A new view on charge and color breaking minima in the MSSM

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DISCLAIMER: No $\gamma\gamma$ excess!

MSSM may be ruled out anyway...

...or not? (cf. Djouadi’s talk?)
FRANKLY, I WAS EXPECTING SOMETHING A BIT MORE SOPHISTICATED...
Brout–Englert–Higgs mechanism

\[ SU(3)_c \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_c \times U(1)_{em} \]

Consequence: The (in)famous Higgs boson!

- scalar mass sensitive to high scale physics (hierarchy problem)
- Standard Model vacuum metastable (will eventually decay)

[Degrassi et al. 2012]

- its mass could not be predicted (in the SM)

A viable solution / extension of the SM

- The Minimal Supersymmetric Standard Model (MSSM)
- “predicts” the Higgs mass; defines Higgs potential
- solves the hierarchy problem
- stabilizes the potential
The Minimal Supersymmetric Standard Model: A multi-scalar theory

\[ V = V_F + V_D + V_{\text{soft}} \]

with

\[ V_F = \sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2, \]

\[ V_D = \frac{1}{2} \sum_a g_a^2 \left( \sum_i \phi_i^\dagger T^a \phi_i \right)^2 \]

\[ V_{\text{soft}} = \sum_i m_{\phi_i}^2 |\phi_i|^2 + \sum_{ijk} A_{ik}^{(j)} \phi_i^\dagger \phi_j \phi_k \]

\[ \leftrightarrow \]

The Standard Model: A single scalar theory

\[ V_{\text{SM}} = -\mu^2 H^\dagger H + \frac{\lambda}{4} \left( H^\dagger H \right)^2 \]
A multi-scalar theory

- 2 Higgs doublets
- $2 \times 6$ scalar quarks, $6 + 3$ scalar leptons
- 12 colored and $18 + 2$ charged directions
- charged Higgs directions “safe”

SM Higgs potential: $SO(4)$ symmetry

[Casas et al. 1996]

The hazard

- impossible to minimize directly, analytically
- colored directions sensitive to all kinds of SUSY breaking
- spontaneous breaking of color charge: $\langle \tilde{q} \rangle \neq 0$

The true vacuum

- effective potential: average energy density
- global minimum: true ground state of the theory
The third generation MSSM

\[ \mathcal{W} = \mu H_1 \cdot H_2 + y_t H_2 \cdot Q_L \bar{T}_R - y_b H_1 \cdot Q_L \bar{B}_R \]

- large couplings to Higgs doublets (\(y_t\) and \(y_b\) comparably large)
- large stop contribution (\(X_t, A_t\)) to light Higgs mass needed
- \(\tan \beta\) resummation for \(m_b\) influences \(y_b\)

Properties of the (effective) scalar potential

- no UFB directions (due to quantum corrections)
- \(D\)-terms: (comparably) large contributions \(\phi^4\)
- “dangerous” directions: small quadrilinears + large trilinears

Analytic constraints

- define certain directions in field space: great simplification
- e.g. \(D\)-terms absent: \(|\tilde{Q}_L| = |\tilde{t}_R| = |h_2|\) (possibly miss sth.)
The tree-level scalar potential

\[ V_{\tilde{q},h} = \tilde{t}_L^* (\tilde{m}_L^2 + |y_t h_2|^2) \tilde{t}_L + \tilde{t}_R^* (\tilde{m}_t^2 + |y_t h_2|^2) \tilde{t}_R + \tilde{b}_L^* (\tilde{m}_L^2 + |y_b h_1|^2) \tilde{b}_L + \tilde{b}_R^* (\tilde{m}_b^2 + |y_b h_1|^2) \tilde{b}_R - [\tilde{t}_L^* (\mu^* y_t h_1^* - A_t h_2) \tilde{t}_R + \text{h.c.}] - [\tilde{b}_L^* (\mu^* y_b h_2^* - A_b h_1) \tilde{b}_R + \text{h.c.}] + |y_t|^2 |\tilde{t}_L|^2 |\tilde{t}_R|^2 + |y_b|^2 |\tilde{b}_L|^2 |\tilde{b}_R|^2 + \frac{g_1^2}{8} \left( |h_2|^2 - |h_1|^2 + \frac{1}{3} |\tilde{b}_L|^2 + \frac{2}{3} |\tilde{b}_R|^2 + \frac{1}{3} |\tilde{t}_L|^2 - \frac{4}{3} |\tilde{t}_R|^2 \right)^2 + \frac{g_2^2}{8} \left( |h_2|^2 - |h_1|^2 + |\tilde{b}_L|^2 - |\tilde{t}_L|^2 \right)^2 + \frac{g_3^2}{8} \left( |\tilde{t}_L|^2 - |\tilde{t}_R|^2 + |\tilde{b}_L|^2 - |\tilde{b}_R|^2 \right)^2 + (m_{h_2}^2 + |\mu|^2) |h_2|^2 + (m_{h_1}^2 + |\mu|^2) |h_1|^2 - 2 \text{Re}(B_\mu h_1 h_2). \]
The tree-level scalar potential

