Hadron structure from generalized parton distributions

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1. Introduction

2. Factorization

3. Some basics about GPDs

4. Impact parameter

5. Spin effects

6. Summary

QCD at scales above $\sim 1\,\mathrm{GeV}$

- quarks and gluons become "visible" as degrees of freedom
- but observed particles are still hadrons
  where quarks and gluons are confined
- hadrons as highly relativistic many-body systems
  particle number not conserved (sea quarks, gluons)

two aims/motivations:

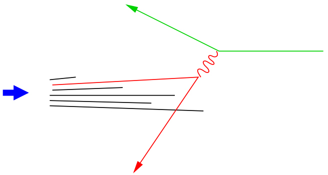
- understand hadron structure from $\mathcal{L}_{\text{QCD}}$
- describe high-energy processes, e.g. at LHC

tool: factorization of dynamics into

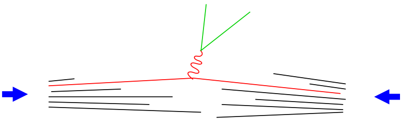
partonic processes	$\otimes$	parton distributions
short times/distances		and related quantities
perturbation theory in α_s		hadron structure at parton level

The parton model

- in processes with large energy-momentum transfer collisions of individual partons possibly involving γ, W, ...
- hadron $\sim$ bunch of almost-collinear, quasi-free partons in frame where it moves fast
- "initial conditions" of collision described by parton distributions in hadron



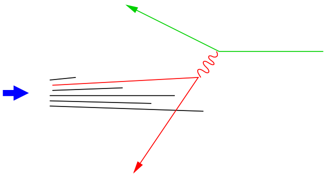
Deep inelastic scattering: $\ell p \to \ell X$



Drell-Yan: $pp \to \ell^+ \ell^- X$

Short-distance factorization in QCD

- implements parton model ideas into quantum field theory
- corrections to partons being free
  perturbation theory at small α_s
- factorization proofs for specific processes and observables in specific kinematics
  exhibit limitations of parton model



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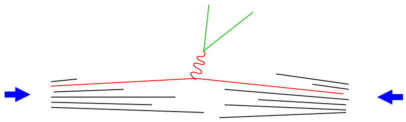
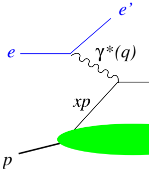


Drell-Yan: $pp \to \ell^+ \ell^- X$

Factorization

- deeply inelastic scattering: $\ell p \to \ell + X$



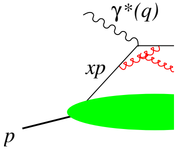
- $Q^2 = -q^2$ and $W^2 = (q+p)^2$ large at fixed Q^2/W^2 $\rightsquigarrow$

  $\sigma_{\text{tot}}(\gamma^* p) \;\propto\;$ scattering on parton
  $\otimes$ parton distribution $f(x)$

- systematic treatment in QCD

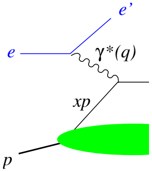
- prove factorization in limit $Q^2 \to \infty$
- hard scattering in perturbation theory
- partonic final states beyond single q

  $\rightsquigarrow \;\; x > x_B \approx \dfrac{Q^2}{W^2 + Q^2}$



Factorization

- deeply inelastic scattering: $\ell p \to \ell + X$

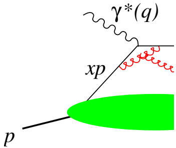


- $Q^2 = -q^2$ and $W^2 = (q+p)^2$ large at fixed $Q^2/W^2 \rightsquigarrow$

  $\sigma_{\text{tot}}(\gamma^* p) \;\propto\;$ scattering on parton $\otimes$ parton distribution $f(x)$

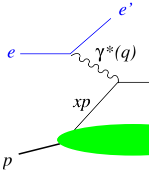
- systematic treatment in QCD

- scale μ to separate parton distribution from hard scattering
- physically: resolution scale separate "object" from "probe"
- DGLAP evolution equation for $\mu \dfrac{d}{d\mu} f(x,\mu)$



Factorization

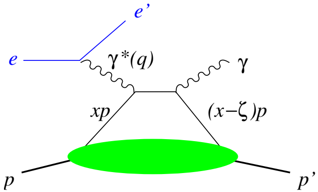
- deeply inelastic scattering: $\ell p \to \ell + X$



