\[ K = -2 \ln(V + \alpha'^3 \xi) - \ln(S + \bar{S}) - \ln \left( -i \int_{CY_3} \bar{\Omega} \wedge \Omega \right) \]

\[ V \sim (T + \bar{T})^{3/2} \quad , \quad \xi \sim -\frac{\chi(CY_3)}{(2\pi)^3} \cdot (S + \bar{S})^{3/2} \]

\[ W = W_0 + A e^{-\alpha T} \quad , \quad W_0 = \int_{CY_3} G_3 \wedge \Omega \]

\[ V \simeq 4AW_0 \frac{a t e^{-at}}{V^2} \cos(a\tau) + \frac{3W_0^2}{4V^3} \sim \frac{2C}{9x^{9/2}} - \frac{e^{-x}}{x^2} \]

- (see also: talk by K. Givens) but we can extend it to \[ h^{1,1} \geq 1 \]
  \[ [h^{1,1} = 2: \text{AbdusSalam, Conlon, Quevedo, Suruliz '07}] \]
- can include the dilaton explicitly, and \[ h^{2,1} > 1 \]
- complex structure moduli
- if tree-level masses from \(|DW_0|^2\) are positive:
  \[ \Rightarrow \frac{\delta S}{S_0} \sim \frac{\xi}{V} \quad \Rightarrow \frac{\delta U_a}{U_{a,0}} \sim \frac{\xi}{V} \]
The overshoot problem in inflation after tunnelling

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(work in progress)
University of Hamburg & DESY Hamburg
- Tunneling feeds the landscape:

- proceeds via CDL instanton
  [Coleman, De Luccia '80]

- nucleates bubbles of negative spatial curvature
equations of motion dictating evolution after passing through CDL tunneling:

\[ \ddot{\phi} + 3H \dot{\phi} = -V'(\phi) \]

\[ H^2 = \frac{1}{3M_P^2} \left( \frac{\dot{\phi}^2}{2} + V(\phi) \right) + \frac{1}{a^2} \]

\[ H^2 = \frac{\rho}{3} \]
- equations of motion dictating evolution after passing through CDL tunneling:

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\ddot{\phi} + 3H \dot{\phi} = -V'(\phi)
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\]

\[
H^2 = \frac{\rho}{3} - \frac{k}{a^2}, \quad k = -1
\]

- slow-down also possible, if matter/radiation present; or non-minimal kinetic terms (Liam’s talk)

- e.g. motion of orthogonal fields, but model & initial condition dependent - while negative curvature is dictated by CDL instanton
• CDL instanton dictates very special initial conditions:

\[ a(t) = t + \mathcal{O}(t^3) \quad \quad \dot{\phi}_0 \equiv \dot{\phi}(t = 0) = 0 \]
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\[ V_0 \gg V_- \]
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• CDL instanton dictates very special initial conditions:

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\[ \phi_0 \equiv \phi(t = 0) \]

\[ |\phi_0| < 1 \]

\[ V_0 \gg V_- \]

\[ \dot{\phi}_f = 0 \]
• CDL instanton dictates very special initial conditions:

\[ a(t) = t + O(t^3) \quad \dot{\phi}_0 \equiv \dot{\phi}(t = 0) = 0 \]

\[ \phi_0 \equiv \phi(t = 0) \]

\[ |\phi_0| < 1 \quad !! \]

\[ V_R(\phi) = V_- \cdot (1 - \sqrt{2\epsilon\phi}) \]

\[ V_L(\phi) = (-1)^n \frac{\lambda_n}{n} \phi^n \]
• on the right side, assuming the field crosses zero at $t = t_f$ and speed $\dot{\phi}_f = \dot{\phi}(t_f)$

$$V_R(\phi) = V_\cdot (1 - \sqrt{2} \epsilon \phi) \quad \dot{\phi} + \frac{3}{t} \dot{\phi} - V_\sqrt{2} \epsilon = 0$$

• slow-roll begins once curvature is sub-dominant, at $t = t_c$:

$$\phi(t_c) = \frac{3}{4\sqrt{2}} \sqrt{\epsilon} + \frac{1}{2} \dot{\phi}_f t_f - \frac{1}{2\sqrt{2}} \sqrt{\epsilon} V_- t_f^2 - \frac{1}{6} \dot{\phi}_f V_- t_f^3 + \frac{1}{12\sqrt{2}} \sqrt{\epsilon} V_-^2 t_f^4$$

$$t_c = \sqrt{\frac{3}{V_-}} \quad \& \text{later we will find:} \quad t_f \sim \begin{cases} \frac{|\phi_0|}{\sqrt{V_0}}, & 1 \leq n \leq 3 \\ \infty, & n \geq 4 \end{cases}$$
• **parametric behaviour on the right side:**

\[ \phi(t_c) = \frac{3}{4\sqrt{2}} \sqrt{\epsilon} + \frac{1}{2} \phi_f t_f + O\left(\frac{V_-}{V_0}\right) \]

