The top quark and Higgs boson masses and the stability of the electroweak vacuum

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Abstract

The ATLAS and CMS experiments observed a particle at the LHC with a mass ≈ 126 GeV, which is compatible with the Higgs boson of the Standard Model. A crucial question is, if for such a Higgs mass value, one could extrapolate the model up to high scales while keeping the minimum of the scalar potential that breaks the electroweak symmetry stable. Vacuum stability requires indeed the Higgs boson mass to be $M_H \gtrsim 129 \pm 1$ GeV, but the precise value depends critically on the input top quark pole mass which is usually taken to be the one measured at the Tevatron, $m_t^{\text{exp}} = 173.2 \pm 0.9$ GeV. However, for an unambiguous and theoretically well-defined determination of the top quark mass one should rather use the total cross section for top quark pair production at hadron colliders. Confronting the latest predictions of the inclusive $p\bar{p} \to t\bar{t} + X$ cross section up to next-to-next-to-leading order in QCD to the experimental measurement at the Tevatron, we determine the running mass in the $\overline{\text{MS}}$-scheme to be $m_t^{\overline{\text{MS}}}(m_t) = 163.3 \pm 2.7$ GeV which gives a top quark pole mass of $m_t^{\text{pole}} = 173.3 \pm 2.8$ GeV. This leads to the vacuum stability constraint $M_H \geq 129.4 \pm 5.6$ GeV to which a ≈ 126 GeV Higgs boson complies as the uncertainty is large. A very precise assessment of the stability of the electroweak vacuum can only be made at a future high-energy electron-positron collider, where the top quark pole mass could be determined with a few hundred MeV accuracy.

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The recent results on Higgs boson searches delivered by the ATLAS and CMS collaborations [1] at the Large Hadron Collider (LHC) show that there is now an established signal (at almost five standard deviations for each experiment) corresponding to a particle with a mass ≈ 126 GeV and with the properties expected for the Standard Model (SM) Higgs boson [2, 3]. A critical question would be whether such a Higgs boson mass value allows to extrapolate the SM up to ultimate scales, while still having an absolutely stable electroweak vacuum [4–6]. Indeed, it is well known that top quark quantum corrections tend to drive the quartic Higgs coupling λ, which in the SM is related to the Higgs mass by the tree-level expression $\lambda = M_H^2/2v^2$ where $v \approx 246$ GeV is the Higgs field vacuum expectation value, to negative values which render the electroweak vacuum unstable.

A very recent analysis, including the state-of-the-art quantum corrections at next-to-next-to-leading order (NNLO) that are relevant in this context gives for the condition of absolute stability of the electroweak vacuum, $\lambda(M_P) \geq 0$, when the SM is extrapolated up to the Planck scale M_P [6]

$$ M_H \geq 129.2 + 1.8 \times \left(\frac{m_t^{\text{pole}} - 173.2 \text{ GeV}}{0.9 \text{ GeV}} \right) - 0.5 \times \left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right) \pm 1.0 \text{ GeV} . \quad (1) $$

This full NNLO calculation is based on three main ingredients that have been calculated only very recently: the two-loop threshold corrections to the quartic coupling λ at the weak scale, $\lambda(\mu) = M_H^2/2v^2 + \Delta\lambda(\mu)$ which involve the QCD and the Yukawa interactions [6, 7], the three-loop leading contributions to the renormalization group evolution of the coupling λ as well as the top quark Yukawa coupling and the Higgs mass anomalous dimension [8], and the three-loop corrections to the beta functions of the three SM gauge couplings taking into account Yukawa and Higgs self couplings [9]. The uncertainty of $\Delta M_H = \pm 1.0$ GeV quoted in eq. (1) reflects the theoretical uncertainty on the Higgs mass bound which, to a good approximation, corresponds to the difference between the results obtained when calculating the bound at next-to-leading order (NLO) and NNLO[1].

The vacuum stability condition eq. (1) critically depends on three basic inputs.

A first parameter is the Higgs boson mass M_H which, from the current excess of data, seems to be in the (wide) range of $M_H \approx 124$-128 GeV [1].

A second one is the strong coupling constant α_s evaluated at the scale of the Z boson mass, with a world average value of [10]

$$ \alpha_s(M_Z) = 0.1184 \pm 0.0007 . \quad (2) $$

The combined theoretical and experimental uncertainty of $\Delta\alpha_s = \pm 0.0007$ generates, at the 2σ level, an uncertainty of $\Delta M_H \approx 1$ GeV on the Higgs mass bound[2].

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[1]Note, that the vacuum stability analysis of Ref. [7] based on the two-loop $\mathcal{O}(\alpha\alpha_s)$ threshold corrections and the three-loop terms in the renormalization group equation [8, 9], arrives at a similar relation for M_H as in eq. (1). The additional improvements of Ref. [6] due to $\mathcal{O}(\alpha_s^2)$ terms shift the bound on M_H by a small amount, of order 0.1 GeV.