\[ V_{\tilde{q},h} = \tilde{t}_L^* (\tilde{m}_L^2 + |y_t h_2|^2) \tilde{t}_L + \tilde{t}_R^* (\tilde{m}_t^2 + |y_t h_2|^2) \tilde{t}_R \\
+ \tilde{b}_L^* (\tilde{m}_L^2 + |y_b h_1|^2) \tilde{b}_L + \tilde{b}_R^* (\tilde{m}_b^2 + |y_b h_1|^2) \tilde{b}_R \\
- [\tilde{t}_L^* (\mu^* y_t h_1^* - A_t h_2) \tilde{t}_R + \text{h.c.}] \\
- [\tilde{b}_L^* (\mu^* y_b h_2^* - A_b h_1) \tilde{b}_R + \text{h.c.}] \\
+ |y_t|^2 |\tilde{t}_L|^2 |\tilde{t}_R|^2 + |y_b|^2 |\tilde{b}_L|^2 |\tilde{b}_R|^2 \\
+ \frac{g_1^2}{8} \left( |h_2|^2 - |h_1|^2 + \frac{1}{3} |\tilde{b}_L|^2 + \frac{2}{3} |\tilde{b}_R|^2 + \frac{1}{3} |\tilde{t}_L|^2 - \frac{4}{3} |\tilde{t}_R|^2 \right)^2 \\
+ \frac{g_2^2}{8} \left( |h_2|^2 - |h_1|^2 + |\tilde{b}_L|^2 - |\tilde{t}_L|^2 \right)^2 \\
+ \frac{g_3^2}{8} \left( |\tilde{t}_L|^2 - |\tilde{t}_R|^2 + |\tilde{b}_L|^2 - |\tilde{b}_R|^2 \right)^2 \\
+ (m_{h_2}^2 + |\mu|^2)|h_2|^2 + (m_{h_1}^2 + |\mu|^2)|h_1|^2 - 2 \text{Re}(B_\mu h_1 h_2). \]

\[ |\tilde{t}_L| = |\tilde{t}_R| = |\tilde{t}|, \ |\tilde{b}_L| = |\tilde{b}_R| = |\tilde{b}| \]
The tree-level scalar potential

$V_{\tilde{q}, h} = \tilde{t}^* (\tilde{m}_L^2 + |y_t h_2|^2) \tilde{t} + \tilde{t}^* (\tilde{m}_t^2 + |y_t h_2|^2) \tilde{t}$

$+ \tilde{b}^* (\tilde{m}_L^2 + |y_b h_1|^2) \tilde{b} + \tilde{b}^* (\tilde{m}_b^2 + |y_b h_1|^2) \tilde{b}$

$- [\tilde{t}^* (\mu^* y_t h_1^* - A_t h_2) \tilde{t} + \text{h.c.}]$

$- [\tilde{b}^* (\mu^* y_b h_2^* - A_b h_1) \tilde{b} + \text{h.c.}]$

$+ |y_t|^2 |\tilde{t}|^2 |\tilde{t}|^2 + |y_b|^2 |\tilde{b}|^2 |\tilde{b}|^2$

$+ \frac{g_1^2}{8} \left( |h_2|^2 - |h_1|^2 + |\tilde{b}|^2 - |\tilde{t}|^2 \right)^2$

$+ \frac{g_2^2}{8} \left( |h_2|^2 - |h_1|^2 + |\tilde{b}|^2 - |\tilde{t}|^2 \right)^2$

$+ (m_{h_2}^2 + |\mu|^2)|h_2|^2 + (m_{h_1}^2 + |\mu|^2)|h_1|^2 - 2 \text{Re}(B_{\mu} h_1 h_2)$.