• **on the left side:**

\[ \ddot{\phi} + \frac{3}{t} \dot{\phi} + (-1)^n \lambda_n \phi^{n-1} = 0 \]

\[ n = 1 : \quad \phi(t) = \phi_0 + \frac{\lambda V_-}{8} t^2 \]

\[ \Rightarrow \quad t_f \sim \frac{|\phi_0|}{\sqrt{V_0}} \]

\[ n = 2 : \quad \phi(t) = 2\phi_0 \frac{J_1(mt)}{mt} \]

\[ \Rightarrow \quad \phi_f \sim \sqrt{V_0} \]
on the left side - cubic potential: [Polianin, Zaitsev “Handbook of solutions for ODEs”]

\[
\ddot{\phi} + \frac{5}{3t} \dot{\phi} - \lambda_3 \phi^2 = 0
\]

\[
\ddot{\phi} + \frac{10}{3t} \dot{\phi} - \lambda_3 \phi^2 = 0
\]

⇒ have known solutions ...

⇒ bound exit speed!

\[
\phi(t) = \frac{(8/3\lambda_3)^{1/3}}{t^{2/3}} p(t^{2/3}(3\lambda_3/8)^{1/3} + \zeta, 0, \xi)
\]

\[
\phi(t) = -\frac{2}{3t^2\lambda_3} \left\{ \left( -\frac{3\lambda_3\xi}{2} \right)^{1/3} t^{2/3} p \left[ i \left( -\frac{3\lambda_3\xi}{2} \right)^{1/6} t^{1/3}, 0, 1 \right] + 1 \right\}
\]

⇒ \( t_f \sim \frac{|\phi_0|}{\sqrt{V_0}} \) \( \Rightarrow \dot{\phi}_f \sim \sqrt{V_0} \)

elliptic Weierstrass function
resulting overshoot for $n = 1 \ldots 3$:

$$\phi(t_c) = \frac{3}{4\sqrt{2}} \sqrt{\epsilon} - \mathcal{O}(1) \phi_0 + \mathcal{O} \left( \frac{V_-}{V_0} \right)$$

\[\Rightarrow\] can show: independent of $\phi_0$
• resulting overshoot for \( n = 1 \ldots 3 \):

\[
\phi(t_c) = \frac{3}{4\sqrt{2}} \sqrt{\epsilon} - O(1) \phi_0 + O\left(\frac{V_-}{V_0}\right)
\]

\(\Rightarrow\) can show: independent of \( \phi_0 \)

for \( n = 1 \), already in: [Freivogel, Kleban, Martinez, Susskind '05]
- on the left side - quartic potential, \( n = 4 \):

\[
\phi(t) = \frac{8\phi_0}{8 + t^2\lambda_4\phi_0^2} \quad t \to \infty \quad 0 \\
\Rightarrow \quad t_f \to \infty
\]

\[
\dot{\phi}(t) = -\frac{16t\lambda_4\phi_0^3}{(8 + t^2\lambda_4\phi_0^2)^2} \quad t \to \infty \quad 0
\]
• on the left side - quartic potential, n = 4:

\[ \phi(t) = \frac{8\phi_0}{8 + t^2\lambda_4\phi^2_0} \quad t \to \infty \Rightarrow 0 \quad \Rightarrow \quad t_f \to \infty \]

\[ \dot{\phi}(t) = -\frac{16t\lambda_4\phi_0^3}{(8 + t^2\lambda_4\phi_0^2)^2} \quad t \to \infty \Rightarrow 0 \]

• thus - no overshoot at all for n = 4!

\( V_- > 0 \) causes vacuum energy domination for some large \( t > 0 \) already on the left for \( \Phi < 0 \) ...
• on the left side - $n \geq 4$ ... we have a first integral for series of equations:

$$\gamma = \frac{n+2}{n-2} \leq 3 \quad \text{for} \quad n \geq 4$$

$$\ddot{\phi} + \frac{\gamma}{t} \dot{\phi} + (-1)^n \lambda_n \phi^{n-1} = 0$$

$$C_0 = \frac{t^{\gamma-1}}{2} \left( \dot{\phi}^2 t^2 + \dot{\phi} \phi t (\gamma - 1) \right) + \left( \frac{\phi t^{\frac{\gamma-1}{2}}}{n} \right)^n$$

[Rosenau '83]

• $C_0 = 0$ at $t = 0$ for arbitrary $\dot{\phi}_0$ ... therefore, if $\Phi$ reaches $\Phi = 0$ at any time $t = t_f > 0$, then

$$\dot{\phi}(t_f) = 0 \quad \text{and} \quad t_f \rightarrow \infty \quad \Rightarrow \text{again no overshoot!}$$
where we allowed to assume curvature domination for $\phi < 0$? 