- $Q^2 = -q^2$ and $W^2 = (q+p)^2$ large at fixed $Q^2/W^2 \rightsquigarrow$

  $\sigma_{\text{tot}}(\gamma^* p) \;\propto\;$ scattering on parton
  $\otimes$ parton distribution $f(x)$

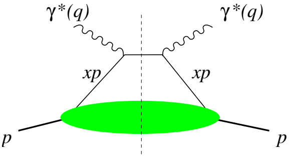
- deeply virtual Compton scattering: $\ell p \to \ell \gamma p$

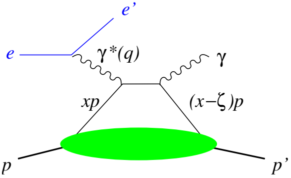


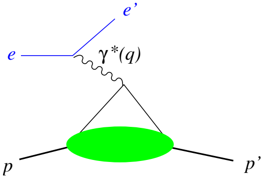
- Q^2 and W^2 large, $t = (p-p')^2$ small $\rightsquigarrow$

  $\mathcal{A}(\gamma^* p \to \gamma p) \;\propto\;$ scattering on parton
  $\otimes$ Generalized Parton Distribution $f(x,\zeta,t)$

D. Müller et al '94
X. Ji '96; A. Radyushkin '96
J. Collins et al '96



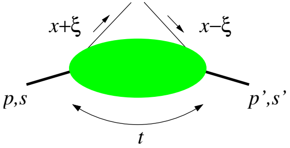




$f(x,0,0)$		$f(x,\zeta,t)$		$\int dx\, f(x,\zeta,t) = F(t)$
parton density	$\Leftarrow$	GPD	$\Rightarrow$	form factor

- define $f(x,\zeta,t)$ as matrix element $\langle p' \,|\, \mathcal{O} \,|\, p\rangle$
  $\mathcal{O} =$ operator with quark/gluon fields at light-like distance
- renormalization of $\mathcal{O}$ $\leftrightarrow$ evolution eq. for $\mu \dfrac{d}{d\mu} f(x,\zeta,t;\mu)$

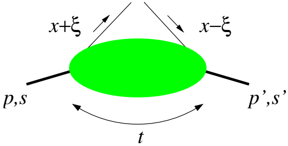
Definition and properties of GPDs



$$\int dz^- \, e^{ixP^+z^-} \langle p', s' | \, \bar{q}(-\tfrac{1}{2}z) \, \gamma^+ q(\tfrac{1}{2}z) \, |p, s\rangle \Big|_{z^+=0,\, \boldsymbol{z}=\boldsymbol{0}}$$

- light-cone variables $v^\pm = \frac{1}{\sqrt{2}}(v^0 \pm v^3)$
  transverse components $\boldsymbol{v} = (v^1, v^2)$
  - for fast right-moving particle: $p^0 \approx p^3$
    $$\Rightarrow \; p^+ \approx \sqrt{2}\, p^3, \quad p^- \ll p^+$$
  - boost along z: $v^+ \to v^+ \alpha, \quad v^- \to v^-/\alpha$

Definition and properties of GPDs



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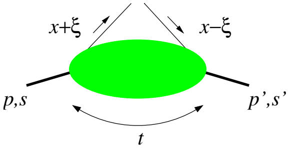
$$\propto H^q \; \bar{u}(p', s')\gamma^+ u(p, s) + E^q \; \bar{u}(p', s') \, \tfrac{i}{2m_p} \sigma^{+\alpha}(p' - p)_\alpha \, u(p, s)$$

- kin. variables in H^q, E^q:
  x, ξ momentum fractions w.r.t. $P = \frac{1}{2}(p + p')$
  in DVCS and light meson production:
  $\xi = x_B/(2 - x_B)$, x integrated over
  t can trade for transverse momentum transfer $\boldsymbol{\Delta} = \boldsymbol{p}' - \boldsymbol{p}$