$|\tilde{t}_L| = |\tilde{t}_R| = |\tilde{t}|, |\tilde{b}_L| = |\tilde{b}_R| = |\tilde{b}|$
The tree-level scalar potential

\[ V_{\tilde{q},h} = \tilde{t}^\ast (\tilde{m}_L^2 + |y_t h_2|^2) \tilde{t} + \tilde{t}^\ast (\tilde{m}_t^2 + |y_t h_2|^2) \tilde{t} \\
+ \tilde{b}^\ast (\tilde{m}_L^2 + |y_b h_1|^2) \tilde{b} + \tilde{b}^\ast (\tilde{m}_b^2 + |y_b h_1|^2) \tilde{b} \\
- [\tilde{t}^\ast (\mu^* y_t h_1^* - A_t h_2) \tilde{t} + \text{h.c.}] \\
- [\tilde{b}^\ast (\mu^* y_b h_2^* - A_b h_1) \tilde{b} + \text{h.c.}] \\
+ |y_t|^2 |\tilde{t}|^2 |\tilde{t}|^2 + |y_b|^2 |\tilde{b}|^2 |\tilde{b}|^2 \\
+ \frac{g_1^2}{8} \left( |h_2|^2 - |h_1|^2 + |\tilde{b}|^2 - |\tilde{t}|^2 \right)^2 \\
+ \frac{g_2^2}{8} \left( |h_2|^2 - |h_1|^2 + |\tilde{b}|^2 - |\tilde{t}|^2 \right)^2 \\
+ (m_{h_2}^2 + |\mu|^2)|h_2|^2 + (m_{h_1}^2 + |\mu|^2)|h_1|^2 - 2 \text{Re}(B_{\mu} h_1 h_2). \]

\[ |\tilde{t}_L| = |\tilde{t}_R| = |\tilde{t}|, \quad |\tilde{b}_L| = |\tilde{b}_R| = |\tilde{b}| ; \quad |\tilde{b}| = |h_1| = |\phi_1|, \quad |\tilde{t}| = |h_2| = |\phi_2| \]
The tree-level scalar potential

\[ V_{\tilde{q},h} = \phi_2^* \left( \tilde{m}_L^2 + |y_t \phi_2|^2 \right) \phi_2 + \phi_2^* \left( \tilde{m}_t^2 + |y_t \phi_2|^2 \right) \phi_2 \]
\[ + \phi_1^* \left( \tilde{m}_L^2 + |y_b \phi_1|^2 \right) \phi_1 + \phi_1^* \left( \tilde{m}_b^2 + |y_b \phi_1|^2 \right) \phi_1 \]
\[ - \left[ \phi_2^* \left( \mu^* y_t \phi_1^* - A_t \phi_2 \right) \phi_2 + \text{h.c.} \right] \]
\[ - \left[ \phi_1^* \left( \mu^* y_b \phi_2^* - A_b \phi_1 \right) \phi_1 + \text{h.c.} \right] \]
\[ + |y_t|^2 |\phi_2|^2 |\phi_2|^2 + |y_b|^2 |\phi_1|^2 |\phi_1|^2 \]
\[ + (m_{h_2}^2 + |\mu|^2) |\phi_2|^2 + (m_{h_1}^2 + |\mu|^2) |\phi_1|^2 - 2 \text{Re}(B_\mu \phi_1 \phi_2). \]

|\tilde{t}_L| = |\tilde{t}_R| = |\tilde{t}|, |\tilde{b}_L| = |\tilde{b}_R| = |\tilde{b}| ; |\tilde{b}| = |h_1| = |\phi_1|, |\tilde{t}| = |h_2| = |\phi_2|
The tree-level scalar potential

\[
V_{\tilde{q}, h} = \phi_2^* \left( \tilde{m}_L^2 + |y_t \phi_2|^2 \right) \phi_2 + \phi_2^* \left( \tilde{m}_t^2 + |y_t \phi_2|^2 \right) \phi_2 \\
- \left[ \phi_2^* \left( - A_t \phi_2 \right) \phi_2 + \text{h.c.} \right] \\
+ |y_t|^2 |\phi_2|^2 |\phi_2|^2 \\
+ \left( m_{h_2}^2 + |\mu|^2 \right) |\phi_2|^2 \\
\]

\[
|\tilde{t}_L| = |\tilde{t}_R| = |\tilde{t}|, \quad |\tilde{b}_L| = |\tilde{b}_R| = |\tilde{b}|; \quad \tilde{b} \equiv h_1 \equiv \phi_1, \quad |\tilde{t}| = |h_2| = |\phi_2| 
\]
Minimize the potential

\[ V(\phi) = m^2 \phi^2 - A\phi^3 + \lambda \phi^4, \]

with \( m^2 = m_{h_2}^2 + |\mu|^2 + \tilde{m}_L^2 + \tilde{m}_t^2, \) \( A = -A_t \) and \( \lambda = 3y_t^2. \)
Minimize the potential

\[ V(\phi) = m^2 \phi^2 - A\phi^3 + \lambda\phi^4, \]

with \( m^2 = m_{h_2}^2 + |\mu|^2 + \tilde{m}_L^2 + \tilde{m}_t^2, A = -A_t \) and \( \lambda = 3y_t^2. \)