[2]This value of α_s is obtained from a large set of measurements with significant spreads between them. This issue will be discussed later.

The most critical ingredient in eq. (1) is the top quark pole mass, identified with the one measured at the Tevatron by the CDF and D0 collaborations[3] [12],

$$m_t^{\mathrm{exp}} = 173.2 \pm 0.9 \text{ GeV} . \qquad (3)$$

Indeed, a change of the input m_t value by 1 GeV will lead to a $\Delta M_H \approx \pm 2$ GeV variation of the Higgs mass bound. Allowing for a 2σ variation of the top quark mass value alone, one obtains the upper bound $M_H \geq 125.6$ GeV. Hence, if the Higgs mass were exactly $M_H = 125$ GeV, the absolute stability of the electroweak vacuum up to the Planck scale would be excluded at the 95% confidence level, while the value $M_H = 126$ GeV would instead allow for the stability of the vacuum at the same confidence level.

Thus, the "fate of the universe" [5], i.e. whether the electroweak vacuum is stable or not up to the largest possible high-energy scale, critically relies on a precise determination of the Higgs boson and top quark masses (besides the strong coupling constant). While it is expected that the clean $H \to \gamma\gamma$ and $H \to ZZ \to 4\ell^{\pm}$ (with $\ell = e, \mu$) decay channels and the excellent energy resolution for photons and charged leptons should allow to ultimately measure the Higgs boson mass with a precision of $\mathcal{O}(100)$ MeV [13], severe theoretical and experimental problems occur in the case of the top quark mass[4].

An immediate problem is that the top quark mass parameter measured at the Tevatron (and to be measured at the LHC) via kinematical reconstruction from the top quark decay products and comparison to Monte Carlo simulations, is not necessarily the pole mass which should enter the stability bound eq. (1). Besides the fact that the reconstruction of the coloured top quark four momentum from its uncolored decay products introduces an intrinsic uncertainty due to the non-perturbative mechanism of hadronisation that can be hardly quantified, there is an important conceptual problem. Strictly speaking, a theoretical prediction of a given measured observable is required to extract a parameter of a model in a meaningful way and this prediction should be made beyond the leading-order approximation for which a renormalisation scheme can be fixed. Obviously, this is not the case for the mass currently measured at the Tevatron which is merely the mass parameter in a Monte Carlo program with which the kinematical fit of the top decay products is performed and which does resort to any given renormalisation scheme.

Furthermore, it is well known that the concept of an "on-shell" or "pole" quark mass has intrinsic theoretical limitations as quarks are colored objects and, as such, do not appear as asymptotic states of the S-matrix because of color confinement [15]. In addition, because of the so-called infrared renormalons, such a pole mass is plagued with an intrinsic non-perturbative ambiguity of the order of Λ_{QCD} amounting to a few hundred MeV, and it cannot be "measured" with an accuracy better[5] than $\mathcal{O}(\Lambda_{\mathrm{QCD}})$ [15].

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[3]In contrast to Ref. [6], we do not average the m_t value determined at the Tevatron with that obtained at the LHC by the ATLAS and CMS collaborations which presently have much larger uncertainties [11].

[4]This situation is similar to that occurring in the context of the electroweak precision tests and the indirect determination of the Higgs mass, where the definition of the top mass also plays a key role. However, while the impact of m_t is relatively modest in the global electroweak fits as the resulting Higgs mass value has a large uncertainty, $\Delta M_H \approx 30$ GeV [14], it is extremely strong for the stability bound.

[5]A precise quantitative statement is rather hard to make and more work in this direction is needed. Very few studies have been devoted to the relations between the "Monte Carlo", the experimentally measured and the "pole" quark mass. In Ref. [16], the uncertainties due to non-perturbative color reconnection effects

So-called short distance top quark masses, such as the one defined in the modified minimal substraction ($\overline{\text{MS}}$) scheme at a scale μ, $m_t^{\overline{\text{MS}}}(\mu)$, offer remedy to these problems. The $\overline{\text{MS}}$ mass realizes the concept of a running mass which depends on the hard scale μ of the process in complete analogy to the running coupling $\alpha_s(\mu)$. A determination of $m_t^{\overline{\text{MS}}}(\mu)$ is then possible from the mass dependence of any observable which is rather precisely measured and, at the same time, theoretically predicted beyond the leading order (LO) approximation in QCD perturbation theory. An immediate choice for the determination of $m_t^{\overline{\text{MS}}}(\mu)$ is the total production cross section for top quark pairs, $\sigma(t\bar{t}+X)$. It has been measured both at the Tevatron and the LHC with an accuracy of better than 10% and it is known to very good approximation at NNLO in QCD in the convenient $\overline{\text{MS}}$ renormalisation scheme [18–21]. The most recent combinations of inclusive cross section measurements at the Tevatron performed by the CDF and D0 collaborations yield a value [22, 23],

$$\sigma(p\bar{p} \to t\bar{t}+X) = 7.56\,^{+0.63}_{-0.56}\text{ pb (D0)} \text{ and } 7.50\,^{+0.48}_{-0.48}\text{ pb (CDF)}. \qquad (4)$$