Definition and properties of GPDs



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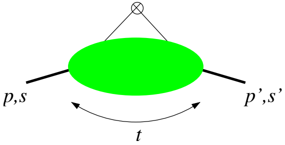
- proton spin structure:
  $H^q \leftrightarrow s = s'$ for $p = p'$ recover usual densities:
  $$H^q(x, \xi = 0, t = 0) = \begin{cases} q(x) & x > 0 \\ -\bar{q}(-x) & x < 0 \end{cases}$$
  $E^q \leftrightarrow s \neq s'$ decouples for $p = p'$
- similar definitions for polarized quarks and for gluons

Definition and properties of GPDs



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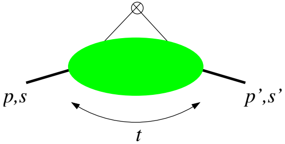
- Mellin moments: $\int dx \, x^n \;\rightarrow\;$ local operators $\bar{q}\,(\overleftrightarrow{D}^+)^n \gamma^+ q$
  $\rightarrow\;$ form factors
- calculations in lattice QCD
- $\sum_q e_q \int dx \, H^q(x, \xi, t) = F_1(t)$ Dirac f.f.
  $\sum_q e_q \int dx \, E^q(x, \xi, t) = F_2(t)$ Pauli f.f.

Definition and properties of GPDs



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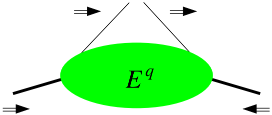
- $\int dx \, x \;\to$ energy-momentum tensor

  Ji's sum rule $\quad \frac{1}{2}\int dx \, x(H^q + E^q) = J^q(t)$

  $J^q(0) =$ total angular momentum carried by quark flavor q (helicity and orbital part)

Definition and properties of GPDs



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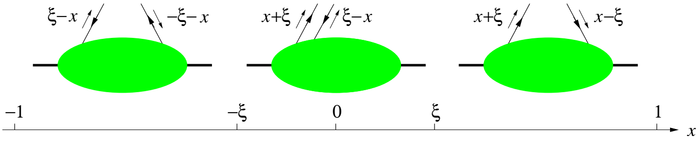
- $E^q \neq 0$ needs orbital angular momentum between partons

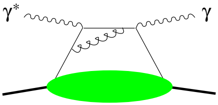
Partonic interpretation

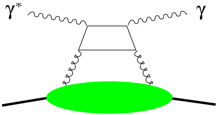


- $|x| > \xi$ similar to parton densities
  correlation $\psi^*_{x-\xi}\,\psi_{x+\xi}$ instead of probability $|\psi_x|^2$
  $|x| < \xi$ coherent emission of $q\bar{q}$ pair
- regions related by Lorentz invariance
  spacelike partons incoming in some frames, outgoing in others

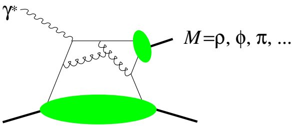
Key processes involving GPDs

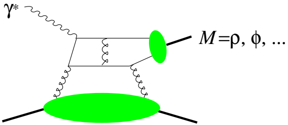
- deeply virtual Compton scattering
  best control over theory D. Müller et al.



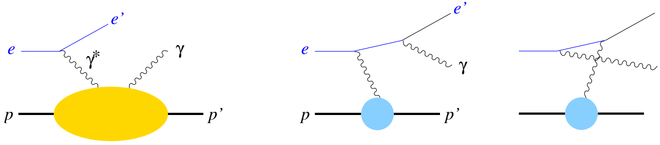


- meson production: large Q^2 or heavy quarks
  help for separating quark flavors and gluons
  radiative and power corrections important, even for $Q^2 \sim 10\,\mathrm{GeV}^2$





From cross sections to GPDs



- virtual Compton scattering + Bethe-Heitler process
  interference term $\rightsquigarrow$ $\mathcal{A}(\gamma^* p \to \gamma p)$ including its phase
- relation $\mathcal{A}(\gamma^* p \to \gamma p) \leftrightarrow \sigma(\ell p \to \ell \gamma p)$ becomes
  simple in relevant kinematics (Q^2 and W^2 large, t small)
- detailed information from angular distrib's and polarization
  isolate interference in beam charge asymmetry

M.D., T.Gousset, B.Pire, J.Ralston '97
A.Belitsky, A.Kirchner, D.Müller '01

- measurements from JLAB, HERMES, HERA Collider
  hopefully soon from COMPASS, eventually from EIC

