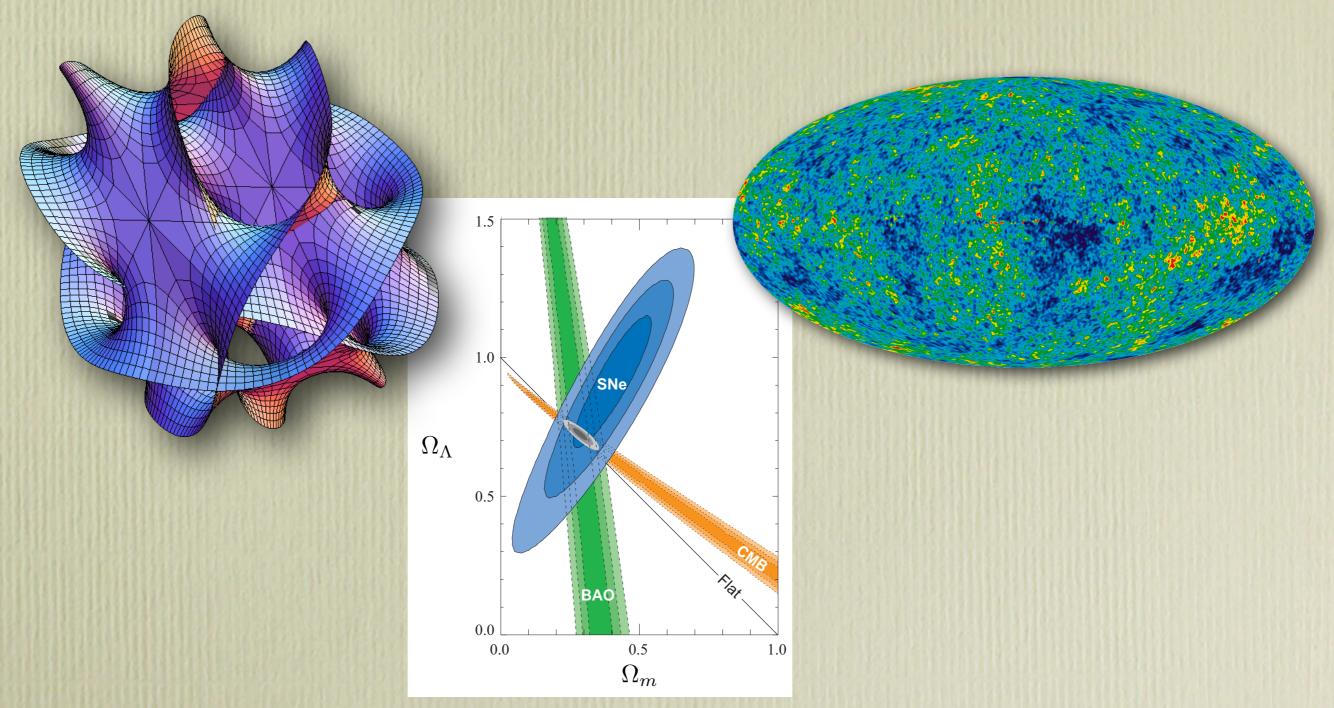
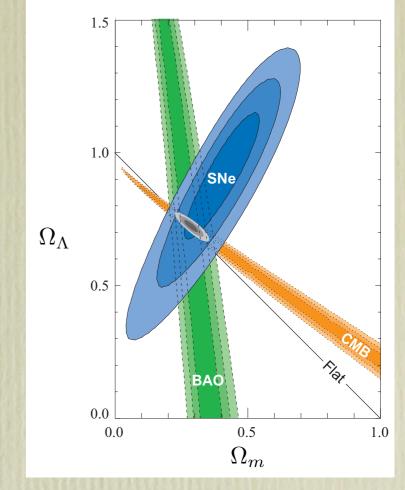
Towards explicit dS vacua...



Jan Louis, Markus Rummel, Roberto Valandro & AW: (arXiv:1107.2115 & arXiv:1208.3208)
University of Hamburg & DESY Hamburg

- the observation:
 - the Universe expands speeding up $-\Lambda > 0$... we live in de Sitter space!

a task:



there is a <u>necessary dS condition</u> in supergravity - positive sectional curvature of the Kahler potential

[Covi, Gomez-Reino, Gross, Louis, Palma, Scrucca]

- it may be worthwhile to get dS from a 'clean' system:
 - dS vacua purely from topological data determining the F-term 4d supergravity in F-theory/IIB
 - find explicit examples in F-theory on compact elliptic CY 4-folds

dS vacua from F-theory/type IIB ...

Try to use just the closed string moduli sector of a 4d N=I
 F-theory compactification on an elliptically fibered CY 4-fold

dilaton S

 $h^{1,1}$ volume moduli T_i

 $h^{2,1}$ complex structure moduli U_a

to get a dS vacuum from spontaneous F-term breaking

- key ingredient: the leading perturbative $O(\alpha^{3})$ correction
- leads to <u>'Kähler uplifted'</u> dS vacua checked for one volume modulus + dilaton S and one complex structure U

[Balasubramanian & Berglund '04][AW '06]

the general setup ...

Kähler potential (KK-reduction on a CY 3-fold):

$$K = -2\ln(\mathcal{V} + \alpha'^3 \xi) - \ln(S + \bar{S}) - \ln\left(-i\int \Omega \wedge \bar{\Omega}\right)$$
$$\xi = \xi(S, \bar{S}) \sim -\chi(CY_3) \cdot (S + \bar{S})^{3/2}$$

- superpotential
 - 3-form fluxes stabilize the c.s. moduli & the dilaton
 - Euclidean D3-branes or 7-brane stacks contribute non-perturbatively

$$W = \int_{CY_3} G_3 \wedge \Omega + \sum_{i} A_i e^{-a_i T_i}$$

the general setup ...