Answer:

\[ \phi_0 = 0, \quad \phi_\pm = \frac{3A \pm \sqrt{9A^2 32\lambda m^2}}{8\lambda}. \]

Condition to be safe from non-standard (i.e. non-trivial) minima:

\[ V(\phi_\pm) > 0 \quad \iff \quad m^2 > \frac{A^2}{4\lambda} \]
Minimize the potential

\[ V(\phi) = m^2 \phi^2 - A\phi^3 + \lambda\phi^4, \]

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Answer:

\[ \phi_0 = 0, \quad \phi_\pm = \frac{3A \pm \sqrt{9A^232\lambda m^2}}{8\lambda}. \]

Condition to be safe from non-standard (i.e. non-trivial) minima:

\[ V(\phi_\pm) > 0 \implies m^2 > \frac{A^2}{4\lambda} \]

Well-known constraints \([Gunion, Haber, Sher ’88]\)

\[ |A_t|^2 < 3y_t^2 (m_{h_2}^2 + |\mu|^2 + \tilde{m}_L^2 + \tilde{m}_t^2) \]

\[ |A_b|^2 < 3y_b^2 (m_{h_1}^2 + |\mu|^2 + \tilde{m}_L^2 + \tilde{m}_b^2) \]

for the limiting cases \( |\tilde{t}_L| = |\tilde{t}_R| = |h_2| \) and \( |\tilde{b}_L| = |\tilde{b}_R| = |h_1| \)!
\[ A^2 = 4\lambda m^2 \]
$A^2 > 4\lambda m^2$
\[ A^2 < 4\lambda m^2 \]
Problem solved?

### Problem already known for a while
- problem noticed
- “$A$-parameter bounds”
- classification of all dangerous directions
- including flavor violation

### Stability \( \neq \) no Instability \( \Rightarrow \) Metastability

- Vacuum tunneling

### The tool
- VeVacious
  - finds all (?) tree-level minima
  - minimizes scalar potential in the vicinity at one loop
  - calculates bounce action / tunneling times
What has changed since the mid 90s?

1. We have discovered the Higgs!
2. No sign of SUSY so far...
3. $m_h = 125 \text{ GeV}$  
   (all SUSY literature during LEP era expected it to be $\lesssim 100 \text{ GeV}$)
4. Consequently: large radiative corrections!
5. Large stop mixing needed? Heavy SUSY spectrum?  
   (or hidden in some hardly accessible valley)
6. Approach today:
   - Less focused on unified models
   - Still certain scenarios
   - $\tan \beta$ resummation for bottom quark mass (large $\tan \beta$)
   - Low $\tan \beta$ favored (for $M_A \lesssim 800 \text{ GeV}$, direct search $A \rightarrow \tau \tau$)
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Semi-analytical bounds/exclusions important for fast processing!
The practical (phenomenological) viewpoint

**My “pMSSM”**

- no unification (more than $m_0$, $m_{1/2}$, $A_0$, $\tan \beta$ and sign $\mu$)
- free parameters: (although similar choice as in CMSSM)
  - $\tilde{m}_L^2 = \tilde{m}_t^2 = \tilde{m}_b^2 = M_{\text{SUSY}}^2$
  - $\mu$, $\tan \beta$
  - $A_t$, $A_b$ (not necessarily equal)
- $m_{h_{1,2}}^2$ determined from ew breaking, $B_{\mu}$ related to $M_A$
- no RG running needed = parameters taken at the SUSY scale

**Why no RG-improvement?**

- SUSY scale parameters; limits on this parameters
- destabilization of ew vacuum *around* SUSY scale
- no Planck scale vevs! (maybe there are... in addition)
- in the spirit of the pMSSM as phenomenologically as possible
- only small RG modifications, qualitative features unchanged
More freedom!

Less constraints, more parameters, more fields, more vevs...

Restriction to certain directions too restrictive!

- give up $|\tilde{q}_L| = |\tilde{t}_R| = |h_2|$
- allow for $h_1 \neq 0$ and $\tilde{b} \neq 0$
- “break” $\tilde{q}_L \to \tilde{t}_L + \tilde{b}_L$
- back to full scalar potential!