At the LHC in the run with a center-of-mass energy of $\sqrt{s}=7$ TeV the ATLAS and CMS collaborations have each measured a combined cross section [24, 25] of

$$\sigma(pp \to t\bar{t}+X) = 177^{+11}_{-10}\text{ pb (ATLAS)} \text{ and } 165.8^{+13.3}_{-13.3}\text{ pb (CMS)}. \qquad (5)$$

The first issue that we will address in the present paper is the comparison of the cross section measurements above with the theory predictions, which will allow us to extract the $\overline{\text{MS}}$ top quark mass $m_t^{\overline{\text{MS}}}$ and use it subsequently to derive the pole top quark mass and, hence, the vacuum stability bound eq. (1) in an unambiguous way. To that end we update the analyses of Refs. [19, 26] using the latest sets of parton distribution functions (PDFs) at NNLO [27–30] and, most importantly, the new NNLO QCD contributions to $\sigma(p\bar{p} \to t\bar{t} + X)$ in the high-energy limit [20] and for the $q\bar{q} \to t\bar{t} + X$ channel [21].

In a second part of this paper, we recall that a self-consistent and precise determination of the top quark mass can best be performed at a high-energy electron-positron collider, especially when scanning the kinematical threshold for $t\bar{t}$ pair production. The accuracy that can be achieved on short distance masses such as the 1S-threshold mass amounts to $\Delta m_t^{1S} \approx 100$ MeV [31, 32]. Together with a Higgs mass measurement with a comparable accuracy or less and a more precise determination of the strong coupling α_s this would ultimately allow to verify the stability bound in the SM at the few per mille level.

Let us briefly summarize how to obtain the top quark pole mass m_t^{pole} from the total production cross section $\sigma(p\bar{p}/pp \to t\bar{t} + X)$ at hadron colliders. This observable has been computed to very good approximation at NNLO in QCD based on the large threshold logarithms [18, 19] which provide sufficiently precise phenomenological predictions in the parton kinematic range covered by the Tevatron and the LHC with a center-of-mass energy of $\sqrt{s} = 7$ TeV. Most recently, the exact NNLO result for contributions to the $q\bar{q} \to$

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in the hadronisation process and from the ambiguities in the top quark mass definition were estimated to be of order ± 0.5 GeV each. In Ref. [17], the combined effect of color-reconnection, underlying events and shower in the Monte Carlo programs was estimated in a toy model to generate an uncertainty of ± 1.5 GeV on the reconstructed top quark mass at the Tevatron. These effects are not included in the central value and error for the top quark mass quoted in eq. (3).

$t\bar{t}+X$ channel [21] and the constraints imposed by the high-energy factorization have been derived [20]. This knowledge suffices to predict $\sigma(p\bar{p}\to t\bar{t}+X)$ at Tevatron in eq. (4) and $\sigma(pp\to t\bar{t}+X)$ at the LHC in eq. (5) with a few percent accuracy[6].

Conventionally, higher order computations in QCD employ the pole mass scheme for heavy quarks. It is straightforward though, to apply the well-known conversion relations [36] which are known even beyond NNLO in QCD to derive the total cross section as a function of the $\overline{\text{MS}}$ mass [19,37]. As a benefit of such a procedure, one arrives at theoretical predictions for hard scattering cross sections with better convergence properties and greater perturbative stability at higher orders in the case of the $\overline{\text{MS}}$ mass. We use the cross section predictions obtained with the program HATHOR (version 1.3) [37] at NNLO accuracy with the latest improvements of Refs. [20,21]. These are combined with modern sets of PDFs, ABM11 [27], JR09 [28], MSTW08 [29], and NN21 [30] and account for the full theoretical uncertainties, i.e., the scale variation as well as the (combined) PDF and α_s uncertainty. From eq. (4), we obtain the values given in Table 1 when the CDF and D0 cross section measurements are combined.