• F-Term scalar potential in 4D N=1 supergravity:

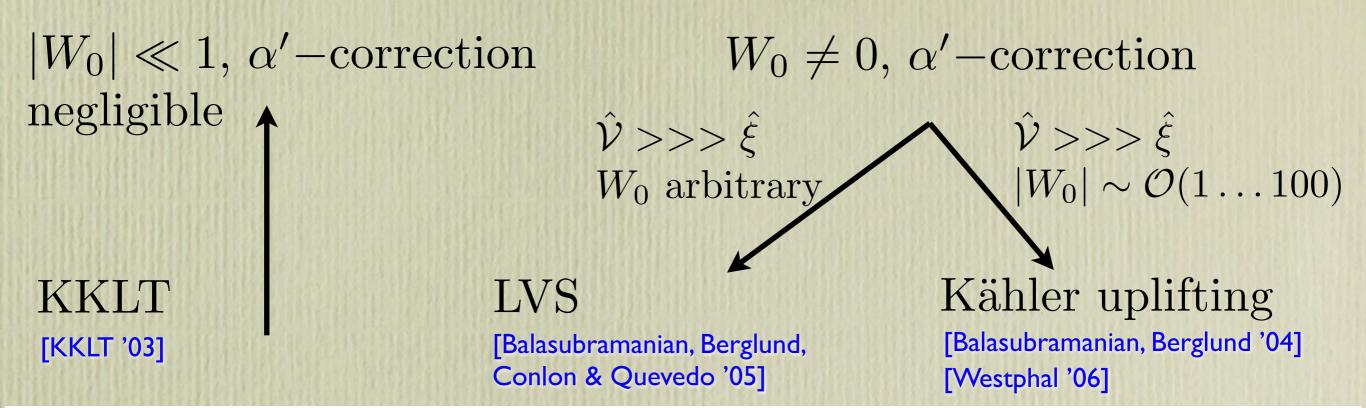
$$V_F = e^K \left(K^{S\bar{S}} |D_S W|^2 + K^{a\bar{b}} D_a W \overline{D_b W} + K^{i\bar{\jmath}} D_i W \overline{D_j W} - 3|W|^2 \right)$$

dilaton S

 $h^{1,1}$ volume moduli T_i

 $h^{2,1}$ complex structure moduli U_a

some relationships ...



- D3 branes
- F-terms from matter fields [Lebedev, Nilles, Ratz'06]
- F-terms from metastable vacua in gauge theories
 - [Intriligator, Seiberg, Shih'07]

- D3 branes
 - D-terms [Burgess, Kallosh, Quevedo'03, Haack, Krefl, Lüst, Van
 - Proyen, Zagermann'06]
- ► F-terms from dilaton dep.
- F-terms from Kähler moduli + α' -correction sufficient for dS
- [& Cicoli et al. 2012]
 - non-pert. effects [Cicoli, Maharana, Quevedo, Burgess'12]

de Sitter vacua from 'Kahler uplifting' at large volume

• 4d N=I supergravity - scalar potential: [De Alwis & Givens '11]

$$V = e^{K} \left\{ K^{T\bar{T}} [a^{2}e^{-2aT_{r}} + (-ae^{-aT_{r}}\overline{W}K_{T} + c.c)] + 3\xi \frac{\xi^{2} + 7\xi\mathcal{V} + \mathcal{V}^{2}}{(\mathcal{V} - \xi)(\xi + 2\mathcal{V})^{2}} |W|^{2} \right\}$$

• expand to leading order in ξ/V and e^{-aT} :

$$V \simeq 4AW_0 \frac{ate^{-at}}{V^2} \cos(a\tau) + \frac{3\xi W_0^2}{4V^3} \sim \frac{2C}{9x^{9/2}} - \frac{e^{-x}}{x^2}$$

$$\mathcal{V} \sim \frac{1}{\sqrt{\kappa}} (T + \bar{T})^{3/2} , \ x = at , \ T = t + i\tau , \ C \sim \frac{-W_0}{A} \xi a^{3/2} \sqrt{\kappa}$$

de Sitter vacua from 'Kahler uplifting' at large volume

there is a de Sitter existence window for C:

- C is bounded from below:

