Principal properties of the velocity distribution of dark matter particles on the outskirts of the Solar System

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ABSTRACT
The velocity distribution of the dark matter particles on the outskirts of the Solar System remains unclear. We suggest to determine it using experimentally found properties of the oldest halo objects. Indeed, the oldest halo stars and globular clusters form a collisionless system, as well as dark matter particles do, and they evolved in the same gravitational field. If we accept this analogy, we can show that the velocity distribution of the dark matter particles should be highly anisotropic and have a sharp maximum near \( \nu \sim 500 \, \text{km/s} \). The distribution is totally different from the Maxwell one.

We analyze the influence of the distribution function on the results of dark matter detection experiments. It is found that the direct detection signal should differ noticeably from the one calculated from the Maxwell distribution with \( \langle \nu \rangle \approx 220 \, \text{km/s} \), which is conventional for direct detection experiments (the ratio depends on the detector properties and typically falls within the range between 6 and 0.2). Moreover, the sharp distinction from the Maxwell distribution can be very essential to the observations of dark matter annihilation.

Key words: cosmology: dark matter, elementary particles, methods: analytical.

1 INTRODUCTION
One of the most evident manifestations of the dark matter existence is the detection of huge invisible halos (with density profiles \( \rho \sim r^{-2} \)) surrounding galaxies (Marochnik & Suchkov 1996). Our Galaxy also has such a halo. We symbolize the orbital radius of the Solar System, \( \nu_\odot \), and the escape velocity at this radius by \( \nu_{\text{esc}} \), respectively. We also denote the radial and tangential components of a dark matter particle velocity by \( \nu_r \) and \( \nu_\theta \), respectively. Velocity distribution of the dark matter particles (hereafter DMPs) inside the halo is poorly known; it is usually supposed to be Maxwell with a cut-off when \( \nu > \nu_{\text{esc}} \) (Cerdeño & Green 2010).

\[
\begin{equation}
 f(\nu) = \frac{N}{(\sqrt{\pi} \nu_\odot)^3} \exp\left(-\frac{\nu^2}{\nu_\odot^2}\right), \quad \nu < \nu_{\text{esc}}
\end{equation}
\]

We accept \( \nu_\odot = 8 \, \text{kpc} \), \( \nu_\odot = 220 \, \text{km/s} \), \( \nu_{\text{esc}} = 643 \, \text{km/s} \). \( N \) is a normalizing constant, and for the chosen parameters \( N \approx 1.001 \). It is worthy of noting that \( \nu_{\text{esc}} \) remains almost constant throughout the halo.

Distribution (1) faces with difficulties. In fact, in the framework of collisionless dynamics it can be naturally obtained from profile \( \rho \sim r^{-2} \), subject to the condition, however, that function \( f \) is isotropic, i.e. \( f \) depends only on \( |\nu| \) (so called isothermal model). This assumption seems highly improbable. Indeed, if (1) is true, the majority of the dark matter particles has large specific angular momentum \( \mu \equiv \Omega \chi = [\nu \times r] \). The average angular momentum of the particles (and, consequently, of the halo) is zero. However, the root-mean-square momentum is \( \sqrt{\langle \mu^2 \rangle} \approx 1800 \, \text{kpc} \cdot \text{km/s} \) at \( r_\odot \). Moreover, since in this model \( \mu \sim r \nu_{\text{orb}} \) and \( \nu_{\text{orb}} \) is constant in the halo if \( \rho \sim r^{-2} \), the root-mean-square angular momentum reaches an incredibly huge value \( \sqrt{\langle \mu^2 \rangle} \approx 4 \cdot 10^4 \, \text{kpc} \cdot \text{km/s} \) at the edge of the halo (\( r \sim 200 \, \text{kpc} \)). Meanwhile, according to modern cosmological conceptions, not only the total angular momentum of the halo but also the momentum of each particle should have been negligibly small on the linear stage of the structure formation (Gorbunov & Rubakov 2011). The halo could gain some angular momentum later, as a result of tidal perturbations or merging of smaller halos; numerical simulations show, however, that it cannot be large (Macciò et al. 2007).

