolution algorithm

An observed astronomical image $I(x, y)$ with the right ascension (RA) on the x -axis and the declination (Dec) on the y -axis is given by

$$I(x, y) = A(x, y)(O \otimes P)(x, y) + A(x, y)B(x, y) \quad (1)$$

where $(O \otimes P)(x, y)$ is the convolution of the true source image $O(x, y)$ and the point spread function (PSF) $P(x, y)$. $B(x, y)$ is an additional noise term and $A(x, y)$ the camera acceptance over the field of view. The PSF describes the response of the Cherenkov telescope to a point source while the noise term takes into account that measurements in the VHE- γ -ray domain are not background free. The deconvolution is applied to a so-called excess-map, which is given by:

$$E(x, y) = (O \otimes P)(x, y) \quad (2)$$

* Corresponding author.

E-mail address: ira.jung@physik.uni-erlangen.de (I. Jung).

This map is obtained by first subtracting the estimated background ($A(x, y)B'(x, y)$) from the image $I(x, y)$ and then correcting the obtained image ($I(x, y) - A(x, y)B'(x, y)$) by the acceptance over the field of view. Deconvolution techniques allow to restore the original image under the assumption that the PSF is known.

2.1. The Richardson–Lucy algorithm

The Richardson–Lucy algorithm is a Bayesian-based iterative algorithm developed for Poisson distributed background [8,9] that converges to the maximum likelihood solution [9].

The deconvolved image of the $n + 1$ -th iteration $O^{n+1}(x, y)$ is given by

$$O^{n+1}(x, y) = \left[\frac{E(x, y)}{(P \otimes O^n)(x, y)} \times P^T(x, y) \right] O^n(x, y) \quad (3)$$

P^T is the transposed point spread function. The first estimation $O^0(x, y)$ is assumed to be the uniform distribution.

Like many iterative deconvolution techniques, the RLA has the disadvantage, that for structures with low significance, the statistical fluctuations are interpreted as real structures leading to large fluctuations in the deconvolved image. In the following we call these fluctuations *artifacts*. The number of iterations and the significance of the source determine how severe the production of artifacts becomes which may ultimately render deconvolution useless. Therefore, the study of this influence using Monte Carlo simulations is an important part of this investigation which is described in the following section.

3. Simulation of VHE γ -ray maps

A simple Monte Carlo simulation was developed to identify the optimal number of iterations of the algorithm and to quantify the improvement of the angular resolution for a given source morphology, background level, and source significance.

The output of the simulation are maps of γ -ray excess events which serve as the images $E(x, y)$ for the deconvolution. In γ -ray astronomy, these so-called *excess maps* are obtained after several analysis steps (see e.g. [10] for details of a typical γ -ray analysis). After calibration of the data, γ -ray selection cuts are applied yielding a reduction of the cosmic-ray background by a factor of ~ 100 . The remaining background of γ -ray-like events is determined and subtracted to extract the γ -ray signal (excess). Given a number of counts N_{on} in a test region and N_{off} in a background region, the γ -ray excess is defined as

$$N_{\text{excess}} = N_{\text{on}} - \alpha N_{\text{off}}. \quad (4)$$

The normalization parameter α takes into account differences between the test region and the background region of solid angle, exposure time, zenith angle, and acceptance and is defined as

$$\alpha = \frac{\int_{\text{on}} A_{\text{on}}^{\gamma}(\theta_x, \theta_y, \Phi_z, t) d\theta_x d\theta_y d\Phi_z dt}{\int_{\text{off}} A_{\text{off}}^{\gamma}(\theta_x, \theta_y, \Phi_z, t) d\theta_x d\theta_y d\Phi_z dt}. \quad (5)$$

$A_{\text{on/off}}^{\gamma}$ are the system acceptances of γ -ray-like events in the field of view (FoV) after trigger, selection and analysis cuts. (θ_x, θ_y) is a **position in the FoV** and Φ_z the zenith angle. The acceptance is radially symmetric with respect to the system pointing position. There exists a variety of algorithms to estimate the background [11]. Typical values for the α -factor are in the order of about 0.07, the value chosen in this paper. To obtain an *excess-map*, i.e. an image of the γ -ray excess on the sky, the excess is calculated at each position of the map according to Eq. (4). This map is then corrected for acceptance differences over the FoV.

Free parameters of the *excess map* simulation are the source morphology, the background level and the source significance calculated according to Eq. (7) in [12]. The PSF was taken to be the PSF of the H.E.S.S. experiment at 20° zenith angle. The noise is assumed to be Poisson distributed. The course of the simulation is the following: first, the source morphology is parameterized (source-model), a constant background is added and then convolved with the system acceptance resulting in a model for the N_{on} event distribution (On-model). According to the selected background level and the significance, the number of events N_{on} is calculated and the events are distributed randomly according to the On-model while taking the PSF into account. After adding Poisson noise the obtained map represents the distribution of N_{on} of Eq. (4).