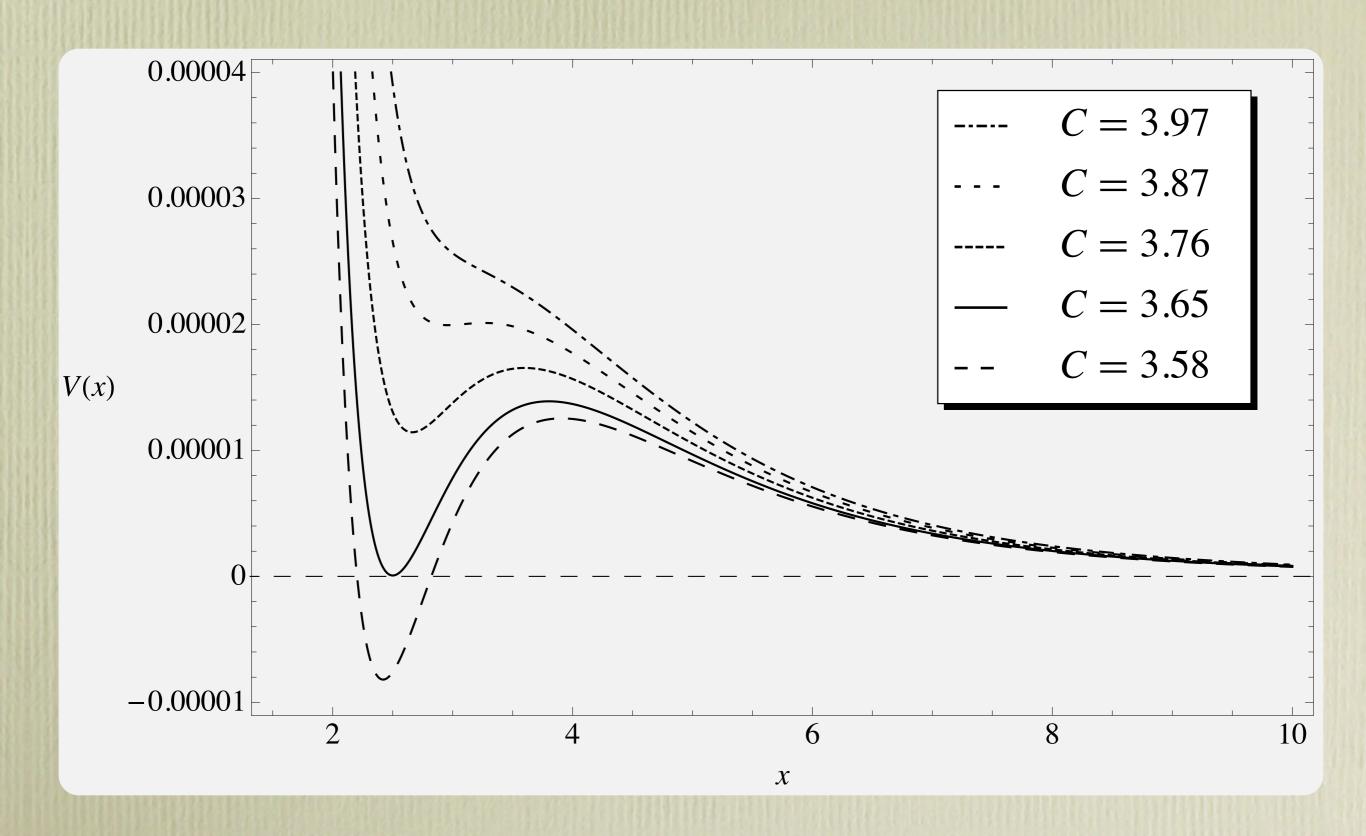
$$V = V' = 0$$
 (Minkowski point)

- C is bounded from above:

$$V' = V'' = 0$$
 (run-away limit)

$$\Rightarrow$$
 $C \simeq 3.75 \pm 0.1$, $x \sim 2.5$

de Sitter vacua from 'Kahler uplifting' at large volume



this works for arbitrary h^{1,1} on Swiss-cheese CYs!

resulting scalar potential

[Rummel & AW 'II]

$$V = \frac{4W_0}{\mathcal{V}^2} \left(atAe^{-at}\cos(a\tau) + \sum_{i=2}^{h^{1,1}} a_i t_i A_i e^{-a_i t_i} \cos(a_i \tau_i) \right) + \frac{3\xi W_0^2}{4\mathcal{V}^3} + \sum_{i=2}^{h^{1,1}} \frac{2\sqrt{2}}{3} \frac{a_i^2 A_i^2}{\mathcal{V}^2} \frac{\sqrt{t_i}}{\gamma_i} e^{-2a_i t_i} \mathcal{V}$$

all $V_{\tau_I \tau_I} > 0$ if $W_0 < 0$ at $\tau_I = 0$;

of $\mathcal{O}(\xi^2/\mathcal{V}^2)$ at the minimum

or if
$$W_0 > 0$$
 at $\tau_I = \pi/a_I$

 $V_{t_I\tau_J}=0$ at these points, thus all axions are massive

• Starting point: S and U_a at SUSY locus, T stabilization breaks SUSY

$$W = W_0 + Ae^{-aT} = C_1 - C_2 S + Ae^{-aT}$$

expand S and U_a in ξ/\mathcal{V} & check that shifts are small - first the dilaton ...

$$\Rightarrow V \sim e^{K}(K^{S\bar{S}}|D_{S}W_{0}|^{2}) + K^{T\bar{T}}|D_{T}W|^{2} + K^{T\bar{S}}D_{T}W\overline{D_{S}W_{0}})$$

$$\delta V \text{ perturbs S away from } S_{0} \dots$$

$$S_0 = -\frac{C_1}{C_2} : D_S W_0|_{S_0} = 0 , \quad \frac{\partial V}{\partial S} = 0 \Rightarrow \frac{\delta S}{S_0} \sim \frac{\xi}{\mathcal{V}}$$

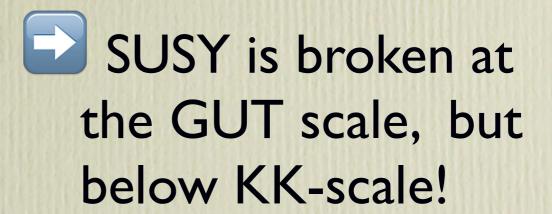
[c.f. also: Gallego & Serone]

