$k_T$, anti-$k_T$ & SIScone jets and $\alpha_S @$ HERA

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on behalf of the H1 and ZEUS collaborations

- introduction
- measurements of $k_T$ multijets at low $Q^2$
- measurement of inclusive $k_T$, anti-$k_T$ and SIScone jets at high $Q^2$
- comparison of data to NLO
- running $\alpha_s$ and $\alpha_s(M_Z)$ from jets
HERA: ep collider, basic kinematics

- \( \sqrt{s} = 318 \text{ GeV} \)
- 2001/2002 luminosity upgrade \( \rightarrow \) HERA-2
- \( \sim 0.5 \text{ fb}^{-1} \) of data collected per experiment

- virtuality of the exchanged boson:
  \( Q^2 = -q^2 = -(k-k')^2 = s_{xy} \)
- Bjorken scaling variable:
  \( x = Q^2/2p \cdot q \)
- inelasticity:
  \( y = p \cdot q/p \cdot k \)
Jet production in DIS @ HERA

2 large scales in DIS: $Q$ (2-125 GeV) & $P_{T\text{jet}}$ (5-80 GeV)

Typical choices for pQCD calculations are:

- $\mu_f = Q$
- $\mu_r = Q$ or $P_{T\text{jet}}$ (ZEUS)
- $\mu_r = \sqrt{[(Q^2 + (P_{T\text{jet}})^2)/2]}$ (H1)

$$d\sigma_{n\text{jet}} = \sum_{i=q,\bar{q},g} dx f_i(x, \mu_f) d\hat{\sigma}_i(x, \alpha_s^{n-1}(\mu_r), \mu_r, \mu_f) (1 + \delta_{\text{had}})$$

- $f_i$: pdf of parton $i$ in proton
- $\hat{\sigma}_i$: matrix element $i$, calculable in pQCD
Jet production in DIS @ HERA

*tracks and calorimetric energy deposits are measured in the laboratory*

Jet finding is usually performed in the Breit frame (in analogy to $e^+e^-$)

- QPM process generates no $p_T$

- only QCD processes generate $p_T$
Jet finding: $k_T$, anti-$k_T$ & SIScone

Requirements for comparing jet cross sections with pQCD:
- factorization $\Rightarrow$ in DIS perform measurement in Breit frame
- collinear & infrared safe jet algorithm $\Rightarrow k_T$, anti-$k_T$ & SIScone

Sequential recombination algorithms:

$$d_{ij} = \min(k_{T_i}, k_{T_j})^{2p} \frac{\Delta R^2}{R^2} \text{ and } d_{iB} = k_{T_i}^{2p}$$

with $\Delta R^2 = (\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2$

- $p = 1 \Rightarrow k_T$ at HERA typically $R=1.0$
- $p = -1 \Rightarrow$ anti-$k_T$

SIScone:
seedless iterative cone with split merge (0.75)
finds stable cones, i.e. cone axis = momentum sum of particles
H1: Multijet cross sections at low $Q^2$

- arXiv:0911.5678, HERA-1 data, 44 pb$^{-1}$
- DIS phase space: $5 < Q^2 < 100 \text{ GeV}^2$, $0.2 < y < 0.7$
- jet phase space: $-1.0 < \eta_{\text{jet,lab}} < 2.5$
  - incl. jets, 2-jet, 3-jet: $p_T > 5 \text{ GeV}$ (Breit)
  - 2-jet & 3-jet: $M_{1,2} > 18 \text{ GeV}$
- cross sections are measured as function of $Q^2$, $p_T$ ($<p_T>$) and $\xi$
- main experimental uncertainties:
  - jet energy scale 2% $\Rightarrow \Delta \sigma / \sigma = 4-10%$
  - uncertainty in acceptance $\Rightarrow \Delta \sigma / \sigma = 2-15%$
- NLO calculation: NLOJET++
  - MSbar scheme for 5 massless quark flavors,
  - $\mu_f = \mu_r = \sqrt{(Q^2 + p_{T,jet}^2)/2}$
  - PDFs: CTEQ6.5M
H1: Multijet cross sections at low $Q^2$