Localizing partons: impact parameter

- states with definite light-cone momentum p^+ and transverse position (impact parameter):

$$|p^+, \boldsymbol{b}\rangle = \int d^2\boldsymbol{p}\, e^{-i\boldsymbol{b}\boldsymbol{p}}\, |p^+, \boldsymbol{p}\rangle$$

  formal: eigenstates of 2 dim. position operator D. Soper '77

- can exactly localize proton in 2 dimensions
  no limitation by Compton wavelength
- and stay in frame where proton moves fast
  $\rightsquigarrow$ parton interpretation
- different from localization in 3 spatial dimensions
  well-known for form factors; also for GPDs Belitsky, Ji, Yuan '03

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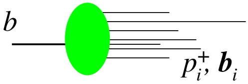
$$|p^+, \boldsymbol{b}\rangle = \int d^2\boldsymbol{p}\, e^{-i\boldsymbol{b}\boldsymbol{p}}\, |p^+, \boldsymbol{p}\rangle$$

- $\boldsymbol{b}$ is center of momentum of the partons in proton



$$\boldsymbol{b} = \frac{\sum_i p_i^+ \boldsymbol{b}_i}{\sum_i p_i^+} \qquad (i = q, \bar{q}, g)$$

consequence of Lorentz invariance: transverse boosts

$$k^+ \to k^+ \qquad \boldsymbol{k} \to \boldsymbol{k} - k^+ \boldsymbol{v}$$

nonrelativistic analog: Galilei invariance $\xrightarrow{\text{Noether}}$ center of mass

Impact parameter distributions

- from $\langle p^+, \boldsymbol{p}' \,|\, \mathcal{O} \,|\, p^+, \boldsymbol{p} \rangle$ to $\langle p^+, \boldsymbol{b} \,|\, \mathcal{O} \,|\, p^+, \boldsymbol{b} \rangle$

  $$\rightsquigarrow \quad q(x, b^2) = (2\pi)^{-2} \int d^2\boldsymbol{\Delta}\, e^{-i\boldsymbol{b}\boldsymbol{\Delta}}\, H^q(x, \xi = 0, t = -\boldsymbol{\Delta}^2)$$

  gives distribution of quarks with
  - longitudinal momentum fraction x
  - transverse distance b from proton center M. Burkardt '00

- average impact parameter

  $$\langle b^2 \rangle_x = \frac{\int d^2b\, b^2\, q(x, b^2)}{\int d^2b\, q(x, b^2)} = 4\, \frac{\partial}{\partial t} \log H(x, \xi = 0, t)\Big|_{t=0}$$

- can generalize to $\xi \neq 0$ M.D. '02

  t dependence of hard exclusive processes

  $\rightsquigarrow$ spatial distribution of partons

  e.g. from $ep \to ep\,\rho$, $ep \to ep\,\phi$, $ep \to ep\,J/\Psi$, DVCS

Impact parameter distributions

- from $\langle p^+, \boldsymbol{p}' \,|\, \mathcal{O} \,|\, p^+, \boldsymbol{p} \rangle$ to $\langle p^+, \boldsymbol{b} \,|\, \mathcal{O} \,|\, p^+, \boldsymbol{b} \rangle$

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- integrated over x $\rightsquigarrow$ form factor

$$\langle b^2 \rangle = \frac{\int dx \int d^2b\; b^2\, q(x, b^2)}{\int dx \int d^2b\; q(x, b^2)} = 4\, \frac{\partial}{\partial t} \log F_1(t) \Big|_{t=0}$$

Apples, oranges, and other fruits

form factor	distribution	$\langle b^2 \rangle$
F_1^p	$\sum_q e_q\,(q-\bar{q})$	$(0.66\,\mathrm{fm})^2$
G_E^p		$(0.71\,\mathrm{fm})^2 = (0.66\,\mathrm{fm})^2 + \dfrac{\kappa_p}{m_p^2}$
G_A	$\Delta u + \Delta\bar{u} - (\Delta d + \Delta\bar{d}\,)$	$(0.52$ to $0.54\,\mathrm{fm})^2$