Some results of stellar dynamics are frequently used in order to show that the dark matter particles could gain a large angular momentum during the Galaxy evolution. The parallels between DMP and stellar dynamics, however, are not universally true. The point is that stars are compact objects, and their gravitational field can be strong, at least,
locally. On the contrary, the small-scale gravitational field of the dark matter is always small (Baushev, 2003). Therefore important relaxation mechanisms of stellar systems like close pair approaches or an interaction with the interstellar medium are completely ineffective for DMPs. So called violent relaxation (Lynden-Bell, 1967) is perhaps the only way to impart significant angular momentum to the dark matter particles. However, it acts also on the halo stars; moreover, its efficiency decreases with radius, and it should stronger affect the star distribution since the stellar halo is more compact.

To summarize: all halo objects initially had \( v_\rho \approx 0 \) and later gained some angular momentum because of various processes like relaxation or tidal effects. All the mechanisms increased \( \langle v_\rho^2 \rangle \) of the halo stars, at least, as much as of the DMPs, while some mechanisms affected only the stars, and not the DMPs. Consequently, velocity distribution of the oldest stellar halo population should be closer to stars, and not the DMPs. Consequently, velocity distribution of the oldest stellar halo population should be closer to stars, and not the DMPs. Consequently, velocity distribution of the oldest stellar halo population should be closer to stars, and not the DMPs. Consequently, velocity distribution of the oldest stellar halo population should be closer to stars, and not the DMPs. Consequently, velocity distribution of the oldest stellar halo population should be closer to stars, and not the DMPs. Consequently, velocity distribution of the oldest stellar halo population should be closer to stars, and not the DMPs.

Modern observations of the oldest halo stars - subdwarfs - confirm the above reasoning (Smith et al., 2003). Their tangential dispersion \( \sigma(v_\rho) \approx \sigma_0 \approx 80 \text{ km/s}, \) which corresponds to \( \sqrt{\langle r^2 \rangle} \approx 900 \text{ kpc} \cdot \text{km/s}, \) is two times lower than in (1). Moreover, the distribution widely differs only from the Maxwellian: \( \sigma(v_\rho) \) is much larger than \( \sigma(v_\rho). \) Consequently, \( \sigma(v_\rho) \) of the dark matter particles on the outskirts of the Solar System does not exceed \( \sigma_0 \approx 80 \text{ km/s} \) and can be even smaller. Second, the observations show that the halo stars have not yet relaxed and their orbits are rather prolate. On the other hand, if distribution (1) is correct, the ellipticity of the majority of DMP orbits is small, and then the dark matter is the only class of halo objects that move almost circularly. It seems much more natural to assume the opposite, and we come to the premises we use throughout this article:

1) The specific angular momentum \( \mu \) of, at least, the main part of the particles is fairly small, and their orbits are rather prolate (below we will express this supposition quantitatively).

2) The Galactic halo is stationary and spherically symmetrical. The latter supposition is not quite accurate; a part of dark matter can form a so-called thick disk (Read et al., 2008), moreover, the influence of the star disk also takes place. However, our assumption is quite acceptable for an estimative consideration.

3) In accordance with observations (Dutton et al., 2011), we assume that the density profile of the Galaxy is \( \rho \propto r^{-2} \) up to some large enough radius \( R. \) The profile cannot be valid for an arbitrary large \( r, \) since in the opposite case the halo mass would be infinite. Starting from some radius (we denote it by \( R), \) the halo density drops much faster. As we will see, this is the main parameter defining the DMP velocity distribution. The profile \( r^2 \) has an another demerit: the predicted annihilation signal (which is proportional to \( \rho^2 \)) diverges when \( r \to 0. \) However, as it will be shown below, the velocity distribution of DMPs on the outskirts of the Solar System weakly depends on the density profile inside the solar orbit. Therefore we will not discuss a very complex question of the dark matter profile near the galactic centre.

4) We assume that the dark matter mass outside of the radius \( R \) is negligibly small as compared with the total halo mass. This supposition is the most discussable; however, there are strong arguments in favour of it. First, experimental data show (Binney & Tremaine, 2008) that \( \rho \approx r^{-2} \) up to very large distances. Second, in order that the total halo mass is finite, the density must fall much faster than \( r^{-2} \) (at least, faster than \( r^{-3} \)) at large distances. For instance, one of the most popular Einasto profile predicts an exponential density decreasing (Einasto, 1965). Third, the outer regions of the halo are respectively weakly bound in the gravitational field and can easily be torn away by tidal effects.

These four suppositions turn out to be sufficient to derive the velocity distribution more or less unambiguously. All our calculations are, of course, focused on determination of the DMP velocity distribution on the outskirts of the Solar System.