Simplify your life

- $h_2 = \phi$
- $|\tilde{t}_L| = |\tilde{t}_R| = |\tilde{t}| = \alpha|\phi|$
- $h_1 = \eta\phi$
- $|\tilde{b}_L| = |\tilde{b}_R| = |\tilde{b}| = \beta|\phi|$
- all fields and parameters real, $\alpha, \beta > 0, \eta \in \mathbb{R}$
- $\text{SU}(3)_c$-flatness: $\tilde{t}_L = \tilde{t}_R$ and $\tilde{b}_L = \tilde{b}_R$
A simple view of a complicated object

\[ h_2 = \phi, \quad |\tilde{t}| = \alpha|\phi|, \quad h_1 = \eta \phi, \quad |\tilde{b}| = \beta|\phi| \]

\[ V_\phi = (m_{h_2}^2 + \eta^2 m_{h_1}^2 + (1 + \eta^2)\mu^2 - 2B_\mu \eta 
+ (\alpha^2 + \beta^2)\tilde{m}_L^2 + \alpha^2 \tilde{m}_t^2 + \beta^2 \tilde{m}_b^2) \phi^2 
- 2 (\alpha^2 (\mu y_t \eta - A_t) + \beta^2 (\mu y_t - \eta A_b)) \phi^3 + (\alpha^2 y_t^2 + \beta^4 y_b^2) \phi^4 
+ \left( \frac{g_1^2 + g_2^2}{8} (1 - \eta^2 + \beta^2 - \alpha^2)^2 + 2\alpha^2 y_t^2 + 2\beta^2 y_b^2 \right) \phi^4 \]

\[ \equiv M^2(\eta, \alpha, \beta) \phi^2 - A(\eta, \alpha, \beta) \phi^3 + \lambda(\eta, \alpha, \beta) \phi^4, \]

with

\[ M^2 = m_{h_2}^2 + \eta^2 m_{h_1}^2 + (1 + \eta^2)\mu^2 - 2B_\mu \eta 
+ (\alpha^2 + \beta^2)\tilde{m}_L^2 + \alpha^2 \tilde{m}_t^2 + \beta^2 \tilde{m}_b^2, \]

\[ A = 2\alpha^2 \eta \mu y_t - 2\alpha^2 A_t + 2\beta^2 \mu y_b - 2\eta \beta^2 A_b, \]

\[ \lambda = \frac{g_1^2 + g_2^2}{8} (1 - \eta^2 + \beta^2 - \alpha^2)^2 
+ (2 + \alpha^2)\alpha^2 y_t^2 + (2\eta^2 + \beta^2)\beta^2 y_b^2. \]
The same but different ("A-parameter bounds")

\[ A^2 < 4 \lambda M^2 \]

\[ \downarrow \]

\[ 4 \min_{\{\eta, \alpha, \beta\}} \lambda(\eta, \alpha, \beta) M^2(\eta, \alpha, \beta) > \max_{\{\eta, \alpha, \beta\}} (A(\eta, \alpha, \beta))^2 \]

\[ h_u = \tilde{b}, \ h_d^0 = 0 \quad \text{[WGH'15]} \]

\[ m_{H_u}^2 + \mu^2 + \tilde{m}_Q^2 + \tilde{m}_b^2 > \frac{(\mu y_b)^2}{y_b^2 + (g_1^2 + g_2^2)/2} \]

\[ |h_d|^2 = |h_u|^2 + |\tilde{b}|^2, \ \tilde{b} = \alpha h_u \quad \text{[WGH'15]} \]

\[ m_{11}^2 (1 + \alpha^2) + m_{22}^2 \pm 2 m_{12}^2 \sqrt{1 + \alpha^2} + \alpha^2(\tilde{m}_Q^2 + \tilde{m}_b^2) > \frac{4 \mu^2 \alpha^2}{2 + 3 \alpha^2} \]
Illustrating the exclusion limits

[Bobrowski, Chalons, WGH, Nierste ’14]

\[ M_{\text{SUSY}} = 2\text{TeV} \]
\[ M_{\text{SUSY}} = 1\text{TeV} \]

\[ \mu/\text{TeV} \]
\[ \tan \beta \]

Higgs potential \((h_u \text{ and } h_d)\), new CC conserving min @ 1-loop
Illustrating the exclusion limits $MSUSY = 1 \text{ TeV}$