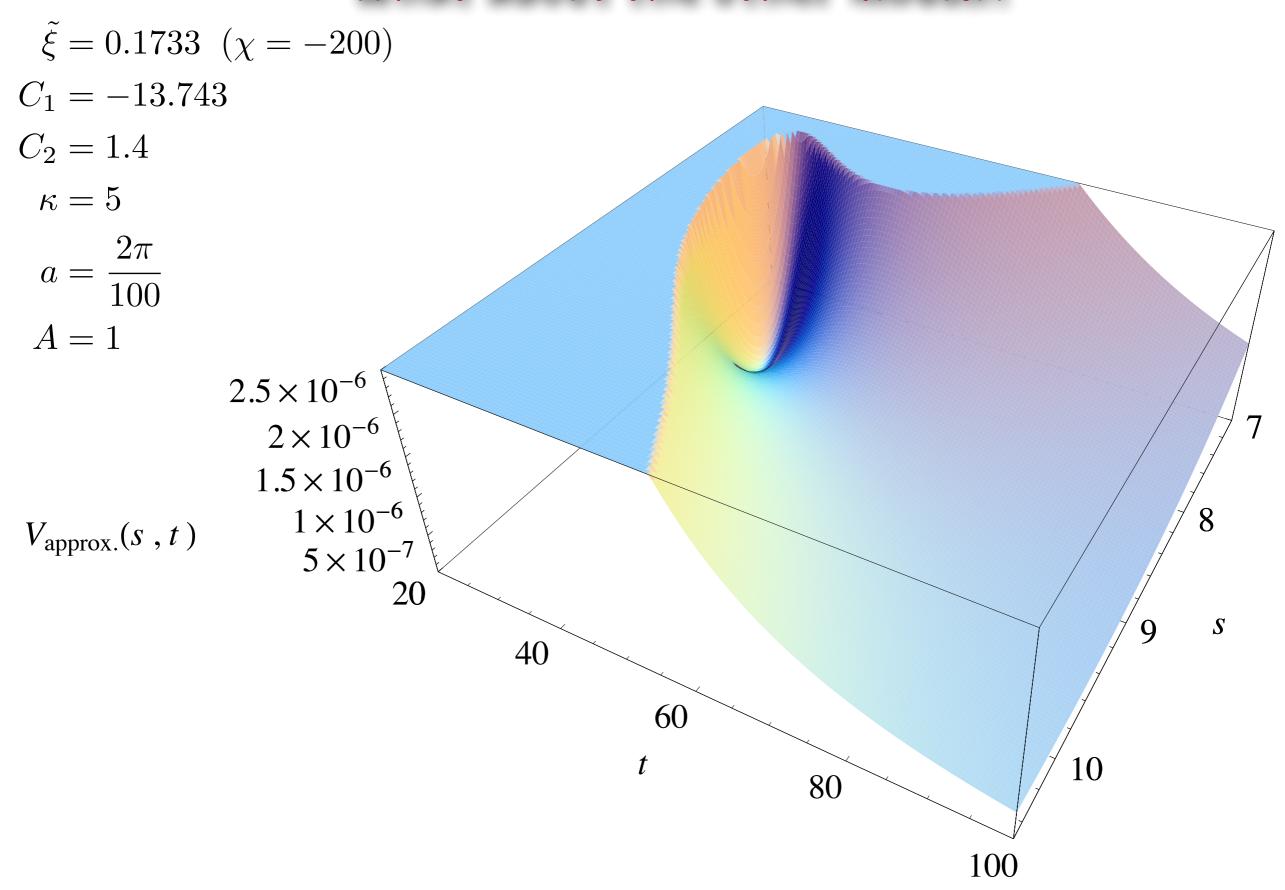
mass scales:

$$m_t^2 \sim rac{\hat{\xi}}{\mathcal{V}^3}$$
 $m_ au^2 \sim rac{\hat{\xi}}{\mathcal{V}^3}$

$$m_{\mathrm{Re}\,S}^2 \sim \frac{1}{\mathcal{V}^2}$$
 $m_{\mathrm{Im}\,S}^2 \sim \frac{1}{\mathcal{V}^2}$

$$m_{3/2}^2 = e^K |W|^2 \sim \frac{1}{\mathcal{V}^2}$$
 $m_{KK}^2 = \frac{1}{L^2 \alpha'} \sim \frac{1}{\mathcal{V}^{4/3}}$





• similarly for the complex structures Ua ...

$$K = \dots - \ln \left(\int_{CY_3} \Omega(U_a) \wedge \bar{\Omega}(\overline{U_a}) \right)$$

 C_1, C_2 become functions of U_a

$$\Rightarrow V \sim e^{K}(K^{a\bar{b}}D_{U_{a}}W_{0}\overline{D_{U_{b}}W_{0}}) + K^{T\bar{T}}|D_{T}W|^{2} + K^{T\bar{S}}D_{T}W\overline{D_{S}W_{0}})$$

$$V_{0} \qquad \qquad \delta V \text{ perturbs } U \text{ away from } U_{0} \dots$$