- in form factor integral parton distributions have average $x \sim 0.2$
- generalized gluon dist. at $x = 10^{-3}$ $\rightsquigarrow$ $\langle b^2 \rangle = (0.57$ to $0.60\,\mathrm{fm})^2$
  from J/Ψ photoproduction at HERA

note:

$4\,\frac{\partial}{\partial t}\log G(t)\big|_{t=0} =$ squared impact parameter

$6\,\frac{\partial}{\partial t}\log G(t)\big|_{t=0} =$ squared radius

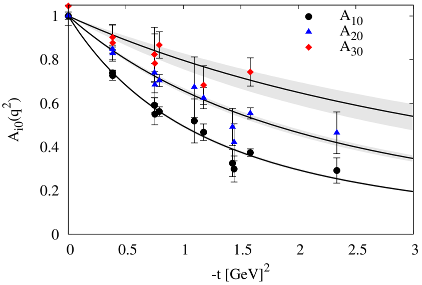
numbers: G_E and F_1 from Particle Data Group; G_A from Bernard et al. '01

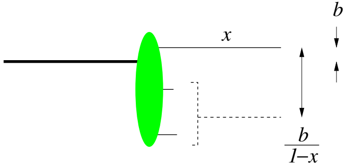
Lattice calculations

- results for moments

$$A_{n,0}(t) = \int dx\, x^{n-1} H(x, \xi = 0, t) = \int d^2\boldsymbol{b}\; e^{i\boldsymbol{b\Delta}} \int dx\, x^{n-1} q(x, b^2)$$

QCDSF Collab. '06: $A_{n0}(t)/A_{n0}(0)$





$b/(1-x) =$ distance of selected parton from spectator system

- steeper t slope for larger n

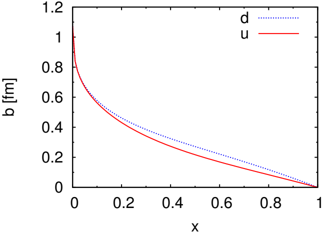
$\rightsquigarrow$ decrease of $\langle b^2 \rangle_x$ with x

Analysis of form factor data

M.D., T.Feldmann, R.Jakob, P.Kroll '04; cf. also M.Polyakov et al. '04

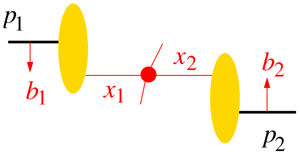
- ansatz for $H^q(x,\xi,t)$ consistent with parton densities and electromagnetic form factor data

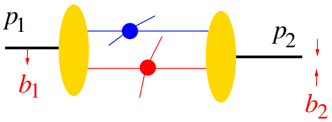
valence quarks $q - \bar{q}$



- $\bar{b}$ decreases with x
  same trend as in lattice results
- averaged over x have $\langle b^2 \rangle = (0.66\,\mathrm{fm})^2$
- tendency: d wider than u
  but not conclusive

Aside: multiple parton interactions





- hard inclusive process, e.g. $pp \to \mathrm{jet\ jet} + X$
  - no impact parameter dependence
    integrate over b_1 and b_2 independently
- secondary soft or hard interactions
  - do not affect inclusive cross section, but change event structure
    will affect many analyses at LHC
  - sensitive to transverse distance between partons
- information from GPDs can help description of mult. interactions
  - b dependence and its interplay with momentum fraction x

Now add spin

- $E \leftrightarrow$ nucleon helicity flip $\langle\downarrow|\mathcal{O}|\uparrow\rangle$

  $\leftrightarrow$ transverse pol. difference $|X\pm\rangle = \frac{1}{\sqrt{2}}(\,|\uparrow\rangle \pm |\downarrow\rangle\,)$

  $$\langle X{+}|\mathcal{O}|X{+}\rangle - \langle X{-}|\mathcal{O}|X{-}\rangle = \langle\uparrow|\mathcal{O}|\downarrow\rangle + \langle\downarrow|\mathcal{O}|\uparrow\rangle$$