2 CALCULATIONS

For our Galaxy we accept \( R = 210 \text{ kpc}, \) which corresponds to the total mass of the Galaxy \( M = 2.3 \cdot 10^{12} M_\odot. \) The module of gravitational potential on the edge of the halo is equal \( \Phi = GM/R. \) It is easy to see that \( \Phi = v^2_\rho/2R. \) Since \( \rho \approx r^{-2}, \) the mass inside some radius can be found as \( \frac{\pi}{2} M, \) gravitational field is equal \( \ddot{g} = G M/R^2, \) and we obtain the gravitational potential inside the halo:

\[
\phi = -\Phi \left( 1 + \ln \frac{R}{r} \right)
\]  \hspace{1cm} (2)

Let us start our consideration from the case when the DMPs have no angular momentum at all. Then their trajectories are radial \( (v = |v_\rho|), \) and the task becomes one-dimensional. Therefore the particle distribution in the halo can be entirely described by a single function \( \psi(r, v), \) so that \( \psi(r, v)dr dv \)
gives the total mass of dark matter in the element of phase space $d\rho dv$. $\psi$ differs from the standard distribution function only by a insignificant multiplier — the DMP mass $m_\chi$. Since we suppose that the dark matter particles are propelled only by the gravitational force, $m_\chi$ is not important for us, the calculations are formally valid even for the dark matter consisting of primordial black holes, and such a definition of $\psi$ allows us to avoid of the undesirable dependence on the DMP mass. The halo density is bound with the function $\psi$ by a trivial relation

$$\eta \equiv 4\pi r^2 \rho(r) = \int_0^\infty \psi(r,v)dv$$ \hspace{1cm} (3)

Here we introduced a more convenient variable $\eta$ instead of $\rho$, $\eta = \text{const}$ if $\rho \sim r^{-2}$. In our case $\eta = M/R$, and we obtain a determining condition on $\psi$:

$$\int_0^\infty \psi(r,v)dv = \frac{M}{R}$$ \hspace{1cm} (4)

Each dark matter particle executes a radial oscillation around the galactic centre. We denote by $r_0$ the maximum distance it moves off the centre. Its velocity in potential $\Phi$ is equal to

$$|v_r| = v = \sqrt{2\Phi \ln \frac{r_0}{r}}$$ \hspace{1cm} (5)

and we obtain useful equations:

$$r_0 = r \exp\left(\frac{v^2}{2\Phi}\right), \quad \frac{d\rho}{dv} = \frac{v\rho}{\Phi}, \quad T = r_0 \sqrt{\frac{\pi}{2\Phi}}$$ \hspace{1cm} (6)

Here $T$ is the time required for the particle to fall from $r_0$ to the centre. We introduce a distribution function $\xi$ of the particles throughout parameter $r_0$, so that $\xi(r_0)dr_0$ is the total mass of DMPs which apoapsis lies in the interval $[r_0;r_0 + dr_0]$. The $r$-coordinate of these particles varies between 0 and $r_0$, and they give a yield into the halo density over all this interval. Indeed, the fraction of time the DMPs from the subsystem under consideration pass in an interval $[r;r + dr]$ is equal to $dt/T = dr/(vT)$. Since the total particle mass of the subsystem is $\xi(r_0)dr_0$, the contribution to the halo mass in interval $dr$ is equal to

$$dM = \frac{\xi(r_0)}{vT} dv dr_0$$ \hspace{1cm} (7)

Velocity interval $dv$ that is covered by the particles of the subsystem at radius $r$ is $\frac{d\rho}{dv} dv = dr_0$. Substituting this to (7) and taking into account that $dM = \psi(r,v) dv dv$, we obtain the general equation for $\psi(r,v)$:

$$\psi(r,v) = \frac{\xi(r_0)}{vT} \frac{d\rho}{dv}$$ \hspace{1cm} (8)

Now we substitute here equations (5) and (6):

$$\psi(r,v) = \sqrt{\frac{2}{\pi \Phi}} \left[r \exp\left(\frac{v^2}{2\Phi}\right)\right]$$ \hspace{1cm} (9)