CDF&D0	ABM11	JR09	MSTW08	NN21
$m_t^{\overline{\text{MS}}}(m_t)$	$162.0\,^{+2.3}_{-2.3}\,^{+0.7}_{-0.6}$	$163.5\,^{+2.2}_{-2.2}\,^{+0.6}_{-0.2}$	$163.2\,^{+2.2}_{-2.2}\,^{+0.7}_{-0.8}$	$164.4\,^{+2.2}_{-2.2}\,^{+0.8}_{-0.2}$
m_t^{pole}	$171.7\,^{+2.4}_{-2.4}\,^{+0.7}_{-0.6}$	$173.3\,^{+2.3}_{-2.3}\,^{+0.7}_{-0.2}$	$173.4\,^{+2.3}_{-2.3}\,^{+0.8}_{-0.8}$	$174.9\,^{+2.3}_{-2.3}\,^{+0.8}_{-0.3}$
(m_t^{pole})	$(169.9\,^{+2.4}_{-2.4}\,^{+1.2}_{-1.6})$	$(171.4\,^{+2.3}_{-2.3}\,^{+1.2}_{-1.1})$	$(171.3\,^{+2.3}_{-2.3}\,^{+1.4}_{-1.8})$	$(172.7\,^{+2.3}_{-2.3}\,^{+1.4}_{-1.2})$

Table 1: The value of the top quark mass $m_t^{\overline{\text{MS}}}(m_t)$ in GeV at NNLO in QCD determined with four sets of NNLO PDFs from the measurement of $\sigma(p\bar{p} \to t\bar{t} + X)$ at the Tevatron when the CDF and D0 results quoted in eq. (4) are combined. The set of uncertainties originate from the experimental error on $\sigma(p\bar{p} \to t\bar{t} + X)$ (first error) and from the variation of the factorization and renormalization scales from $\frac{1}{2}m_t \leq \mu_F = \mu_R \leq 2m_t$ (second error). The resulting pole mass m_t^{pole} in the second line is obtained from a scheme transformation to NNLO accuracy, using the program RunDec and the value of $\alpha_s(M_Z)$ of the given PDF set. For comparison, in the third line in parentheses, (m_t^{pole}) is also given as extracted directly from the measured cross section.

The values for $m_t^{\overline{\text{MS}}}(m_t)$ in Table 1 determined from the combined Tevatron cross sections carry an uncertainty of $\Delta_{\text{exp}} m_t^{\overline{\text{MS}}}(m_t) \approx \pm 2.3$ GeV due to the experimental errors in eq. (4). The residual scale dependence of the theory prediction for $\sigma(p\bar{p} \to t\bar{t} + X)$, which is determined in the interval $\frac{1}{2}m_t \leq \mu_F = \mu_R \leq 2m_t$ as effects due to $\mu_F \neq \mu_R$ are small at NNLO [19,37], results in an error of $\Delta_{\text{scale}} m_t^{\overline{\text{MS}}}(m_t) \approx \pm 0.7$ GeV illustrating the great stability of the perturbative expansion at NNLO in QCD when using the running $\overline{\text{MS}}$ mass.

The second line in Table 1 lists the pole mass values m_t^{pole} at NNLO obtained from the values for the $\overline{\text{MS}}$ mass $m_t^{\overline{\text{MS}}}(m_t)$ using the scheme transformation given in Ref. [36]

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[6]We do not account here for the electroweak radiative corrections at NLO [33]. For light Higgs bosons with $M_H \approx 126$ GeV, these are vanishingly small at the Tevatron and give a negative contribution of $\mathcal{O}(2\%)$ at the LHC. Bound state effects and the resummation of Coulomb type corrections have been shown to be small at the Tevatron as well [34]. Likewise, we do not include the electroweak radiative corrections derived in Ref. [35] in the conversion of the pole mass m_t^{pole} to the running mass $m_t^{\overline{\text{MS}}}(\mu)$.

as implemented in the program `RunDec` [38] together with the $\alpha_s(M_Z)$ value of the given PDF set. For comparison, the third line in Table 1 quotes in parentheses the value of m_t^{pole} determined by a direct extraction from the NNLO theory prediction using the on-shell scheme. The differences of $\mathcal{O}(+2)$ GeV with the values in the second line obtained from converting the $\overline{\text{MS}}$ mass indicate the importance of higher order corrections beyond NNLO in QCD if using the pole mass scheme. This is to be contrasted with the observed very good apparent convergence of the perturbative predictions already at NNLO in the running mass $m_t^{\overline{\text{MS}}}(m_t)$ scheme, see Ref. [19].