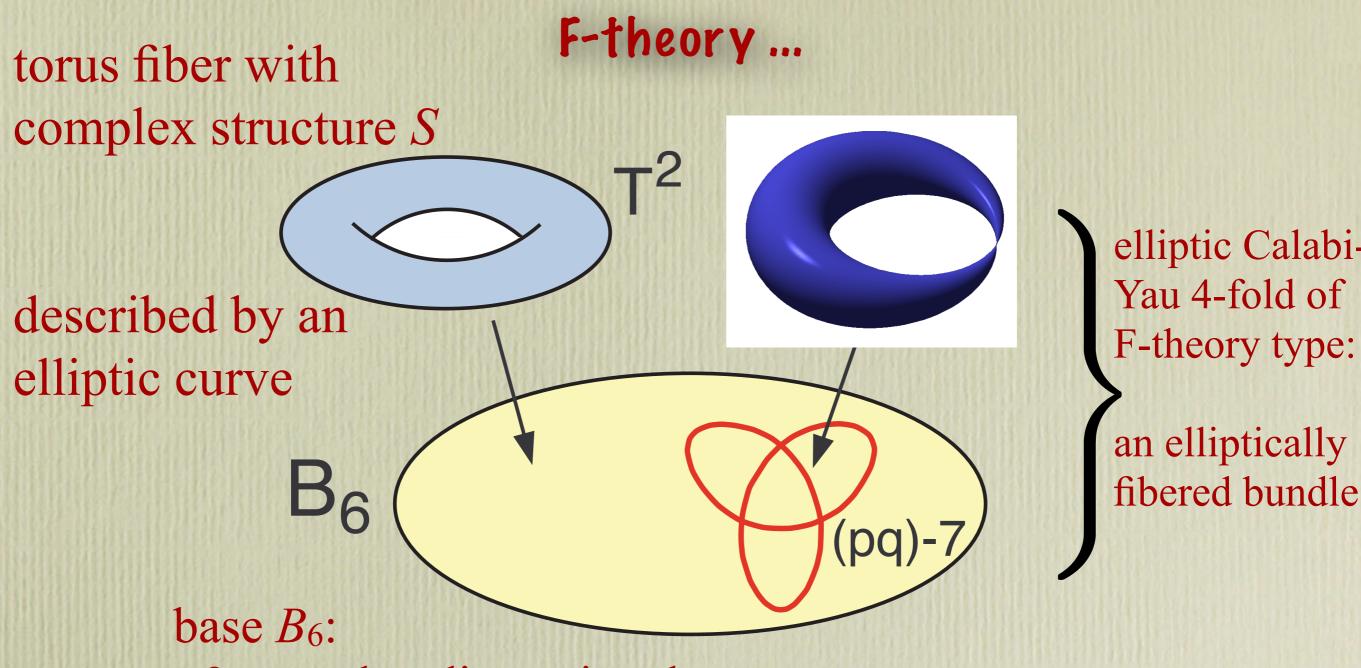
$$|U_{a,0}|: |D_{U_a}W_0|_{U_{a,0}} = 0 , \qquad \frac{\partial V}{\partial U_a} = 0 \implies \frac{\partial U_a}{U_{a,0}} \sim \frac{\xi}{\mathcal{V}}$$

[c.f. also: Gallego & Serone]

need
$$(\partial_a \partial_{\bar{b}} V_0) > 0$$

but this is guaranteed:

- flux alone gives a no-scale vacuum
- V_0 positive semidefinite
 - → stable
- no-scale breaking is volume suppressed
 - stability survives



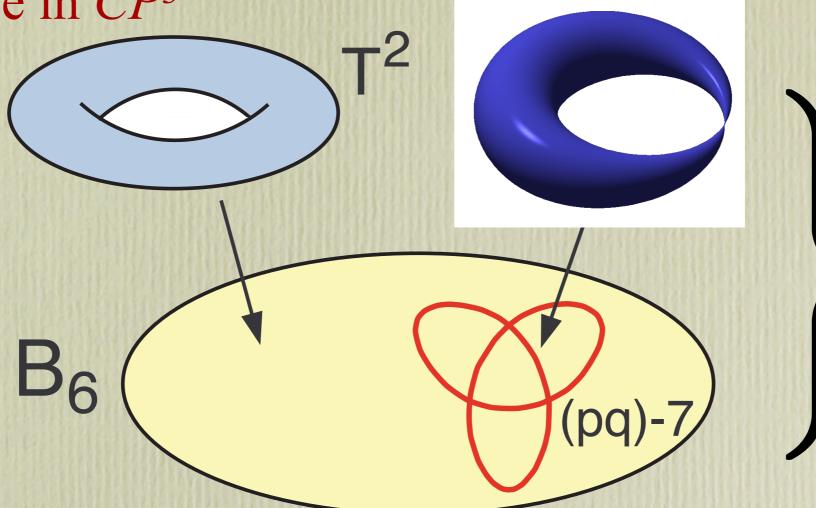
- 3-complex dimensional compact space
- simplest case: complex projective space CP^4
- more general class: hypersurfaces described by polynomial equations in an ambient toric variety

described by an

elliptic curve in CP^3

w/ coord. (X,Y,Z)





elliptic Calabi-Yau 4-fold of F-theory type:

an elliptically fibered bundle

elliptic curve of fiber described a 'Weierstrass model':

$$Y^2 = X^3 + f X Z^4 + g Z^6$$

f and g are sections of $4\bar{K}$ and $6\bar{K}$

(deg. 4 and 6 polynomials in the coord.s on B_6)

 $\bar{K} = c_1(B_6)$ is the anti-canonical bundle of B_6

described by an F-theory ...

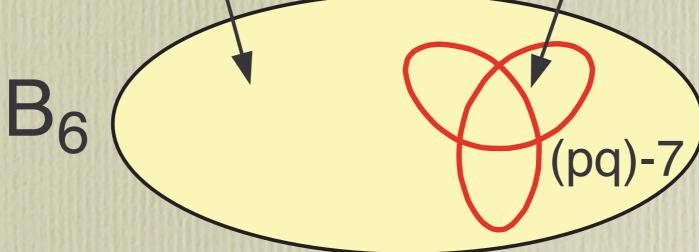
elliptic curve in CP³

w/ coord.