We can easily find function $\xi$ if we suppose that the halo boundary is sharp, i.e. the density obeys the law $\rho \sim r^{-2}$ up to radius $R$ and is equal to zero just after it. Then the maximum velocity the dark matter particles may have at radius $r$ is

$$v_{\text{max}} = \sqrt{2\Phi \ln \frac{R}{r}} = v_\odot \sqrt{\frac{2\ln \frac{R}{r}}{r}}$$ \hspace{1cm} (10)

It is a matter of direct verification to prove that function

$$\xi(r_0) = \frac{M}{\sqrt{2\pi R \ln \frac{r}{r_0}}}$$ \hspace{1cm} (11)

satisfies condition (3). This function has a peculiarity at $r_0 = R$. So the main part of DMPs comes to us from the very edge of the halo. The distribution function is

$$\psi(r,v) = \frac{2M}{\pi R \sqrt{v_{\text{max}}^2 - v^2}}$$ \hspace{1cm} (12)

where $v_{\text{max}}$ is defined by (10). $v \in [0;v_{\text{max}}]$. The distribution through the radial velocity $v_r$ can be easily obtained from (12): $\psi(r,v_r) = \psi(r,-v_r) = \psi(r,v)/2$, where $v_r \in [-v_{\text{max}};v_{\text{max}}]$. Normalized velocity distribution on the outskirts of the Solar System is equal to

$$f(v) = \frac{2}{\pi \sqrt{v_{\text{max}}^2 - v^2}} \approx \frac{2}{\pi \sqrt{(2.2v_\odot)^2 - v^2}}$$ \hspace{1cm} (13)

The distribution has a peculiarity at $v_{\text{max}} \approx 2.55v_\odot \approx 562$ km/s, which is a result of the supposition that the density after $r = R$ immediately drops to zero. In actuality there is a characteristic length $l$ of density decreasing out of $R$, for instance, if the decreasing is exponential $\rho \propto \exp(-r/r_d)$ we can use $r_d$ as $l$. Then the cusp at $v_{\text{max}}$ transforms into a smooth peak. Since the free fall acceleration at $r = R$ is $g = GM/R^2$, we can easily estimate the width of the peak:

$$\delta v \approx 0.5 \sqrt{\frac{gl}{v_{\text{max}}}} = \frac{l}{4R \ln \frac{R}{r_0}}$$ \hspace{1cm} (14)

Near the Solar System $\delta v \approx (l/R) \cdot 50$ km/s. It is natural to assume that $l \ll R$ (actually, it follows from supposition 4 in the Introduction), so the distribution is still very narrow.

To illustrate the above reasoning, let us consider the following density profile:

$$dM/dr = \begin{cases} \text{const}, & \text{if } r \leq R; \\ \text{const} \cdot \exp\left(-\frac{r - R}{r_d}\right), & \text{if } r > R \end{cases}$$ \hspace{1cm} (15)

In this case, we cannot determine the distribution $\xi(r)$ analytically. However, if $r_d \ll R$, we can neglect the change in the gravitational field on the scale of $r_d$ and then find an approximate solution. Fig. 11 represents distributions (18) (for the sharp halo edge, solid line) and for density profile (16) (dotted line, we accepted $r_d = 0.1R = 21$ kpc). One can see that in the case of smooth halo edge the peculiarity at $v_{\text{max}} \approx 562$ km/s is transformed into a smooth peak, which is, however, quite narrow, and the general shape of the distribution remains the same. We will discuss the influence of the halo density profile on the velocity distribution in the beginning of the Discussion section.

The angular momentum of the dark matter particles hardly can be exactly equal to zero. If a particle possesses some specific momentum $\mu$, its velocity in gravitational field is equal to:

$$v_\rho = \frac{\mu}{r}; \quad |v_r| = \sqrt{2\Phi \ln \frac{r_0}{r} - \mu^2 \left(\frac{1}{r^2} - \frac{1}{r_0^2}\right)}$$ \hspace{1cm} (16)

Since (18) is distinct from (3), distribution (11) strictly speaking, is not valid anymore. However, we may use it if the difference between (16) and (3) is small. Mathematically
it can be written as:

\[ 2\Phi \ln \frac{r_0}{r} \gg \mu^2 \left( \frac{1}{r^2} - \frac{1}{r_0^2} \right) \] (17)