$V_{\text{eff}} / \text{GeV}^4$

$\nu_u$

$2 \times 10^8$

$4 \times 10^8$

$6 \times 10^8$
Illustrating the exclusion limits $MSUSY = 1 \text{ TeV}$

$V_{\text{eff}} / \text{GeV}^4$

-2 x 10^8
-1 x 10^8
0
2 x 10^8
4 x 10^8
6 x 10^8

$\nu_u$

200
400
600

CCB in MSSM
Illustrating the exclusion limits MSUSY = 1 TeV

CCB sbottom vev, \( h_d = -\sqrt{|h_u|^2 + |	b|^2} \)

[WGH: PLB752 7 (2016)]
Illustrating the exclusion limits $\mu/\text{GeV}$ vs $\tan\beta$

CCB sbottom vev, $h_d = +\sqrt{|h_u|^2 + |\tilde{b}|^2}$

[Reference: WGH: PLB752 7 (2016)]
Include more field freedom, extent exclusion bounds.
More fields, more freedom, stronger (!) bounds \( MSUSY = 1 \text{ TeV} \)

\[ \mu/\text{GeV} \]
\[ \tan \beta \]

Only \( h_u \) and \( \tilde{b} \); \( A_b = 0 \)
More fields, more freedom, stronger (!) bounds $\text{MSUSY} = 1 \text{ TeV}$

W. G. H. CCB in MSSM

only $h_u, h_d, \text{ and } \tilde{b}; A_b = 0$
More fields, more freedom, stronger (!) bounds $\text{MSUSY} = 1 \text{ TeV}$

Only $h_u, h_d, \tilde{t},$ and $\tilde{b}; A_b = 0$
More fields, more freedom, stronger (!) bounds $\text{MSUSY} = 1\,\text{TeV}$

- $\mu/\text{GeV}$
- $\tan\beta$

Only $h_u, h_d, \tilde{t},$ and $\tilde{b}; A_b = A_t = -1500\,\text{GeV}$
Optimization continued

“$A$-parameter” bounds

\[ 4\lambda(\eta, \alpha, \beta) M^2(\eta, \alpha, \beta) > (A(\eta, \alpha, \beta))^2 \]

- no unique solution (many combinations possible)
- no clear analytic bound (no robust exclusion)
- numerical exclusions possible
- may run into deep minimum at $\eta, \alpha, \beta \gg 1$
- find pathologic configurations

Global minimum (true vacuum) vs. local minima and saddle points

How much excluded is an excluded point?

Tunneling $\neq$ Tunneling?
The quantum tunneling effect

\[ V(\phi) \]

\[ \phi_M \]

\[ \phi_+ \]

\[ \phi_- \]

W. G. H. CCB in MSSM
The quantum tunneling effect

Decay probability (per unit volume)

\[ \frac{\Gamma}{V} = A e^{-B/\hbar} \]

[Coleman '77]

Some approximation...

\[ B = \frac{2\pi^2}{3} \left[ \left( \Delta \phi_+ \right)^2 - \left( \Delta \phi_- \right)^2 \right]^2 \frac{\Delta V_+}{\Delta V} \]

[Duncan, Jensen '92]

Metastability bound

\[ B \gtrsim 400 \]

- value of \( B \) depends on path
- different conclusions for different \( \eta, \alpha, \beta \)
Tomography of the scalar potential—sbottom direction

$\tilde{b} = 0 h_u, h_d = \eta h_u$
Tomography of the scalar potential—sbottom direction

\[ \tilde{b} = 0.1 h_u, \ h_d = \eta h_u \]
Tomography of the scalar potential—sbottom direction

\[ \tilde{b} = 0.2h_u, \quad h_d = \eta h_u \]
Tomography of the scalar potential—$\tilde{b}$bottom direction

$\tilde{b} = 0.4h_u, h_d = \eta h_u$
Tomography of the scalar potential—sbottom direction

\[ \tilde{b} = 0.5 h_u, \quad h_d = \eta h_u \]
Tomography of the scalar potential—sbottom direction

\[ \tilde{b} = 0.6h_u, \ h_d = \eta h_u \]
Tomography of the scalar potential—$\tilde{b}$ direction

$\tilde{b} = 0.7 h_u, \ h_d = \eta h_u$
Tomography of the scalar potential—sbottom direction

$\tilde{b} = 0.8h_u$, $h_d = \eta h_u$
Tomography of the scalar potential—sbottom direction

\[ \tilde{b} = 0.9 h_u, \ h_d = \eta h_u \]
Tomography of the scalar potential—$\tilde{b}$bottom direction