- measurements are well described by NLO
- exp. uncertainty 6-11%
- theo. uncert., dominated by renorm. scale uncertainty: 30% (lowest $Q^2$ and $p_T$) to 10% (highest $Q^2$ and $p_T$)
- pdf uncertainty: 6 to 2%
- low predictive power of NLO at low $Q^2$ and/or low $p_T$ ⇒ orders beyond NLO are needed to match the precision of the data
H1: 3-jet/2-jet ratio in $Q^2$, $<p_T>$

- in ratio norm. errors cancel & other syst. uncertainties reduced by 50%
- reduced sensitivity to renorm. scale variation in theory
- good description of ratio by NLOjet++

analysis on 9 x stats of HERA-2 in progress
H1: $\alpha_s$ from low & high $Q^2$ jets

- at low $Q^2$, extraction of $\alpha_s(M_Z)$ from double diff. incl. jet, 2 and 3-jet cross sections using the $k_T$ jet finder:
  $$\alpha_s(M_Z) = 0.1160 \pm 0.0014 \text{ (exp.)} \pm 0.0016 \text{ (pdfs)}$$

- at high $Q^2$, extraction of $\alpha_s(M_Z)$ from double diff. normalized incl. jet, 2 and 3-jet cross sections using the $k_T$ jet finder:
  $$\alpha_s(M_Z) = 0.1168 \pm 0.0007 \text{ (exp.)} \pm 0.0016 \text{ (pdfs)}$$

central value of $\alpha_s(M_Z)$ using anti-$k_T$ is within 0.6%

remarkable agreement between low and high $Q^2$ extraction & with QCD expectations

ZEUS: Incl. $k_T$, anti-$k_T$, SIScone jets

- HERA-1 data, 82 pb$^{-1}$, DESY-10-034, arXiv:1003.2923
- DIS phase space: $Q^2 > 125$ GeV$^2$, $|\cos \gamma_h| < 0.65$
- jets are found in the Breit frame using anti-$k_T$ and SIScone
- jet phase space: at least one jet with $-2 < \eta < 1.5$, $E_T > 8$ GeV in Breit frame
- incl. jet cross sections are measured as a function of $Q^2$ and $E_T$
- main experimental uncertainties:
  - jet energy scale 1% ($E_{T,\text{lab}} > 10$ GeV) to 3% for lower $E_{T,\text{lab}}$ $\Rightarrow \Delta \sigma / \sigma \approx 5$
  - uncertainty in acceptance $\Rightarrow \Delta \sigma / \sigma \approx 4$
- NLO calculations: DISENT & NLOjet++
  - MSbar scheme for 5 massless quark flavors
  - $\mu_f = Q$ and $\mu_r = E_T$
- PDFs: ZEUS-S parametrization
ZEUS: Incl. $k_T$, anti-$k_T$, SIScone jets

- Data and NLO are in good agreement
- Hadronization corrections are smallest for $k_T$ and largest for SIScone
ZEUSS: Incl. k\(_T\), anti-k\(_T\), SIScone jets

- theoretical relative uncertainties

- theory uncert. as a funct. of Q\(^2\) varies from 3-7% (3-10%) for the anti-k\(_T\) (SIScone)

- NLO using k\(_T\) and anti-k\(_T\) have similar precision, with SIScone slightly less precise
Ratio of incl. jet cross sections based on different jet algorithms:

- incl. jet cross sections currently calculated up to $O(\alpha_s^2)$
- differences of incl. jet cross sections using different jet algorithms can however be predicted up to $O(\alpha_s^3)$ using NLOjet++

\[
\frac{d\sigma_{\text{anti-}k_T}}{d\sigma_{k_T}} = 1 + \frac{d\sigma_{\text{anti-}k_T}}{d\sigma_{k_T}} = 1 + \frac{C\alpha_s^3}{A\alpha_s + B\alpha_s^2}
\]

\[
\frac{d\sigma_{\text{SIScone}}}{d\sigma_{k_T}} = 1 + \frac{d\sigma_{\text{SIScone}}}{d\sigma_{k_T}} = 1 + \frac{D\alpha_s^2 + E\alpha_s^3}{A\alpha_s + B\alpha_s^2}
\]

note: for the cancellations to work, the differences are calculated on an event by event basis
ZEUS: xsect ratios of data & of NLO

- in $E_T$ the ratios differ from unity by $< 3.6\%$, except at highest $E_T$ (10%) 
- in $Q^2$ they differ by $< 3.2 \%$
- data ratios are well described by predictions up to $O(\alpha_s^3)$
- in ratio, theoretical uncertainty mainly due to hadronization uncertainty
ZEUS: determination of $\alpha_s(M_Z)$

- Use data on $d\sigma/dQ^2$ for $Q^2 > 500$ GeV$^2$ (to minimize error on $\alpha_s(M_Z)$)
- NLO calculation using DISENT
- PDFs: ZEUS-S parametrizations for five different values of $\alpha_s(M_Z)$
- Main uncertainties on $\alpha_s(M_Z)$
  - Jet energy scale $\rightarrow$ 1.9 to 2%
  - Terms beyond NLO $\rightarrow$ 1.5% (method by Jones et al.)
  - PDFs $\rightarrow$ 0.7 to 0.8%
  - Hadronization $\rightarrow$ 0.8% ($k_T$), 0.9% (anti-$k_T$), 1.2% (SIScone)

$$k_T: \quad \alpha_s(M_Z) = 0.1207 \pm 0.0014 \text{ (stat.)}^{+0.0035}_{-0.0033} \text{ (exp.)}^{+0.0022}_{-0.0023} \text{ (th.)}$$
$$\text{anti}-k_T: \quad \alpha_s(M_Z) = 0.1188 \pm 0.0014 \text{ (stat.)}^{+0.0033}_{-0.0032} \text{ (exp.)}^{+0.0022}_{-0.0022} \text{ (th.)}$$
$$\text{SIScone:} \quad \alpha_s(M_Z) = 0.1186 \pm 0.0013 \text{ (stat.)}^{+0.0034}_{-0.0032} \text{ (exp.)}^{+0.0025}_{-0.0025} \text{ (th.)}$$

The values are very similar, differences comparable to terms beyond NLO.
Summary

- Multijet cross sections for $Q^2 < 100 \, \text{GeV}^2$ in good agreement with expectations from NLO
- Consistent $\alpha_s(M_Z)$ & running from low and high $Q^2$ multijet cross sections
- First measurements of incl. jet cross sections using anti-$k_T$ and SIScone
- The measured cross sections have similar shapes & normalization, and they agree well with NLO
- Calculations have similar precision, only SIScone is slightly less precise
- $k_T$, anti-$k_T$ and SIScone lead to similar values of $\alpha_s(M_Z)$ with similar precision.

Calculations beyond NLO in DIS are needed!
Thank you!

Further results from H1 & ZEUS:

http://www-h1.desy.de/publications/H1_sci_results.shtml

http://www-zeus.desy.de/zeus_papers/zeus_papers.html
Jet finding: $k_T$, anti-$k_T$ & SIScone
**H1: 2-jets & $\alpha_s$**

### 2-Jet Cross Section

- **$5 < Q^2 < 7$ GeV$^2$**
  - **$10 < Q^2 < 15$ GeV$^2$**
- **$7 < Q^2 < 10$ GeV$^2$**
  - **$15 < Q^2 < 20$ GeV$^2$**
- **$20 < Q^2 < 30$ GeV$^2$**
  - **$30 < Q^2 < 40$ GeV$^2$**
- **$40 < Q^2 < 100$ GeV$^2$**

### $\alpha_s$ from Inclusive Jet Cross Sections

- **H1 data**
- **$\alpha_s$ fit to $\alpha_{jet}$**
- **Theory $\otimes$ PDF**

### $\alpha_s$ from 2-Jet Cross Sections

- **H1 data**
- **$\alpha_s$ fit to $\alpha_{2\text{-jet}}$**
- **Theory $\otimes$ PDF**
NLOjet++ provides also a good description of the 3-jet cross section in NLO, i.e. $O(\alpha_s^3)$. 