As we have already discussed in the Introduction, specific angular momentum of the majority of the dark matter particles hardly can be larger than 900 kpc · km/s. Substituting \( r_0 = R \), \( r = r_0 \) to the inequality, we can see that its right part is equal to 110^2 km^2/s^2, while its left part is \( \sim 550^2 \) km^2/s^2. Hence the inequality asserts near the Solar System, the influence of the angular momentum on the radial dynamics is still negligible for the majority of DMPs, and we can use (11) up to \( r_0 \) as before. Thus the DMP distribution throughout \( v_r \) in this approximation coincides with (13). However, the particles have also some distribution throughout \( v_\phi \). For simplicity we will suppose that \( f \propto \exp(-v_\phi^2/2\sigma_0^2) \) where \( \sigma_0 = 80 \) km/s, though the distribution can be much narrower. Then the normalized DMP distribution near the Solar System can be closely approximated by

\[ f(v) = \frac{\exp \left( \frac{-v_r^2}{2\sigma_0^2} \right)}{2\pi^2\sigma_0^2 \sqrt{v_{\text{max}}^2 - v_r^2}} \] (18)

where \( v_r \in [-v_{\text{max}}; v_{\text{max}}] \), \( v_{\text{max}} = 562 \) km/s. Distribution (18) is strongly anisotropic and actually describes two colliding beams of particles.

### 3 DISCUSSION

Fig. 1 represents distributions (13) and (1) (solid and dashed lines, respectively). One can see that (13) is much narrower and has much higher average velocity. The physical reason of it is obvious: in the case of Maxwell distribution (1) the particles move almost circularly, which is why only a few of DMPs from the edge of the halo reach the Solar orbit. On the contrary, in the case considered in this article the majority of DMPs comes from the halo edge and thus are much more accelerated by the gravitational field. Consequently, the question of what of the distributions, (1) or (13), is correct, can be reduced to whether the particles from the halo edge can reach the Solar orbit or not. In addition to the arguments presented in the Introduction we note that, according to (16), a particle falling from \( r = R \) should have a specific angular momentum \( \mu \sim 4000 \) kpc · km/s, lest the particle can reach \( r = 8 \) kpc. This value is huge, it far exceed not only the characteristic momentum of halo objects, but even the momentum of the disk, and thus looks very unlikely. So particles from the edge of the halo freely reach the Earth, and their spectrum should be closer to (13).

A similar consideration allows us to examine the dependence of velocity distribution (13) on the density profile. Our assumption of the existence of a large region with \( \rho \propto r^{-2} \) is approximately correct for massive spiral galaxies, such as the Milky-Way (Dutton et al. 2010). However, we obtained (13) on the additional assumption that the edge of the halo is more or less sharp. Meanwhile, the outer region of the halo can have a density profile steeper than \( r^{-2} \), but not steep enough to be considered as a cutoff. As an instance, one can consider a double power-law halo (Lisanti et al. 2011). How can it influence on the velocity profile? The answer depends on the mass fraction of this steeper region with respect to the total mass of the halo. If the fraction is small, the distribution differs little from (13), as we demonstrated with distribution (13). Let us consider the case when the fraction is significant. We indicate the radius where the profile gets steeper than \( r^{-2} \) by \( R \); hereafter we will name 'outer halo' the region out of \( R \). As we could see, in the model with sharp halo edge the majority of the particles comes from the edge of the halo. Expressing this fact mathematically, distribution \( \xi(r_0) \) is small for \( r_0 < R \) and goes to infinity at the edge of the halo (11). It is easy to show that in the case of the presence of a massive outer halo dark matter particles mainly come from it, and the fraction of the particles with \( r_0 < R \) is relatively small. Let us consider a system of particles with \( r_0 > R \). According to (4), their contribution to the halo mass in interval \( dr \) depends on \( r \) only as \( v_{\text{out}}^{-1}(r) \), \( v_{\text{out}}^{-1}(r) \), however, changes rather slowly inside the region where \( \rho \propto r^{-2} \), since the potential there (see (2)) depends on \( r \) only logarithmically. Therefore, the particles falling from the edge of halo provide almost the same contribution to the halo mass on each radius inside the region \( dM \approx \text{const} \), which corresponds to \( \rho \approx r^{-2} \). Function \( \xi(r_0) \) should be chosen so that it reproduces the density profile, in particular, it should provide \( \rho \propto r^{-2} \). However, as we could see, the particles from the outer halo by themselves give a very similar profile, and we need relatively few of particles with \( r_0 < R \) in order to make it exactly \( r^{-2} \). Consequently, \( \xi(r_0) \) is small for \( r_0 < R \) and a significant fraction still comes from the halo edge. Thus, this property of the distribution does not depend on the exact density profile, being only a result of the assumption of strong anisotropy of the velocity distribution and of the flatness of potential (2).