(X,Y,Z)



vanishing fiber describes a 7-brane



degenerate fiber: zeros of discriminant of 'Weierstrass model':

$$\Delta = 27 \, g^2 + 4 \, f^3$$

order of zero = rank of gauge group = # (stacked 7-branes) order of zeros in f,g = gauge group type $A_n (SU(n))$, $D_n (SO(n))$, $E (E_6,E_7,E_8)$

- search for:
 - elliptically fibered CY 4-folds
 - with good Sen limit to weakly coupled type IIB
 - Kähler uplifting → large-rank 7-brane gauge group

• tool:

- 'mini'-landscape of 7,602 four-folds of F-theory type from the Kreuzer-Skarke classification with distinct Hodge number pairs $(h^{1,1},h^{2,1})$
- 3,040 of those give type IIB on a CY3 with negative Euler number

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'Mini' landscape: 97,036 models of the F-theory type: [Kreuzer, Skarke '00]

Ambient toric variety described by (resolution of) weight system

$$\mathbb{CP}_{n_1 n_2 n_3 n_4 n_{\xi}}$$
: $0 < n_1 \le n_2 \le n_3 \le n_4 < n_{\xi} = \sum_{i=1}^4 n_i$

$$\frac{u_1 \ u_2 \ u_3 \ u_4 \ \xi}{n_1 \ n_2 \ n_3 \ n_4 \ n_5} \quad \text{with} \quad 0 < n_1 \le n_2 \le n_3 \le n_4 \le n_5$$

$$(u_1,...,u_4,\xi) \sim (\lambda^{n_1}u_1,...,\lambda^{n_4}u_4,\lambda^{n_5}\xi), \quad \text{with } \lambda \in \mathbb{C}^*$$

• the toric divisors D_i , D_{ξ} are hypersurfaces (4-cycles) given by holomorphic equations:

$$D_i: \{u_i = 0\} \qquad D_{\xi}: \{\xi = 0\}$$

 for N=I supersymmetry in 4D we need to orientifold - I possible orientifold projection:

$$\sigma: \quad \xi \mapsto -\xi$$

 a CY 3-fold is then a hypersurface describing a double-cover of B₆ in the complex-4d toric ambient space

$$\xi^2 = P_{(2\sum_{i=1}^4 n_i, \dots)}$$

if its degree = $2n_{\xi}$ and

$$n_{\xi} \equiv n_5 = \sum_{i=1}^{4} n_i \quad \Rightarrow \quad \bar{K}(CY_3) = c_1(CY_3) = 0$$

 can do this in F-theory with a Weierstrass model fibered over the toric base B₆:

$$\frac{u_1 \ u_2 \ u_3 \ u_4}{n_1 \ n_2 \ n_3 \ n_4} \qquad \mathbb{CP}^3_{n_1 n_2 n_3 n_4}$$

• rewrite the Weierstrass equation in Tate form:

$$Y^{2} + a_{1}XYZ + a_{3}YZ^{3} = X^{3} + a_{2}X^{2}Z^{2} + a_{4}XZ^{4} + a_{6}Z^{6}$$

 a_i are functions of the u_i on the base such, that they are sections of $i\bar{K}$

• to construct the 7-brane singularity for a given gauge group on divisor D_j , a_i have to scale in u_j with certain weights

$$a_i = u_j^{w_i} a_{i,w_i}$$

Kodaira classification gives weights:

	a_1	a_2	a_3	a_4	a_6	Δ
Sp(N)	0	0	\overline{N}	\overline{N}	2N	2N
SU(2N)	0	1	N	N	2N	2N
SU(2N+1)	0	1	N	N+1	2N + 1	2N + 1
SO(4N+1)	1	1	N	N+1	2N	2N + 3
SO(4N+2)	1	1	N	N+1	2N + 1	2N + 3
SO(4N+3)	1	1	N+1	N+1	2N + 1	2N + 4
SO(4N+4)	1	1	N+1	N+1	2N + 1	2N + 4

• the toric base B₆ of the 4-fold has anti-canonical class

$$\bar{K} = [D_{\xi}] = \sum_{i=1...4} [D_i] = n_{\xi}[D_1] \Rightarrow n_{\xi} = \sum_{i=1...4} n_i$$

• because a_i is in $i\bar{K}$ we have:

$$a_i = u_j^{w_i} a_{(in_{\xi} - w_i n_j, \dots)}, \quad j = 1 \dots 4$$

• because the $a_{i,w}$ must be holomorphic:

$$N = rank(gauge\ group) = w_i \le \frac{i \, n_{\xi}}{n_i}$$

strongest constraint from a₃, a₆:

$$N_{lg} \le \frac{3n_{\xi}}{n_j}$$

• Sen limit - rescale the a_i:

$$a_3 \mapsto \epsilon \, a_3$$
,

$$a_4 \mapsto \epsilon a_4$$
,

$$a_3 \mapsto \epsilon \, a_3 \,, \qquad a_4 \mapsto \epsilon \, a_4 \,, \qquad a_6 \mapsto \epsilon^2 \, a_6$$

such that:

$$g_s \sim -\frac{1}{\log|\epsilon|} \to 0$$
 as $\epsilon \to 0$

 then we get (complete square & cube in Tateextended Weierstrass equation):

$$f = -\frac{1}{48}(h^2 - 24\epsilon\eta), \qquad g = -\frac{1}{864}(-h^3 + 36\epsilon h\eta - 216\epsilon^2\chi)$$

$$h = a_1^2 + 4a_2$$
, $\eta = a_1a_3 + 2a_4$, $\chi = a_3^2 + 4a_6$

• the discrimant becomes:

$$\Delta = \frac{1}{16} \left(\epsilon^2 h^2 P_{D7} + 8\epsilon^3 \eta^3 + 27\epsilon^4 \chi^2 - 9h\epsilon^3 \eta \chi \right) \sim \frac{1}{16} \epsilon^2 h^2 P_{D7} + \mathcal{O}(\epsilon^3)$$

where
$$P_{D7} = -\frac{1}{4}(\eta^2 - h\chi)$$

• **O7**:
$$h = 0$$

• D7:
$$P_{D7} = 0$$

• CY 3-fold is a double cover of B_6 :

$$X_3: 0 = \xi^2 - h = \xi^2 - (a_1^2 + 4a_2)$$

• so, we are now in type IIB ...