There is one particular aspect in the chosen procedure, though, which requires attention. The Tevatron cross section data [22,23] acquire a weak dependence on the top quark mass in the extrapolation from the recorded events in the fiducial volume to the total cross section. This is induced by comparison to Monte Carlo simulations and the values quoted in eq. (4) assume a mass of $m_t = 172.5$ GeV. This systematic uncertainty of $\sigma(p\bar{p} \to t\bar{t} + X)$ has been published by the D0 collaboration [23] as a parametrization in m_t. For CDF, it has not been published for the value in eq. (4) based on the combination of data at 4.6 fb^{-1} luminosity in [22]. It has, however, been quoted as a shift for $\sigma(p\bar{p} \to t\bar{t} + X)$ of approximately $\Delta\sigma/\sigma \approx -0.01\Delta m_t/\text{GeV}$ in a previous combination of data at 760 pb^{-1} luminosity [39]. In order to account for this additional source of systematic uncertainty one can identify this parameter m_t with the on-shell mass and check that the pole mass values in Table 1 are consisted with $m_t = 172.5$ GeV within $\Delta_{\text{sys}} m_t \approx \pm 1$ GeV. This assumption is motivated by the fact, that the NLO computations applied in the experimental analysis, e.g., `MC@NLO` [40] or `MCFM` [41] contain perturbative matrix elements at NLO in QCD using the pole mass scheme for the top quark. At the moment however, we are lacking further quantitative information. Therefore it is reassuring to see that the potential shifts of $\Delta_{\text{sys}} m_t \approx \pm 1$ GeV are contained well within the experimental error on m_t^{pole} in Table 1.

The largest residual uncertainty in the extraction of $m_t^{\overline{\text{MS}}}(m_t)$ in Table 1 resides in the dependence on the PDFs as can be seen by comparing the central values for the sets ABM11, JR09, MSTW and NN21. Although the $q\bar{q}$ parton luminosity is quite well constrained in the kinematical range of interest at the Tevatron, the differences in the individual global fits (value of $\alpha_s(M_Z)$ etc.) lead to a spread in the central value of $m_t^{\overline{\text{MS}}}(m_t) \approx 162.0$ GeV to 164.4 GeV. This is larger than the combined PDF and α_s uncertainty of any individual set not quoted in Table 1 which amounts to an additional error of $\Delta_{\text{PDF}} m_t^{\overline{\text{MS}}}(m_t) \approx \pm 0.7$ GeV, except for JR09, where one finds $\Delta_{\text{PDF}} m_t^{\overline{\text{MS}}}(m_t) \approx \pm 1.4$ GeV. Yet, within the PDF uncertainty the values of $m_t^{\overline{\text{MS}}}(m_t)$ in Table 1 are largely consistent at the level of 1σ.

Combining the $m_t^{\overline{\text{MS}}}(m_t)$ values in Table 1 from the combined Tevatron measurements, we obtain the central value of the $\overline{\text{MS}}$ mass and its associated uncertainty at NNLO

$$m_t^{\overline{\text{MS}}}(m_t) = 163.3 \pm 2.7 \text{ GeV}, \tag{6}$$

which is equivalent to the top quark pole mass value of

$$m_t^{\text{pole}} = 173.3 \pm 2.8 \text{ GeV}, \tag{7}$$

where all errors were added in quadrature including the $\Delta_{\text{sys}} m_t \approx \pm 1$ GeV discussed above. Note that, although the total error is a factor of four larger, the central value is remarkably close to that of m_t^{exp} in eq. (3) determined from the top decay products at the Tevatron.

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When injected in eq. (1), the value of the top pole mass above[7] leads to the upper bound for vacuum stability to be realized (ignoring the theoretical and the experimental uncertainties on the Higgs mass and on α_s)

$$M_H \geq 129.4 \pm 5.6 \text{ GeV}, \qquad (8)$$

in which the Higgs mass values $M_H \approx$ 124-127 GeV, indicated by the ATLAS and CMS searches, comply in contrast to the case where the mass value of eq. (3) from kinematical reconstruction at the Tevatron is used instead. Note also that the uncertainties are much larger, a factor of approximately 3, than if eq. (3) were used.

Let us now turn to the LHC. The top quark mass extracted from the combined AT-LAS and CMS measurements in eq. (5) is given in Table 2. While the uncertainty $\Delta_{\text{exp}} m_t^{\overline{\text{MS}}}(m_t) \approx \pm 2.3$ GeV due to the experimental errors is similar to the Tevatron measurement the theoretical uncertainty due to the scale variation is mostly larger, i.e., $\Delta_{\text{scale}} m_t^{\overline{\text{MS}}}(m_t) \approx \pm 1.1$ GeV. The most striking observation in Table 2 is certainly the very large spread in the central value of $m_t^{\overline{\text{MS}}}(m_t) \approx 159.0$ GeV to 166.7 GeV depending on the chosen PDF set. The combined PDF and α_s uncertainty of the individual sets in Table 2 is in the range $\Delta_{\text{PDF}} m_t^{\overline{\text{MS}}}(m_t) \approx \pm 1.0$ GeV to 1.4 GeV and $\Delta_{\text{PDF}} m_t^{\overline{\text{MS}}}(m_t) \approx \pm 2.4$ GeV for JR09. This leads to consistency between the central values for $m_t^{\overline{\text{MS}}}(m_t)$ for each PDF set (comparing Tevatron in Table 1 and LHC in Table 2) but the ones obtained for the different PDF sets at the LHC are not compatible with each other within the errors.