To derive N_{off} , the calculated background events are divided by the α -factor and are randomly distributed according to the acceptance while taking the PSF into account. Application of Eq. (4) with corrections for the acceptance leads to the simulated *excess map*.

The RLA is only applicable to images with exclusive positive entries. In case of negative values, the *excess map* is therefore shifted by the minimal value and back shifted after deconvolution.

To properly determine the error on the simulation results, 100 simulations were performed for each set of simulation parameters. The RMS values of the parameter distribution serve as error estimates for the specific parameter related to the deconvolution procedure.

4. Systematic studies

Based on the simulations, systematic studies on the influence of the number of iterations on the severity of artifacts and on the improvement of angular resolution have been performed, as well as on the source morphology and the significance.

To quantify the quality of the deconvolved image the relative error

$$RE = \sqrt{\frac{\sum_{i,j}^{n_i, n_j} (N_{\text{excess}}(x_i, y_j) - N_{\text{model}}(x_i, y_j))^2}{\sum_{i,j}^{n_i, n_j} N_{\text{model}}(x_i, y_j)^2}} \quad (6)$$

between the *excess map* $N_{\text{excess}}(x_i, y_j)$ and the model $N_{\text{model}}(x_i, y_j)$ was calculated. x_i and y_i stand for the center of the right ascension and the declination bin, respectively.

4.1. Study of the angular resolution

In a first step, the angular resolution as a function of the number of iterations was studied. An appropriate measure for the angular resolution is the 68% containment radius r_{68} of the image of a point source. Fig. 1 left side shows r_{68} as a function of the number of iterations for different source significances of 20σ , 40σ , 60σ , 80σ and 100σ , while the background level was kept constant at 2900 background events in the source region. The number of background events corresponds to the one of a typical galactic source with 10 h of observations at a zenith angle of 20° with the H.E.S.S. telescope system. The dependence on the source strength is very weak. After the first iteration step, the resolution is worse than the initial one, because $O^0(x, y)$ (see Eq. (3)) is assumed to be a uniformly distributed source. With the third iteration, an improvement of about 20% compared to the initial PSF is achieved and after about 6 iterations an improvement of about 40% is obtained. In Fig. 1 right side the improvement in angular resolution with respect to the initial angular resolution before deconvolution is given for zenith angles between 5° and 40° . This overall behavior depends only slightly on the zenith angle.

Fig. 1. The 68% containment radius of the image of a point source is a measure for the angular resolution. On the left side its behavior as a function of number of iterations for different source significances is shown. Black stars, open triangles, filled triangles pointing downward, open crosses and filled triangles pointing upwards stand for 20σ , 40σ , 60σ , 80σ , and 100σ , respectively. On the right side the improvement of the angular resolution relative to the initial one given for different zenith angles. The significance is 40σ . Black stars, open triangles, filled triangles pointing downward, open crosses, filled triangles, pointing upwards, open circle and filled squared stand for 5° , 10° , 15° , 20° , 25° , 30° , 35° and 40° respectively.

Fig. 2. Given are the excess-maps of a ring-like structure deconvolved with the optimal number of iterations (3) on the left and after 20 iterations with obvious artefacts on the right.

4.2. Studies of the influence of the artifacts on the deconvolution

The RLA tends to produce artifacts for sources with low signal-to-noise ratio. The severity depends strongly on the number of iterations, i.e. after a certain number of iterations artifacts begin to dominate over the actual morphology. Fig. 2 shows the excess map of a ring-like morphology with outer- and inner radii of 0.4° and 0.2° after application of the optimal number of iterations (3) and after 20 iterations. The formation of artifacts in the latter case is obvious. Therefore, it is important to determine the number of iterations for which the relative error RE is minimal, the optimal iteration (N_{opt}). Fig. 3 presents RE as a function of the number of iterations for a source with ring-like morphology. The outer and inner radii were 0.4° and 0.2° , respectively, as in Fig. 2. RE decreases with increasing number of iterations, reaches a minimum at 3 iterations and then starts to increase again. This minimum represents the optimal number of iterations N_{opt} combining the minimal number of artifacts with the best fit between source-model and deconvolved image.

Different source morphologies (ring and elliptical) and their dependence on the source strength have been analysed in detail. Examples are shown in Figs. 4 and 5 presenting the optimal number of iterations N_{opt} as a function of significance. The graphs are shifted vertically for details see the figure legend. As expected, the optimal number of iterations increases with increasing significance due to the fact that statistical fluctuations are damped,

Fig. 3. Shown is the RE as a function of iteration for a source with ring-like morphology. The outer- and inner radii are 0.4° and 0.2° , respectively.

leading to smaller artifacts. The optimal number of iterations increases when the outer source radius decreases, which can be understood by a decrease of the signal-to-noise ratio with increasing ring area. Only strong sources allow for a high enough numbers of iterations to improve the angular resolution by $\sim 50\%$.