Function \( \xi(r_0) \) unambiguously determines the velocity distribution of the dark matter particles, and the above-mentioned common properties of \( \xi(r_0) \) directly correspond to characteristics of \( f(v) \) For the Milky Way galaxy \( R \) is large (\( R > 100 \) kpc, Dutton et al. 2010), which corresponds to \( v \approx 500 \) km/s for a terrestrial observer. Below this speed the distribution as a whole is similar to (13), and...
it does not depend much on the density profile of the outer halo, since it is created by the particles with \( r_0 < R \). On the contrary, the distribution in region \( v_{\text{esc}} > v > 500 \text{ km/s} \) strongly depends on the density distribution on the edge of the halo and can differ drastically from \( 13 \). However, now we can estimate what distribution \( f(v) \) looks like if profile \( \rho \propto r^{-2} \) is valid only up to \( R \approx 100 \text{ kpc} \), and then the halo has a massive outer region, for instance, a Navarro-Frank-White tail. We can expect that \( f(v) \) is similar to \( 13 \) below \( v = 500 \text{ km/s} \) and is totally defined by the density profile of the outer halo for \( v_{\text{esc}} > v > 500 \text{ km/s} \). Since the outer halo is rather extensive, we can expect that a cusp in \( 13 \) is strongly smoothed, Fig. 1 illustrates all these properties, though we made rather a small modification of the density profile \( 15 \). So the main characteristic features of distribution \( 13 \) are not sensitive to the density profile: the distribution is completely not Maxwell, a significant fraction of particles comes from the edge of the halo forming a high-velocity bump in \( f(v) \).

The difference between \( 1 \) and \( 13 \) is important for various aspects of the dark matter physics. In the case of direct dark matter search the signal, roughly speaking, can be represented as a product of a part depending almost not at all on the DMP distribution and an integral (Belanger, Nezri, & Pukhov 2009)

\[
I(v) = \int_{v_{\min}}^{\infty} \frac{\tilde{f}(v)}{v} dv
\]

Here \( v_{\min} \) is the minimal DMP speed, to which the detector is sensitive, \( \tilde{f}(v) \) is the distribution in the Earth's frame of reference, obtained from \( 1 \) or \( 10 \) by a Galilean transformation (see details in Cerdeño & Green 2011, section 3.3). Because of the Earth's orbital motion \( I \) varies with a year period, and it is this variation that is observed in the direct detection experiments. Fig. 2 shows the ratio between the double amplitudes \( 2A = I_{\text{max}} - I_{\text{min}} \) of direct detection signals calculated for anisotropic distribution \( 13 \) and Maxwell distribution \( 1 \), as a function of \( v_{\min} \). One can see a very significant difference.

The difference between distributions \( 1 \) and \( 13 \) can also be important for the indirect dark matter search. Neutrino observations, for instance, are trying to detect the dark matter annihilation going on in the centre of the Sun. The signal depends on the number of the DMPs captured by the Sun. Very roughly speaking, it is proportional to \( \tilde{f}(0) \). One can see that for distribution \( 13 \) \( \tilde{f}(0) \) is approximately 20 times smaller than for the conventional Maxwell distribution \( 1 \).

If the s-channel of the dark matter annihilation dominates (which is typical), \( \langle \sigma v \rangle \approx \text{const} \), and the signal is not sensitive to DMP distribution. However, if the p-channel prevails, \( \sigma \approx \text{const} \), and the signal is proportional to the averaged velocity of the particle collision, \( \langle v_c \rangle \approx 0.8 v_{\text{esc}} \) in the case of distribution \( 13 \) and \( \langle v_c \rangle = \sqrt{8/\pi} v_{\text{orb}} \) in the case of Maxwell distribution \( 1 \). \( v_{\text{max}} \) relates to \( v_{\text{orb}} \) by \( 10 \), and one can see that \( \langle v_c \rangle \) predicted by anisotropic distribution \( 13 \) is much lower at the halo edge and much higher in the central region than the Maxwell one. Near the Solar System, however, they are nearly equal. Finally, Sommerfeld effect is inversely proportional to \( \langle v_c \rangle \), and if it plays any role in the dark matter annihilation, it is also sensitive to the particle distribution.

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