• the D7-brane equation becomes:

$$\eta^2 - \xi^2 \chi = 0$$

• this is a holomorphic D7-brane called Whitney (umbrella) for its shape ...

 a large-rank gauge group requires a large stack of coincident D7-branes - need to factorize in divisor coordinate

$$\eta = u_i^{N_i} \tilde{\eta} \,, \qquad \chi = u_i^{2N_i} \tilde{\chi}$$

and thus

$$u_i^{2N_i} \left(\tilde{\eta}_{(4n_{\xi} - n_i N_i, \dots)}^2 - \xi^2 \tilde{\chi}_{(6n_{\xi} - 2n_i N_i, \dots)} \right) = 0$$

• so, by holomorphy again:

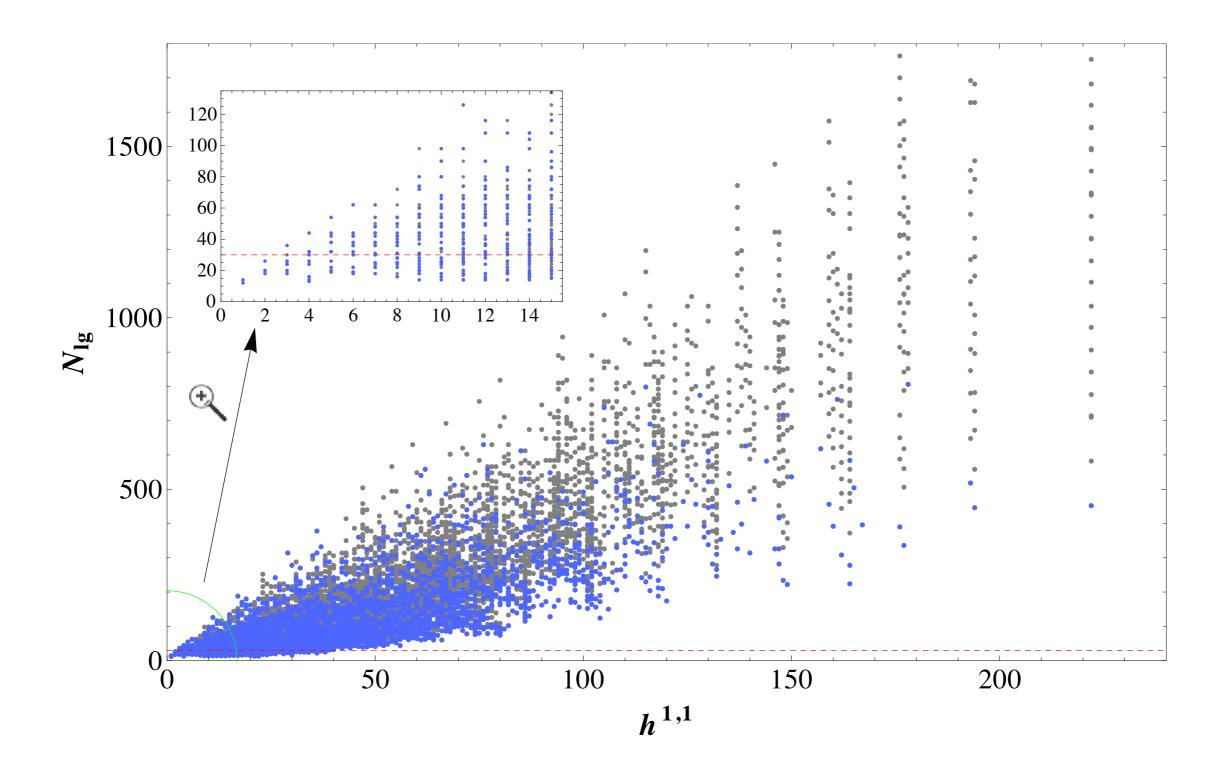
$$N_i \le 3 \frac{n_{\xi}}{n_i}$$

• thus, large rank needs large orientifold class n_{ξ}

• if any $n_i > 1$, then B_6 has singularities

- resolution (blow-up) adds new lines to weight system (GLSM) -> each new line is a new independent divisor, and thus new Kähler modulus
- so maximal rank will increase with $h^{1,1}$

can plot this for F-theory type Kreuzer-Skarke set:



• thus, because (naively):

$$\mathcal{V} \sim N_{lg}^{3/2}$$

volumes > 10³ are possible in a large fraction of CY space

 this ensures control & separation of Kähler and complex structure moduli mass scales → justifies treating the instanton prefactors as constant!

Constraints on a consistent model

- ▶ Contribution of gaugino condensation to the superpotential, $A \neq 0$:
 - Rigid divisor? [Witten'96]
 - Can it be 'rigidified' by gauge flux F? [Martucci'06, Bianchi, Collinucci, Martucci'11]
- Swiss-cheese?

- ► Flux: Freed-Witten anomalies? [Minasian, Moore'96, Freed, Witten'97]
- ▶ $N_1 \gg 1$ enforces factorization of D7 brane equation in coordinates $u_i \neq u_1$ [Cicoli,Mayrhofer,Valandro'11]?