ATLAS&CMS	ABM11	JR09	MSTW08	NN21
$m_t^{\overline{\text{MS}}}(m_t)$	$159.0^{+2.1}_{-2.0}{}^{+0.7}_{-1.4}$	$165.3^{+2.3}_{-2.2}{}^{+0.6}_{-1.2}$	$166.0^{+2.3}_{-2.2}{}^{+0.7}_{-1.5}$	$166.7^{+2.3}_{-2.2}{}^{+0.8}_{-1.3}$
m_t^{pole}	$168.6^{+2.3}_{-2.2}{}^{+0.7}_{-1.5}$	$175.1^{+2.4}_{-2.3}{}^{+0.6}_{-1.3}$	$176.4^{+2.4}_{-2.3}{}^{+0.8}_{-1.6}$	$177.4^{+2.4}_{-2.3}{}^{+0.8}_{-1.4}$
(m_t^{pole})	$(166.1^{+2.2}_{-2.1}{}^{+1.7}_{-2.3})$	$(172.6^{+2.4}_{-2.3}{}^{+1.6}_{-2.1})$	$(173.5^{+2.4}_{-2.3}{}^{+1.8}_{-2.5})$	$(174.5^{+2.4}_{-2.3}{}^{+2.0}_{-2.3})$

Table 2: Same as Table 1 for the measurement of $\sigma(pp \to t\bar{t}+X)$ at the LHC with $\sqrt{s} = 7$ TeV when the ATLAS and CMS results in eq. (5) are combined.

Also the LHC experiments assume in eq. (5) a mass of $m_t = 172.5$ GeV when extrapolating the number of measured events with top quark pairs to the inclusive cross section $\sigma(pp \to t\bar{t}+X)$. However, no information on the m_t dependence of this procedure is given in Refs. [24,25] and the same self-consistency check applied above by comparing to the pole mass value m_t^{pole} in the second line of Table 2 shows that one should expect a significantly larger systematic uncertainty. Thus, at present the determination of $m_t^{\overline{\text{MS}}}(m_t)$ from the inclusive cross section at LHC is very difficult, predominantly because of lacking information on the experimental systematics and because of the strong correlation of the top quark mass with the value of α_s and the gg parton luminosity in the theory predictions. The latter problem could be addressed by combining measurements of different observables, for instance, by using a novel method for the top quark mass determination from the ratio of rates for the process $t\bar{t}$+jets [42].

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[7]One could write directly eq. (1) in terms of the $\overline{\text{MS}}$ top quark mass which is in fact the basic input entering the top Yukawa coupling which is defined in the $\overline{\text{MS}}$ scheme. This will prevent the unnecessary translation to the pole mass both in the stability bound and in the $p\bar{p} \to t\bar{t}$ cross section. Such a formula will be soon provided by the authors of Ref. [6]. We thank Gino Isidori for a discussion on this point.

The ultimate precision on the top quark mass to be reached at the LHC, however, is hard to predict at the moment. A total uncertainty that is a factor of two smaller than the present uncertainty from the Tevatron measurements,

$$\Delta m_t^{\text{pole}}|_{\text{LHC–expected}} \approx 1.5 \text{ GeV} , \tag{9}$$

does not seem to be excluded at present, but more work is needed to reach this level.

A very precise and unambiguous determination of the top quark mass and, hence, the possibility to derive a reliable upper bound on the Higgs mass for which the electroweak vacuum would be stable, can only be performed at an e^+e^- collider ILC with an energy above $\sqrt{s} = 350$ GeV [43]. Indeed, as a consequence of its large total decay width, $\Gamma_t \sim 1.5$ GeV, the top quark will decay before it hadronises making non-perturbative effects rather small and allowing to calculate quite reliably the energy dependence of the $e^+e^- \to t\bar{t}$ production cross section when an energy scan is performed near the $t\bar{t}$ kinematical threshold. The location of the cross section rise allows to extract the value of the 1S-threshold top quark mass, while the shape and normalization provide information on the total width Γ_t and on the strong coupling α_s [48].

The cross section $\sigma(e^+e^- \to t\bar{t})$ at threshold is known up to the next-to-next-to-leading-logarithm (NNLL) using renormalization group improvements and the next-to-next-to-next-to-leading order(N^{3}LO) in the QCD coupling is almost complete [31, 32]. It could ultimately be determined with a theoretical uncertainty of $\Delta\sigma(e^+e^- \to t\bar{t}) \approx 3\%$ (the experimental uncertainties are much smaller) but, as the impact on the threshold top quark mass determination is rather modest, an accuracy on m_t much below 100 MeV can be achieved. This threshold mass can then be translated into the $\overline{\text{MS}}$ top quark mass $m_t^{\overline{\text{MS}}}$(which can be directly used as input in the Higgs mass stability bound equivalent to eq. (1) but in the $\overline{\text{MS}}$ scheme) or the one in the on-shell scheme, m_t^{pole}. The combined experimental and theoretical uncertainty on the mass parameter m_t^{pole} at the ILC determined in this way, i.e., by conversion form a short-distance mass, is estimated to be [31, 32]

$$m_t^{\text{pole}}|_{\text{ILC}} \lesssim 200 \text{ MeV} , \tag{10}$$

i.e., an order of magnitude better than what can be achieved at the Tevatron and the LHC. In other words, the uncertainty in the top quark mass determination will be so small at the ILC that its impact on the stability bound eq. (1) will become very mild. At such level of accuracy, the two parameters which will then be of concern are M_H and α_s.