Fig. 4. The optimal number of iterations N_{opt} as a function of significance for a ring-like source morphology. The graphs are shifted on the ordinate to prevent overlap. The ring width was kept constant at 0.2° while the radii varied. The outer radii are 0.3° (dots, shift -0.2), 0.4° (squares, shift -0.1), 0.5° (triangles pointing upwards, shift 0.0), 0.6° (triangles pointing downwards, shift 0.1) and 0.7° (crosses, shift 0.2).

Fig. 5. The optimal number of iterations N_{opt} as a function of significance for an elliptical source morphology. The graphs are shifted on the ordinate to prevent overlap. The radius was varied. (0.3° (dots, shift -0.2), 0.4° (squares, shift -0.1), 0.5° (triangles pointing upwards, shift 0.0), 0.6° (triangles pointing downwards, shift 0.1) and 0.7° (crosses, shift 0.2).

Another aspect is shown in Figs. 6 and 7. Here RE is given as a function of significance before and after deconvolution with the optimal number of iterations N_{opt} . Filled markers label RE before and open markers after RLA application. Fig. 6 shows the results for a ring-like morphology, Fig. 7 for an elliptical source morphology. One important result is that the behavior of RE after deconvolution is almost independent of the exact source morphology. RE decreases after deconvolution by about a factor of 10, which means that the deconvolved image reveals the true source structure.

5. Correlation studies between excess maps with different angular resolution

In VHE γ -ray astronomy a common question is the underlying process responsible for the VHE γ -ray emission of a source. Correlation studies between different wavelengths can provide information to distinguish between possible processes, e.g. inverse Compton effect and neutral pion decay. In the case that inverse Compton is the dominant process, a correlation between X-rays

Fig. 6. RE versus significance before and after deconvolution for a ring-like source morphology. Filled markers label RE before RLA application and open markers after. The ring width was kept constant at 0.2° while the radii varied. The outer radii are 0.3° (dots), 0.4° (squares), 0.5° (triangles pointing upwards), 0.6° (triangles pointing downwards) and 0.7° (crosses).

Fig. 7. RE versus significance before and after deconvolution for an elliptical source morphology. Filled markers label RE before RLA application and open markers after. The radius was varied. (0.3° (dots), 0.4° (squares), 0.5° (triangles pointing upwards), 0.6° (triangles pointing downwards) and 0.7° (crosses).

and γ -rays is expected. These studies are hampered by differences in angular resolutions of e.g. X-ray and VHE γ -ray instruments. To calculate the correlation the only acceptable scale is given by the worst angular resolution. Since deconvolution improves the resolution, it allows to decrease this scale.

In the following, a detailed study of the correlation between a simulated X-ray and a simulated VHE γ -ray excess map is presented. The same ring-like morphology was used for both. The X-ray map was simulated with the angular resolution of the ASCA satellite and the signal- and background strength were set to the values of the ASCA measurements of RX J1713.7–3946 [13]. For the VHE- γ -ray excess map the simulations follow the description of Section 3 for a source with a significance of 39σ . We examined how the correlation coefficient depends on bin size, that is the area used for its calculation, with and without deconvolution of the VHE- γ -ray excess map varying the area of the ring-like structure by holding the outer radius of the ring constant at 0.4° and changing the inner one between 0.2° and 0.39° .

In Fig. 8 the results are presented. While without deconvolution the correlation coefficient increases significantly with increasing

Fig. 8. The correlation coefficient between simulated X-ray and VHE- γ -ray data before and after the deconvolution of the simulated VHE- γ -ray data as a function of bin size, the area for which the coefficient was calculated. Given are the dependencies for several ring-like sources. The outer ring was kept constant at 0.4° . The width was varied between 0.2° and 0.01° . (0.2° (dots), 0.15° (squares), 0.1° (triangles pointing upwards), 0.05° (triangles pointing downwards) and 0.01° (crosses)).

bin size, deconvolution provides for an almost constant correlation coefficient by improving the resolution especially for small bin sizes. For the interpretation of this fact, one has to keep in mind that the number of iterations was chosen to give the best fit of the model to simulated data, which corresponds to an image with the least amount of fluctuations. The possibility to almost gain independence on the bin size by deconvolution opens the way to the investigation of morphological fine structure down to the angular resolution of the simulated X-ray map.