**H1: 3-jets & $\alpha_s$**
Jet multiplicity & Running $\alpha_s$

Normalised Inclusive Jet Cross Section

- H1: 150 < $Q^2$ < 200 GeV$^2$
- H1: 200 < $Q^2$ < 270 GeV$^2$
- H1: 270 < $Q^2$ < 400 GeV$^2$
- H1: 400 < $Q^2$ < 700 GeV$^2$
- H1: 700 < $Q^2$ < 5000 GeV$^2$
- H1: 5000 < $Q^2$ < 15000 GeV$^2$

$\mu_r = \sqrt{(Q^2+P_T^2)/2}$

- H1 data
- $\alpha_s$ fit to $\sigma_{jet}/\sigma_{NC}$
- Theory uncertainty

$\alpha_s$ vs $P_T$ / GeV

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Theory uncertainty on $\alpha_s(M_Z)$

- method 1: the fit of $\alpha_s(M_Z)$ to the data is repeated with $\mu_r$ scaled by 0.5 and 2 in the NLO calc.; the difference to the result with the nominal scale is taken as uncertainty.

  ➔ the theory uncertainty depends on the data

- method 2: only theory is used (Jones et al., JHEP 122003007), no refit to data
ZEUS: data/NLO for $k_T$ & SIScone

ZEUS

- ZEUS 82 pb$^{-1}$
- anti-$k_T$
- NLO uncertainty
- ratio to NLO
- jet energy scale uncertainty

$125 < Q^2 < 250$ GeV$^2$
$250 < Q^2 < 500$ GeV$^2$
$-2 < \eta_{\ell} < 1.5$
$|\cos\gamma_h| < 0.65$
$500 < Q^2 < 1000$ GeV$^2$
$1000 < Q^2 < 2000$ GeV$^2$
$2000 < Q^2 < 5000$ GeV$^2$
$Q^2 > 5000$ GeV$^2$

ZEUS

- ZEUS 82 pb$^{-1}$
- SIScone
- NLO uncertainty
- ratio to NLO
- jet energy scale uncertainty

$125 < Q^2 < 250$ GeV$^2$
$250 < Q^2 < 500$ GeV$^2$
$-2 < \eta_{\ell} < 1.5$
$|\cos\gamma_h| < 0.65$
$500 < Q^2 < 1000$ GeV$^2$
$1000 < Q^2 < 2000$ GeV$^2$
$2000 < Q^2 < 5000$ GeV$^2$
$Q^2 > 5000$ GeV$^2$
Fitting $\alpha_s(M_Z): \chi^2$

Minimise $\chi^2(\alpha_s(M_Z))$ defined as:

\[
\chi^2 = \tilde{V}^T \cdot M^{-1} \cdot \tilde{V} + \sum_{k} \varepsilon_k^2 \nonumber
\]

- correlated version of $\sum$(difference/error)$^2$
- penalty term for fitted systematics
- "Hessian" method

\[
M = M_{\text{stat.}} + M_{\text{uncor.}}
\]

- correlated for some bins
- uncorrelated systematics

\[
\tilde{V}_i = \sigma_i^{\text{exp.}} - \sigma_i^{\text{theo.}} (1 - \sum_k \Delta_{ik} \varepsilon_k) \nonumber
\]

- bin #
- correlated systematical error #k
- parameter in fit, pull "Hessian" method

Exp. uncertainty of fit defined as $\alpha_s$ interval upto minimum $\chi^2 + 1$