$\tilde{b} = 1.0 h_u$, $h_d = \eta h_u$
Tomography of the scalar potential—sbottom direction

\[ \tilde{b} = 1.1h_u, \ h_d = \eta h_u \]
Tomography of the scalar potential—sbottom direction

\[ \tilde{b} = 1.2 h_u, \ h_d = \eta h_u \]
Tomography of the scalar potential—$s$bottom direction

\[ \tilde{b} = 1.3h_u, \quad h_d = \eta h_u \]
Tomography of the scalar potential—sbottom direction

\[ \tilde{b} = 1.4h_u, \ h_d = \eta h_u \]
$\tilde{b} = 1.5 h_u$, $h_d = \eta h_u$
\[ \tilde{b} = 1.6 h_u, \ h_d = \eta h_u \]
Tomography of the scalar potential—sbottom direction

\[ \tilde{b} = 1.7h_u, \ h_d = \eta h_u \]
Tomography of the scalar potential—sbottom direction

\[ \tilde{b} = 1.8h_u, \ h_d = \eta h_u \]
Tomography of the scalar potential—sbottom direction

\[ \tilde{b} = 1.9 h_u, \ h_d = \eta h_u \]
Tomography of the scalar potential—$h_d$ direction

\[ \tilde{h}_d = -1.5h_u, \tilde{b} = \beta h_u \]
Tomography of the scalar potential—$h_d$ direction

$\tilde{h}_d = -1.4 h_u$, $\tilde{b} = \beta h_u$
Tomography of the scalar potential—$h_d$ direction

$\tilde{h}_d = -1.3 h_u$, $\tilde{b} = \beta h_u$
Tomography of the scalar potential—$h_d$ direction

$$\tilde{h}_d = -1.2 h_u, \quad \tilde{b} = \beta h_u$$
Tomography of the scalar potential—$h_d$ direction

$\tilde{h}_d = -1.1 h_u$, $\tilde{b} = \beta h_u$
Tomography of the scalar potential—$h_d$ direction

$\tilde{h}_d = -1.0h_u, \tilde{b} = \beta h_u$
Tomography of the scalar potential—$h_d$ direction

$$\tilde{h}_d = -0.9h_u, \tilde{b} = \beta h_u$$
Tomography of the scalar potential—$h_d$ direction

$$\tilde{h}_d = -0.8 h_u, \quad \tilde{b} = \beta h_u$$
Tomography of the scalar potential—$h_d$ direction

\[ \tilde{h}_d = -0.7 h_u, \tilde{b} = \beta h_u \]
Tomography of the scalar potential—$h_d$ direction

\[ \tilde{h}_d = -0.6h_u, \quad \tilde{b} = \beta h_u \]
Tomography of the scalar potential—$h_d$ direction

\[ \tilde{h}_d = -0.5h_u, \quad \tilde{b} = \beta h_u \]
Tomography of the scalar potential—$h_d$ direction

$\tilde{h}_d = -0.4h_u$, $\tilde{b} = \beta h_u$
Tomography of the scalar potential—$h_d$ direction

$\tilde{h}_d = -0.3 h_u, \tilde{b} = \beta h_u$
Tomography of the scalar potential—$h_d$ direction

\[ \tilde{h}_d = -0.2h_u, \quad \tilde{\beta} = \beta h_u \]
Tomography of the scalar potential—$h_d$ direction

$\tilde{h}_d = -0.1 h_u$, $\tilde{b} = \beta h_u$
Tomography of the scalar potential—$h_d$ direction

$\tilde{h}_d = 0 h_u$, $\tilde{b} = \beta h_u$
Tomography of the scalar potential—$h_d$ direction

$\tilde{h}_d = 0.1h_u$, $\tilde{b} = \beta h_u$
Tomography of the scalar potential—$h_d$ direction

$\tilde{h}_d = 0.2 h_u$, $\tilde{b} = \beta h_u$
\[ \tilde{h}_d = 0.3h_u, \quad \tilde{b} = \beta h_u \]
Tomography of the scalar potential—$h_d$ direction

$\tilde{h}_d = 0.4h_u$, $\tilde{b} = \beta h_u$
Tomography of the scalar potential—$h_d$ direction

$\tilde{h}_d = 0.5h_u$, $\tilde{b} = \beta h_u$
Tomography of the scalar potential—$h_d$ direction