Constraints on a consistent model

- Stabilization inside the Kähler cone?
- ► Chiral matter at brane intersections that might destroy $A \neq 0$ [Blumenhagen, Moster, Plauschinn'08]?
- ► D3 tadpole: $Q^{D7-\text{stacks}} + Q^{O7} = Q^{\mathcal{F}} + Q^{RR,NS-NS} + Q^{D3-\text{branes}}$?
- ► Complex structure moduli stabilized such that sufficient condition for de Sitter is fulfilled: $W_0 \cdot \text{Re}(S)^{3/2}$ in right interval?

Explicit dS example: \mathbb{CP}_{11169}

Some geometric properties:

$$h^{1,1} = 2, h^{2,1} = 272$$

Divisors:

Explicit dS example: \mathbb{CP}_{11169}

$$\hat{\mathcal{V}} = \sqrt{\frac{2}{3}} \left(\hat{\mathcal{V}}_1 + \frac{1}{3} \hat{\mathcal{V}}_5 \right)^{3/2} - \frac{\sqrt{2}}{9} \hat{\mathcal{V}}_5^{3/2} \Rightarrow \text{'Approx. swiss-chesse'}$$

► Complex structure moduli: $\mathbb{Z}_6 \times \mathbb{Z}_{18}$ modding: $h_{\text{inv.}}^{2,1} = 2$. with known prepotential. [Greene, Plesser'89], [Candelas, Font, Katz, Morrison'94]

G(z) via mirror symmetry in the large complex structure limit:

$$G(z_1, z_2) = \sum_{i+j \le 3} c_{ij} z_1^i z_2^j + \xi + G_{instanton}(e^{-2\pi z_1}, e^{-2\pi z_2})$$

• 1st step - 3-form flux fixes the 2 invariant complex structure moduli supersymmetrically:

Stabilizing the $h_{\text{inv.}}^{2,1}=2$ moduli effectively fixes all other C.S. moduli at $D_iW=0$ since V positive definite up to corrections $\mathcal{O}(\hat{\xi}/\hat{\mathcal{V}})$ [Giryavets, Kachru, Tripathy, Trivedi '03]

• the 270 Z_6 x Z_{18} non-invariant c.s. moduli are stabilized by invariant higher-order terms in W (from invariant higher-order terms in the 4 invariant periods)

Strategy to find $\langle W_0 \rangle$, $\langle S \rangle$ suitable for Kähler uplifting: [DDF'04]

Solve $(W_0, D_S W_0, D_{U_1} W_0, D_{U_2} W_0) = 0$, for the flux quanta $f_1, \ldots, f_6, h_1, \ldots, h_6$

Include instanton corrections in the prepotential: shifts in $\langle W_0 \rangle$, $\langle S \rangle$, $\langle U_1 \rangle$ and $\langle U_2 \rangle$

generates $W_0 \neq 0$, then check VEVs of W_0 and S

 \Rightarrow have to fall into dS window

A solution:

$$(f,h) = (-16,0,0,0,-4,-2;0,0,2,-8,-3,0), Q^{RR,NS-NS} = 66,$$

$$\langle S \rangle = 6.99, \quad \langle U_1 \rangle = 1.01, \quad \langle U_2 \rangle = 0.967, \quad \langle W_0 \rangle = 0.812,$$

$m_{u_1}^2$	$m_{u_2}^2$	m_s^2	$m_{ u_1}^2$	$m_{ u_2}^2$	m_{σ}^2
0.24	$1.8 \cdot 10^{-4}$	$5.6 \cdot 10^{-6}$	0.24	$1.8 \cdot 10^{-4}$	$5.7 \cdot 10^{-6}$

• 2nd step - Kähler moduli:

- ▶ Brane config. $(N_{lg} = 27)$: Sp(24) on D_1 forces SO(24) on D_5 .
- ▶ D_5 rigid, D_1 can be 'rigidified' by gauge flux $\Rightarrow Sp(24) \rightarrow SU(24)$.
- ▶ Brane intersections: Switch on gauge flux $F_{1/5} + c_1(D_{1/5})/2$ to cancel Freed-Witten anomalies and tune F_1 , F_5 and B such that $\mathcal{F}_{1/5} = F_{1/5} B$ is 'trivial' \Rightarrow No chiral matter or D-terms.
- ▶ D3 tadpole: $Q_{D3}^{\text{tot}} = Q_{D3}^{\text{O7s}} + Q_{D3}^{\text{stacks}} + Q_{D3}^{W} = \begin{cases} -104 & \text{for } Q_{D3}^{W} = -81 \\ -96 & \text{for } Q_{D3}^{W} = -73 \end{cases}$

• 2nd step - Kähler moduli:

$$\Rightarrow W = W_0 + A e^{-2\pi/24 T_1} + B e^{-2\pi/22 T_2}, A, B \neq 0.$$

If in the complex structure sector:

$$\langle W_0 \rangle = 0.812, \ \langle S \rangle = 6.99, \ \langle A \rangle = 1.11, \ \langle B \rangle = 1.00.$$

▶ stable dS vacuum with $\langle T_1 \rangle = 10.76$, $\langle T_2 \rangle = 12.15$ and $\hat{\mathcal{V}} = 52$.

Conclusions

 We have explicit constructions of dS vacua in type IIB string theory/F-theory in the 'Kähler uplifting' scenario

- They are fully determined by the 4-fold data:
 - the Euler characteristic
 - choice of ADE-type singularities,
 - and choice of fluxes
- They break SUSY by Kähler moduli F-terms at the GUT scale. No extra source of uplifting is needed
- A whole 'mini'-landscape of explicit examples (Kreuzer-Skarke) is available. One example is shown to satisfy all known F-theory/IIB consistency constraints.