... illustrative example $n=1$:

$$\frac{1}{2} \dot{\phi}^2 + V(\phi) = V_0 - \frac{3}{32} t^2 V^2 \chi^2 \ < \ V_0 \ < \ \frac{3}{t^2} \lambda^2 \ < \ \frac{3}{t^2}$$
where we allowed to assume curvature domination for $\phi < 0$? ... illustrative example $n=1$:

$$\frac{1}{2} \dot{\phi}^2 + V(\phi) = V_0 - \frac{3}{32} t^2 V^2 \lambda^2 < V_0 \frac{3}{t^2_f} < \frac{3}{t^2}$$
where we allowed to assume curvature domination for $\phi < 0$? ... illustrative example $n=1$:

$$\frac{1}{2} \dot{\phi}^2 + V(\phi) = V_0 - \frac{3}{32} t^2 V_-^2 \lambda^2 < V_0 < \frac{3}{t^2} \Rightarrow |\phi_0| < 1 \quad (\text{and: } V_- < V_0)$$
where we allowed to assume curvature domination for $\phi < 0$? ... illustrative example n=1:

\[
\frac{1}{2} \dot{\phi}^2 + V(\phi) = V_0 - \frac{3}{32} t^2 V_-^2 \lambda^2 < V_0 ! \left< \frac{3}{t_f^2} \right. < \frac{3}{t^2}.
\]

\[
t_f \sim \frac{|\phi_0|}{\sqrt{V_0}} \Rightarrow |\phi_0| < 1 \quad \text{(and: } V_- < V_0)\]

\[
\frac{3}{t^2}, t < t_f
\]

\[
V_0 - \frac{3}{32} t^2 V_-^2 \lambda^2
\]
where we allowed to assume curvature domination for $\phi < 0$? ... illustrative example $n=1$:

$$\frac{1}{2} \dot{\phi}^2 + V(\phi) = V_0 - \frac{3}{32} t^2 V_-^2 \lambda^2 < V_0 < \frac{3}{t^2} < \frac{3}{t^2}$$

$$t_f \sim \frac{|\phi_0|}{\sqrt{V_0}} \Rightarrow |\phi_0| < 1 \quad (\text{and: } V_- < V_0)$$

$$\Rightarrow \text{structurally similar arguments hold for } n = 2, 3$$

$$n \geq 4 \text{ can be argued, again similarly, from the first integral}$$
• binomials: \[ V(\phi) = (-1)^m \frac{\lambda_m}{m} \phi^m + (-1)^n \frac{\lambda_n}{n} \phi^n \quad , \quad n > m \]

• match \( \Phi^m \)-solution w/ \( \Phi^n \)-solution, where forces are equal:

\[
(-1)^m \lambda_m \phi^{m-1} \bigg|_{\phi=\phi^*} = (-1)^n \lambda_n \phi^{n-1} \bigg|_{\phi=\phi^*}
\]

• example: linear + quartic potential

\[ \phi(t) = \begin{cases} 
\frac{\lambda_1}{8\lambda_4} t^2 + \frac{8\phi_0}{8+t^2\lambda_4\phi_0^2}, & t < t^* \text{ and } \phi_0 \leq \phi < \phi^* \\
\frac{\lambda_1}{\lambda_4} t^2 + \frac{8(\lambda_{1/3} + \lambda_{4/3} \phi_0)^3}{\lambda_4\phi_0^4} t^{-2} + \lambda_1 \frac{2/3 \lambda_{1/3} + 3\lambda_{4/3} \phi_0}{\lambda_4\phi_0^2}, & t > t^* \text{ and } \phi^* < \phi < 0
\end{cases} \]

\[ \phi(t_c) = \frac{3}{4\sqrt{2}} \sqrt{\epsilon} - \left( \frac{\phi^*}{\phi_0} \right)^{3/2} \cdot \phi_0 \]

• by iteration carries over to polynomials ...
• analytical result by matching approximation
\[ V(\phi) = -\lambda_1 \phi + \frac{\lambda_4}{4} \phi^4 \]

• numerical result
\[ V(\phi) = -\lambda_1 \phi + \frac{\lambda_4}{4} \phi^4 \]
Conclusions

- inflationary regions fed by a CDL tunneling event have negligible overshoot:
  - field slows down due to negative curvature of the bubble
  - attractor-like behaviour:
    exit speed independent from shape & steepness of potential, or initial speed of field

immediate consequence:
- small-field & large-field inflation on par w.r.t. initial cond.
- can envision statistical prediction for $r$ by counting solutions (similar to the c.c.)

- open points: multiple fields (likely carries over, because results do not depend on initial field speed)