$\tilde{h}_d = 0.6 h_u, \tilde{b} = \beta h_u$
Tomography of the scalar potential—$h_d$ direction

\[ \tilde{h}_d = 0.7h_u, \tilde{b} = \beta h_u \]
Tomography of the scalar potential—$h_d$ direction

$\tilde{h}_d = 0.8 h_u$, $\tilde{b} = \beta h_u$
Tomography of the scalar potential—$h_d$ direction

$\tilde{h}_d = 0.9 h_u$, $\tilde{b} = \beta h_u$
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$\tilde{h}_d = 1.0h_u$, $\tilde{b} = \beta h_u$
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$\tilde{h}_d = 1.1 h_u$, $\tilde{b} = \beta h_u$
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$\tilde{h}_d = 1.2 h_u, \tilde{b} = \beta h_u$
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Tomography of the scalar potential—$h_d$ direction

$\tilde{h}_d = 1.5h_u, \tilde{b} = \beta h_u$
What now?

- no pocket-calculator formula :-(
- numerical/graphical exclusion limits in $\mu$-$\tan\beta$-$A_t$-$A_b$
Numerical constraints

MSUSY = 1 TeV

\[ A_t / \text{GeV} = 0 \text{ GeV} \]

W. G. H. CCB in MSSM
Numerical constraints

$A_t/\text{GeV} = 500 \text{ GeV}$

$A_b = 500 \text{ GeV}$

$\mu/\text{GeV}$

$\text{CCB in MSSM}$
Numerical constraints

$A_t/\text{GeV}$ vs $\mu/\text{GeV}$

$A_b = 1000 \text{ GeV}$

$\text{MSUSY} = 1 \text{ TeV}$
Numerical constraints $\mu/\text{GeV}$ vs. $A_t/\text{GeV}$

$A_b = 1500\,\text{GeV}$

W. G. H. CCB in MSSM

MSUSY = 1 TeV
Wrap-up

- charge and color breaking constraints complementary to direct searches
- check for theoretical consistency
- a closer view reveals very strong constraints

Simple analytic formulae — by far not the strongest bounds

\[ |\tilde{t}_L| = |\tilde{t}_R| = |h_u| \]
\[ |A_t|^2 < 3y_t^2 \left( m_{h_2}^2 + |\mu|^2 + \tilde{m}_L^2 + \tilde{m}_t^2 \right) \]

[Gunion, Haber, Sher '88]

\[ |\tilde{b}_L| = |\tilde{b}_R| = |h_u| \]
\[ m_{H_u}^2 + \mu^2 + \tilde{m}_Q^2 + \tilde{m}_b^2 > \frac{(\mu y_b)^2}{y_b^2 + (g_1^2 + g_2^2)/2} \]

[WGH '15]

Actually more involved!
Wrap-up

- charge and color breaking constraints complementary to direct searches
- check for theoretical consistency
- a closer view reveals very strong constraints

**Simple analytic formulae — by far not the strongest bounds**

- $|\tilde{t}_L| = |\tilde{t}_R| = |h_u|$
  \[ |A_t|^2 < 3 y_t^2 (m_{h_2}^2 + |\mu|^2 + \tilde{m}_L^2 + \tilde{m}_t^2) \]
  [Gunion, Haber, Sher '88]

- $|\tilde{b}_L| = |\tilde{b}_R| = |h_u|$
  \[ m_{H_u}^2 + \mu^2 + \tilde{m}_Q^2 + \tilde{m}_b^2 > \frac{(\mu y_b)^2}{y_b^2 + (g_1^2 + g_2^2)/2} \]
  [WGH '15]

- $4\lambda(\eta, \alpha, \beta) M^2(\eta, \alpha, \beta) > (A(\eta, \alpha, \beta))^2$
All the details: arXiv:1606.0xxxx
Backup Slides
Yukawa coupling not directly proportional to mass (same for $y_t$)

\[ y_b = \frac{m_b}{v_d(1 + \Delta_b)} \]

[Hall, Rattazzi, Sarid '94; Carena, Garcia, Nierste, Wagner '99]

\[
\Delta_{b, \text{gluino}} = \frac{2\alpha_s}{3\pi} \mu M_{\tilde{G}} \tan \beta C_0(\tilde{m}_{\tilde{b}_1}, \tilde{m}_{\tilde{b}_2}, M_{\tilde{G}}),
\]

\[
\Delta_{b, \text{higgsino}} = \frac{Y_t^2}{16\pi^2} \mu A_t \tan \beta C_0(\tilde{m}_{\tilde{t}_1}, \tilde{m}_{\tilde{t}_2}, \mu).
\]