- quark density in proton state $|X+\rangle$

  $$q^X(x,\boldsymbol{b}) = q(x,\boldsymbol{b}^2) - \frac{b^y}{m}\frac{\partial}{\partial\boldsymbol{b}^2}\,e^q(x,\boldsymbol{b}^2)$$

  is shifted in y direction M. Burkardt '02

  $e^q(x,b)$ is Fourier transform of $E^q(x,\xi=0,t)$

- semi-classical picture: rotating matter distribution

gives alternative view on Ji's sum rule $L^x = b^y\,p^z$

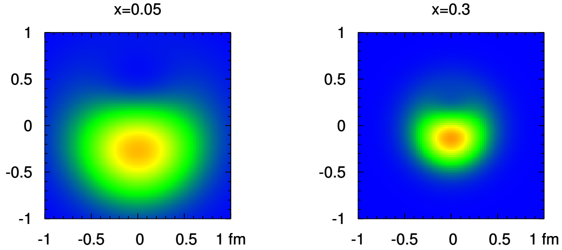
M. Burkardt '05

$(d - \bar{d})$ density in transverse plane

quark density in proton state $|X+\rangle$

$$q^X(x, \boldsymbol{b}) = q(x, \boldsymbol{b}^2) - \frac{b^y}{m} \frac{\partial}{\partial \boldsymbol{b}^2} e^q(x, \boldsymbol{b}^2)$$

is shifted



M.D., Th. Feldmann, R. Jakob, P. Kroll '04

- $\int dx\, E^u(x,0,0) = \kappa^u \approx 1.67$
  $\int dx\, E^d(x,0,0) = \kappa^d \approx -2.03$

  from p and n magnetic moments
  $\rightarrow$ large spin-orbit correlations

$(d - \bar{d})$ density in transverse plane

quark density in proton state $|X+\rangle$

$$q^X(x, \boldsymbol{b}) = q(x, \boldsymbol{b}^2) - \frac{b^y}{m} \frac{\partial}{\partial \boldsymbol{b}^2} e^q(x, \boldsymbol{b}^2)$$

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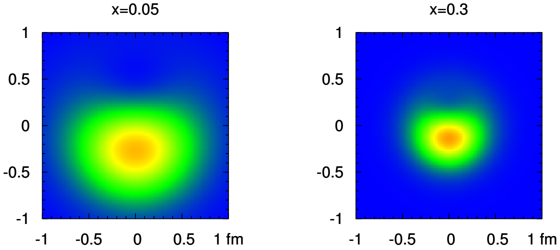
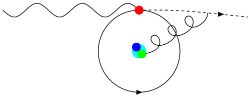


M.D., Th. Feldmann, R. Jakob, P. Kroll '04

- explanation of Sivers effect by chromodynamic lensing
  anisotropic $\boldsymbol{p}_T$ distrib. of π produced from transv. polarized target

$$\left.\begin{array}{l}\text{struck quark interacts with spectators}\\ \text{anisotropic spectator distribution}\end{array}\right\} \Rightarrow \begin{array}{l}\text{anisotropic } \boldsymbol{p}_T \text{ distrib.}\\ \text{of struck quark}\end{array}$$



M. Burkardt '04

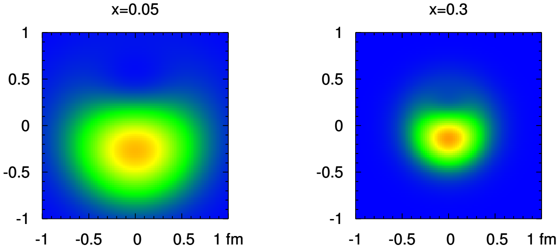
figure from arXiv:0807.2599

$(d - \bar{d})$ density in transverse plane

quark density in proton state $|X+\rangle$

$$q^X(x, \boldsymbol{b}) = q(x, \boldsymbol{b}^2) - \frac{b^y}{m} \frac{\partial}{\partial \boldsymbol{b}^2} e^q(x, \boldsymbol{b}^2)$$

is shifted



M.D., Th. Feldmann, R. Jakob, P. Kroll '04

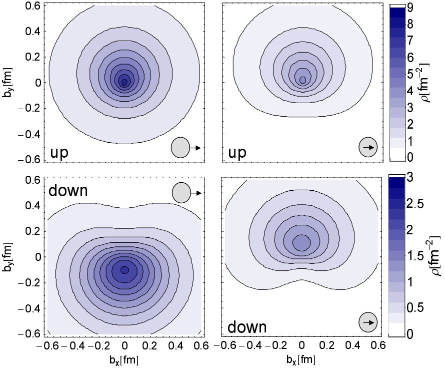
- analogous phenomenon for transv. pol. quarks in unpol. target
- chromodynamic lensing found in spectator models
  Burkardt, Hwang '03; Meissner, Metz, Goeke '07; Gamberg, Schlegel '09