6. Potential of the application to VHE γ -ray shell-type supernova remnants

Supernova remnants (SNR) are discussed as the possible sources of galactic cosmic rays with the assumption that the dominant population in these sources is hadronic and the VHE γ -ray emission is dominated by neutral pion decay. But in reality the situation is still unclear. Newest results of the Fermi γ -ray satellite of the SNR RX J1713.7–3946 [14] disfavour this scenario. Therefore, correlation studies of shell-type SNR down to their smallest structures could help to shed light unto the underlying processes.

The results presented in the previous section enable us to discuss the chances for improvements through deconvolution of the five already detected VHE- γ -ray shell-type supernova remnants. RX J1713.7–3946 is the best studied and brightest shell-type SNR in the VHE domain. The significance and the signal-to-noise ratio of all other detected sources is smaller. Our studies presented in Section 4 have shown that under these conditions the optimal number of iterations is lower and therefore the performance of the deconvolution will be reduced. It is of high interest to examine these extended sources with respect to the question how they might perform with respect to deconvolution. In Table 1 the results of four sources are summarized. Given are the source size and the required significance to obtain an optimal number of iterations of

Table 1

Given are the source sizes, the already observed significance of the extended SNRs detected by the H.E.S.S. experiment as well as the required significance to obtain at least an optimal iteration of three.

Source	Source size [$^\circ$]	Observed significance [σ]	Required significance [σ]	Reference
RX J1713.7–3946	0.65	39	39	[16]
RCW 86	0.45	8.5	30	[17]
SN 1006	0.35	4.9	20	[19]
Vela Junior	1.00	19	68	[4]
HESS J1731–347	0.33	22	20	[18]

at least three and the already observed significance. For the current data sets the sources RX J1713.7–3946 and HESS J1731–347 have a significance high enough to apply the deconvolution method. Marin and Djannati-Atai [15] applied a modified RLA to the RX J1713.7–3946 data set and found, that the correlation between X-ray and VHE data increases.

7. Conclusion

In this paper the application of the RLA to VHE images was studied. It was found that the overall improvement of the angular resolution is of the order of $\sim 40\%$ if the number of the optimal iterations is of the order of 6. The number of optimal iterations (N_{opt}) itself depends strongly on the signal-to-noise ratio and on the significance of the source. For typical VHE sources N_{opt} is in the order of 2 to 3. The study of the five known shell-type supernova remnants has shown that for two of them deconvolution is feasible. We studied the spatial correlation between two simulated excess maps with different angular resolutions. One was chosen to represent an ASCA X-ray map, the other a typical VHE γ -ray excess map. The latter one was deconvolved. We found that the correlation factor increases with increasing size of the spatial bins which are used to calculate the respective correlation coefficients. After deconvolution, the correlation factor is almost independent of the binning down to bin sizes of 0.025° . This result indicates that deconvolution enables us to analyze the spatial structure of the processes within the interior of SNRs.

References

- [1] F.A. Aharonian et al., Rep. Prog. Phys. 71 (2008) 6901.
- [2] J.A. Hinton, W. Hofmann, Ann. Rev. Astron. Astr. 47 (2009) 523.
- [3] F.A. Aharonian et al., Astron. Astrophys. 449 (2006) 223.
- [4] F.A. Aharonian et al., Astrophys. J. 661 (2007) 236.
- [5] F. Acero et al., Astron. Astrophys. 516 (2010) A62.
- [6] A. Abramowski et al., Astron. Astrophys. 531 (2011) A81.
- [7] P.J. Serlemittos et al., Publ. Astron. Soc. Jpn. 47 (1995) 105.
- [8] W.H. Richardson, J. Opt. Soc. Amer. 62 (1972) 55.
- [9] L.B. Lucy, Astron. J. 79 (1974) 745.
- [10] F.A. Aharonian et al., Astron. Astrophys. 457 (2006) 899.
- [11] D. Berge, S. Funk, J. Hinton, Astron. Astrophys. 466 (2007) 1219.
- [12] T. Li, Y. Ma, Astrophys. J. 272 (1983) 317.
- [13] Y. Uchiyama, T. Takahashi, F.A. Aharonian, PASJ 54 (2002) L73.
- [14] A. Abdo et al., Astrophys. J. 734 (2011) 28.
- [15] G. Maurin, A. Djannati-Atai, P. Espigat, in: Proceedings 30th ICRC, vol. 2, 2008, p. 727.
- [16] F.A. Aharonian et al., Astron. Astrophys. 457 (2006) 899.
- [17] F.A. Aharonian et al., Astrophys. J. 692 (2009) 1500.
- [18] F.A. Aharonian et al., Astron. Astrophys. 477 (2008) 353.
- [19] H.E.S.S. Collaboration et al., Astron. Astrophys. 516 (2010) A62.