At a high-energy e^+e^- collider, the Higgs mass can be measured with an accuracy below 100 MeV, and most probably $\Delta M_H \approx 50$ MeV, from the recoil of the Z boson in the Higgs-strahlung process $e^+e^- \to HZ \to H\ell^+\ell^-$ independently of the Higgs decays [43].

At the e^+e^- collider, α_s can be determined with an accuracy close to or better than the one currently adopted (which cannot be considered to be conservative[8]) $\Delta\alpha_s = 0.0007$ [10], in a single measurement; a statistical accuracy of $\Delta\alpha_s = 0.0004$ is for instance quoted in

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[8]The world average $\alpha_s(M_Z)$ value quoted in eq. (2) is based on a comparison of QCD theory predictions at least to NNLO accuracy with data on a variety of measurements including jet rates and event shapes in e^+e^--collisions, deep-inelastic scattering (DIS), Z- and τ-decays as well as entirely non-perturbative predictions based on lattice simulations. The very small uncertainty of $\Delta\alpha_s = \pm0.0007$ is remarkable as recent high precision determinations of $\alpha_s(M_Z)$ have lead to results which are only marginally compatible

Ref. [49]. This can be done either in $e^+e^- \to q\bar{q}$ events on the Z-resonance (the so-called GigaZ option) or at high energies [43] or in a combined fit with the top quark mass and total width in a scan around the $t\bar{t}$ threshold [48].

Assuming for instance that accuracies of about $\Delta m_t \approx 200$ MeV and $\Delta\alpha_s \approx 0.0004$ can be achieved at the ILC, a (quadratically) combined uncertainty of less than $\Delta M_H \approx 0.5$ GeV on the Higgs mass bound eq. (1) could be reached. This would be of the same order as the experimental uncertainty, $\Delta M_H \lesssim 100$ MeV, that is expected on the Higgs mass.

At this stage we will be then mostly limited by the theoretical uncertainty in the determination of the stability bound eq. (1) which is about ± 1 GeV. The major part of this uncertainty originates from the the QCD threshold corrections to the coupling λ which are known at the two-loop accuracy [6, 7]. It is conceivable that, by the time the ILC will be operating, the theoretical uncertainty will decrease provided more refined calculations of these threshold corrections beyond NNLO are performed.

The situation is illustrated in Fig. 1 where the areas for absolute stability, metastability[9] and instability of the electroweak vacuum are displayed in the $[M_H, m_t^{\mathrm{pole}}]$ plane at the 95% confidence level. The boundaries are taken from Ref. [6] but we do not include additional lines to account for the theoretical uncertainty of $\Delta M_H = \pm 1$ GeV (which could be reduced in the future) and ignore for simplicity the additional error from the α_s coupling.

As can be seen, the 2σ blue–dashed ellipse for the present situation with the current Higgs and top quark masses of $M_H = 126 \pm 2$ GeV and $m_t^{\mathrm{pole}} = 173.3 \pm 2.8$ GeV, and in which the errors are added in quadrature, is large enough to cover the three possibilities of absolute stability, metastability and also instability. Assuming the same central values as above, the green–dashed contour shows the impact of an improved accuracy on the top quark and Higgs masses of $\Delta m_t^{\mathrm{pole}} = \pm 1.5$ GeV and $\Delta M_H = \pm 100$ MeV which is expected to be achieved at the LHC with more accumulated data. With the present central values (which might of course change with more accurate measurements), only the metastability and a small area of the stability regions would be covered. The red–solid contour represents the expected situation at the ILC where one could reach accuracies of the order of $\Delta M_H = \pm 50$ MeV on the Higgs mass and $\Delta m_t^{\mathrm{pole}} = \pm 200$ MeV on the top quark mass, if obtained from a short-distance mass, cf., eq. (10). In this case, only one region, the metastability region with the above assumed central values, is covered (even when the theoretical uncertainty on the bound is included).