Calculations in lattice QCD

nucleon: QCDSF Collab., M. Göckeler et al., hep-lat/0612032

$\int dx\, q^X(x, \boldsymbol{b})$ polarized p

$\int dx\, q_T^X(x, \boldsymbol{b})$ polarized q



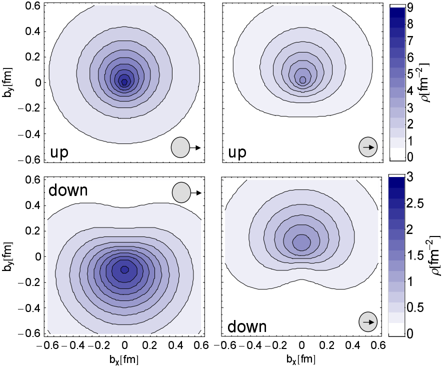
Calculations in lattice QCD

nucleon: QCDSF Collab., M. Göckeler et al., hep-lat/0612032

pion: QCDSF Collab., D. Brömmel et al., arXiV:0708.2294

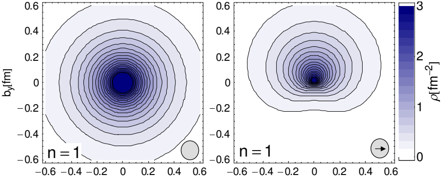
$\int dx\, q^X(x, \boldsymbol{b})$ polarized p

$\int dx\, q_T^X(x, \boldsymbol{b})$ polarized q



$\int dx\, q(x, \boldsymbol{b})$ u in π^+

$\int dx\, q_T^X(x, \boldsymbol{b})$ polarized u in π^+



average shifts:

- $\langle b_y \rangle = 0.154(6)$ fm in proton
- $\langle b_y \rangle = 0.162(11)$ fm in pion

The proton spin budget

- Ji's sum rule: $\frac{1}{2} = J^g + \sum_q J^q$ with Ji '96

$$J^q = \frac{1}{2} \int dx\, x \left(H^q + E^q\right)\Big|_{\substack{t=0\\ \xi=0}} \qquad J^g = \frac{1}{2} \int dx \left(H^g + E^g\right)\Big|_{\substack{t=0\\ \xi=0}}$$

- further decompose $J^q = L^q + \frac{1}{2}\Sigma$ and $J^g = L^g + \Delta g$
  with Σ and Δg from ordinary parton densities
  operator interpretation of L^g nontrivial Wakamatsu '10
- alternative decomp. $\frac{1}{2} = \mathcal{J}^g + \sum_q \mathcal{J}^q$ Bashinski, Jaffe '98

  with $\mathcal{J}^g = \mathcal{L}^g + \Delta g$ and $\mathcal{J}^q = \mathcal{L}^q + \frac{1}{2}\Sigma$
  - $J^q \neq \mathcal{J}^q$, $L^q \neq \mathcal{L}^q$ and $J^g \neq \mathcal{J}^g$ Burkardt, Hikmat '08
  - no known connection of $\mathcal{L}^q$ and $\mathcal{L}^g$ with observables

The proton spin budget

- Ji's sum rule: $\frac{1}{2} = J^g + \sum_q J^q$ with — Ji '96

$$J^q = \frac{1}{2} \int dx\, x\, (H^q + E^q)\Big|_{\substack{t=0\\ \xi=0}} \qquad J^g = \frac{1}{2} \int dx\, (H^g + E^g)\Big|_{\substack{t=0\\ \xi=0}}$$