In conclusion, the present values of the Higgs boson mass as measured at the LHC and

within their quoted errors. This is the case for α_s extractions from e^+e^--annihilation, see e.g., [44] or those based on DIS data [45]. These differences can arise from theory assumptions such as power corrections, hadronisation corrections and so on and, likewise, on the treatment of data, see e.g., Ref. [27] for a comparative study in the case of DIS. In Ref. [46] they have simply been averaged in an arithmetic manner. Therefore, the uncertainty due to α_s attached to M_H in eq. (1), should be considered at present as a lower bound at most. If instead, one adopts the value $\alpha_s(M_Z) = 0.1189 \pm 0.0026$ of Ref. [47] that has been determined from $Z \to q\bar{q}$ data and predicted to N^{3}LO accuracy in QCD (and which can be considered to be safe from short-comings of other analyses) one would have an uncertainty that is ≈ 4 times larger than in the case of the world average eq. (2), generating an uncertainty $\Delta M_H \approx 2$ GeV on the Higgs mass bound eq. (1) at the 1σ level.

[9]This situation occurs when the true minimum of the scalar potential is deeper than the standard electroweak minimum but the latter has a lifetime that is larger than the age of the universe [5]. The boundary for this region is also taken from Ref. [6].
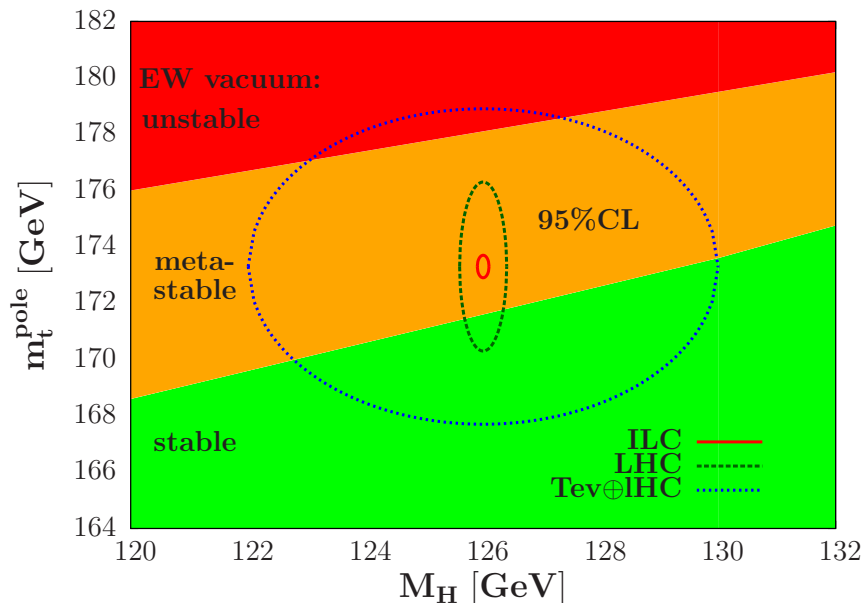
Figure 1: The 2σ ellipses in the $[M_H, m_t^{\text{pole}}]$ plane that one obtains from the current top quark and Higgs mass measurements at the Tevatron and LHC and which can be expected in future measurements at the LHC and at the ILC, when confronted with the areas in which the SM vacuum is absolutely stable, metastable and unstable up to the Planck scale.

the top quark pole mass as determined through a measurement of the cross section for top-quark pair production at the Tevatron, and that we have calculated in this paper to be $m_t^{\text{pole}} = 173.3 \pm 2.8$ GeV, are affected with too large uncertainties which do not allow to draw a firm conclusion on the important question whether the electroweak vacuum is indeed stable or not when the Standard Model is extrapolated up to the Planck scale. The situation will not dramatically improve with a more accurate measurement of the Higgs boson mass at the LHC as the top quark mass, which plays the dominant role in this issue, is not expected to be measured to better than ± 1.5 GeV accuracy even after a significant amount of LHC data. In particular, if the central m_t^{pole} value slightly moves downwards, it will be still undecided if we are in the stable or metastable region. It is only at a linear e^+e^- collider where one could determine in a theoretically unambiguous and experimentally very precise way the top quark mass in a scan near the $e^+e^- \to t\bar{t}$ kinematical threshold, and eventually measure also more accurately the Higgs boson mass and the strong coupling constant α_s, that the "fate of the universe" could be ultimately decided. The importance of a future ILC in this respect has also been stressed in [7].

If the measured central top quark and Higgs boson mass values turn out to be such that one is close to the critical boundary for vacuum stability, which implies that the Higgs self–coupling λ and its β_λ function are very close to zero at the Planck scale, it would open a wide range of interesting possibilities for new physics model building such as asymptotically safe gravitational theories [50] or inflation models that use the standard Higgs particle as the inflaton [51]. It is therefore very important that the intriguing possibility $\lambda(M_P) \approx 0$ for the Higgs self-coupling is verified experimentally in the most accurate manner. This provides a very strong argument in favor of the most unambiguous and accurate determination of the top quark mass which plays a crucial role in this issue.

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