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  operator interpretation of L^g nontrivial — Wakamatsu '10
- alternative decomp. $\frac{1}{2} = \mathcal{J}^g + \sum_q \mathcal{J}^q$ — Bashinski, Jaffe '98
- ambiguities in decomposition reflect difficulty to separate
  - "quark" from "gluon" contrib's in presence of interactions
    gluon field contains physical and unphysical (gauge) d.o.f.
  - "intrinsic" from orbital angular momentum for spin-1 particles

  similar issues already in QED

  there need not be a unique choice

The proton spin budget

- Ji's sum rule: $\frac{1}{2} = J^g + \sum_q J^q$ with — Ji '96

$$J^q = \frac{1}{2}\int dx\, x\,(H^q + E^q)\Big|_{\substack{t=0\\ \xi=0}} \qquad J^g = \frac{1}{2}\int dx\,(H^g + E^g)\Big|_{\substack{t=0\\ \xi=0}}$$

- further decompose $J^q = L^q + \frac{1}{2}\Sigma$ and $J^g = L^g + \Delta g$
  with Σ and Δg from ordinary parton densities
  operator interpretation of L^g nontrivial — Wakamatsu '10
- lattice $\rightsquigarrow$ Σ and J^q — J^g difficult, Δg impossible
  - directly get integrals over x at $\xi = 0$
- exclusive processes $\rightsquigarrow$ GPDs $\rightsquigarrow$ J^q and (more difficult) J^g
  - exclusive (and inclusive) processes: $\int dx$ difficult
  - measure at $\xi \neq 0$
  - but direct access to x dependence of $E^{q,g}(x,x,t)$

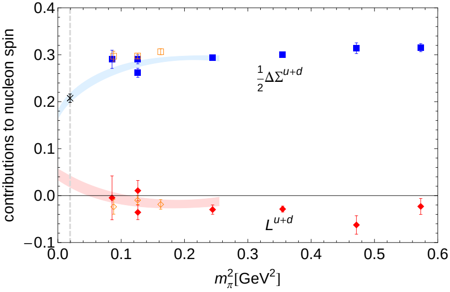
Current estimates of L^q at scale $\mu = 2\,\mathrm{GeV}$

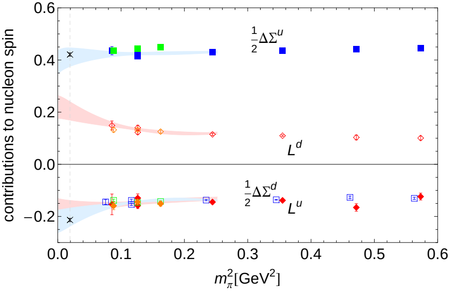
- lattice results — QCDSF, M. Ohtani et al. '07
  $J^u = 0.230(8) \qquad J^d = -0.004(8)$
  $L^{u+d} = 0.025(27)$
- lattice results — LHPC, J.D. Bratt, Ph. Hägler et al. '10
  $J^u = 0.236(6) \qquad J^d = 0.0018(37)$
  $L^{u+d} = 0.056(11)$ or $0.030(12)$
- phenomenological analysis of Pauli form factors — M.D. et al '04
  $J^u_v = 0.19$ to $0.23 \qquad J^d_v = -0.02$ to 0.02
  $L^{u+d}_v = -0.11$ to -0.06
  only valence $(q - \bar{q})$ contrib's from $\sum_q e_q \int dx\, E^q(x,0,t) = F_2(t)$
- still important systematic uncertainties
- trend: $J^u > 0, \quad J^d \approx 0$
  $L^u < 0\,,\ L^d > 0$ and $L^u + L^d \approx 0$

A closer look at lattice results

figures courtesy Ph. Hägler

- all values in $\overline{\text{MS}}$ scheme at $\mu = 2\,\text{GeV}$
- rather weak pion mass dep'ce for $290\,\text{MeV} < m_\pi < 760\,\text{MeV}$





data:

blue and red: LHPC $n_F = 2 + 1$ mixed action, arXiv:1001.3620

orange and green: LHPC domain wall fermions, preliminary (Syritsin et al.)

crosses: $\Delta\Sigma$ from HERMES, hep-ex/0609039

Summary

Generalized parton distributions

- versatile tool to study hadrons in terms of quarks and gluons firmly rooted in QCD
- natural extension of knowledge gained from parton densities and form factors
- important trends already established:
    - correlation of impact parameter and momentum fraction
    - large spin-orbit correlations in valence quark sector
    - tendency of cancellation between u and d in orbital angular momentum
- next generation of experiments and lattice calculations will hopefully bring more detailed and quantitative knowledge