Observational consequences of non-minimally coupled chaotic inflation

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where I want to take you ...

• the start:
  - inflation - short summary

• summary of general chaotic inflation in supergravity

• summary of Higgs inflation

• non-minimal chaotic inflation in supergravity
  - short-cutting into Jordan frame supergravity
  - scalar potential - 2 examples
  - parameter scan -- predictions
Inflation ...
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\[ \ddot{\phi} + 3H \dot{\phi} + V' = 0 \]
Inflation ...

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• inflation: period quasi-exponential expansion of the very early universe

• driven by the vacuum energy of a slowly rolling light scalar field:

\[
e.o.m.: \quad \ddot{\phi} + 3H\dot{\phi} + V' = 0
\]

scale factor grows exponentially: \( a \sim e^{Ht} \) if: \( \dot{\phi} \ll \ddot{\phi} \)

\[
\Rightarrow \quad \epsilon \equiv -\frac{\dot{H}}{H^2} \simeq \frac{1}{2} \left( \frac{V'}{V} \right)^2 \ll 1 \quad , \quad \eta \equiv \frac{\dot{\epsilon}}{\epsilon H} \simeq \frac{V''}{V} \ll 1
\]

with the Hubble parameter \( H^2 = \frac{\dot{a}^2}{a^2} \simeq \text{const.} \sim V \)
Smoking Gun for Inflation ...
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- inflation generates metric perturbations: scalar (us) & tensor
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\[ P_S \sim \frac{H^2}{\epsilon} \sim \left( \frac{\delta \rho}{\rho} \right)^2 \]

\[ \sim k^{n_S - 1} \]
inflation generates metric perturbations: scalar (us) & tensor

\[ P_S \sim \frac{H^2}{\epsilon} \sim \left( \frac{\delta \rho}{\rho} \right)^2 \]

\[ \sim k^n_{S-1} \]

\[ n_S = 1 - 6\epsilon + 2\eta \]
• inflation generates metric perturbations: scalar (us) & tensor

\[ \mathcal{P}_S \sim \frac{H^2}{\epsilon} \sim \left( \frac{\delta \rho}{\rho} \right)^2 \]

and

\[ \mathcal{P}_T \sim H^2 \sim V \]

\[ \sim k^{n_S - 1} \]

\[ n_S = 1 - 6\epsilon + 2\eta \]
Smoking Gun for Inflation ...

- Inflation generates metric perturbations: scalar (us) & tensor

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\[ n_S = 1 - 6\epsilon + 2\eta \]

and

\[ \mathcal{P}_T \sim H^2 \sim V \]

Window to GUT scale & ‘smoking gun’: alternatives (e.g. ekpyrosis) have no tensors.
• **Smoking Gun for Inflation** ...

  **inflation generates metric perturbations:** scalar (us) & tensor

  \[ P_S \sim \frac{H^2}{\epsilon} \sim \left( \frac{\delta \rho}{\rho} \right)^2 \quad \text{and} \quad P_T \sim H^2 \sim V \]

  \[ \sim k^{n_S - 1} \]

  \[ n_S = 1 - 6\epsilon + 2\eta \]

  • **but:** if field excursion sub-Planckian, no measurable gravity waves: [Lyth '97]

  \[ r \equiv \frac{P_T}{P_S} = 16\epsilon \leq 0.003 \left( \frac{50}{N_e} \right)^2 \left( \frac{\Delta \phi}{M_P} \right)^2 \]
general chaotic inflation in supergravity ...
A summary ....
scalar potential \( V(\Phi_i) \) determined by Kahler potential \( K \) and superpotential \( W \):

\[
V(\phi_i) = e^K \cdot \left[ (K_{i\bar{j}})^{-1} D_i W D_{\bar{j}} \bar{W} - 3|W|^2 \right]
\]

with \( D_i W = \partial_i W + W \partial_i K \)

• canonical kinetic terms for:

\[
K = \sum_i \Phi_i \Phi_i \quad \text{or} \quad K = \sum_i \frac{1}{2} (\Phi_i + \Phi_i)^2
\]
• **general chaotic inflation in supergravity** [Kallosh & Linde & Rube ’10]

\[
K = \frac{1}{2} (\bar{\Phi} + \Phi)^2 + \bar{S}S - c_1 (\bar{S}S)^2 - c_2 \bar{S}S(\bar{\Phi} + \Phi)^2
\]

\[
W = S f(\Phi)
\]

• **quartic terms render \(\text{Re}(\bar{\Phi})\), \(S\) massive**

[Kawasaki, Yamaguchi, Yanagida, ’00 ; Lee ’10]

• **general large field potential for inflaton \(\Phi = \text{Im}(\bar{\Phi})\) has shift symmetry & only \(|F_S| > 0\)**

\[
\langle \text{Re} \Phi \rangle = \langle S \rangle = 0 \quad V(\phi) = |f(\phi)|^2
\]
Higgs inflation ...
A summary ...
essential idea: consider a non-minimally coupled scalar field with Higgs potential - called Jordan frame gravity with frame function $\Omega^2(\phi)$

$$S = \int d^4x \sqrt{-g_J} \left[ \Omega^2(\phi) R + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V_J(\phi) \right], \quad V_J(\phi) = \lambda (\phi^2 - v^2)^2$$

one can Weyl transform this into Einstein frame

$$g^J_{\mu\nu} \rightarrow g^E_{\mu\nu} = \Omega^2(\phi) g^J_{\mu\nu}, \quad \Omega^2(\phi) = (1 + \xi \phi^2)$$

$$\Rightarrow S = \int d^4 x \sqrt{-g_E} \left[ R + Z^{-1}(\phi) \partial_\mu \phi \partial^\mu \phi - V_E(\phi) \right]$$

with

$$Z^{-1}(\phi) = \frac{1 + \xi \phi^2 + 6 \xi^2 \phi^2}{(1 + \xi \phi^2)^2}$$
the scalar potential in Einstein frame is

\[ V_E(\phi) = \frac{1}{\Omega^4(\phi)} V_J(\phi) = \frac{\lambda}{4} \left( \phi^2 - v^2 \right)^2 \]

\[ \longrightarrow \quad \frac{\lambda}{4\xi^2} = \text{const.} \]

for \( \phi \gg \frac{1}{\sqrt{\xi}} \gg v \)

\[ Z^{-1}(\phi) \] defines a canonically normalized scalar \( \varphi \)

\[ \frac{d\varphi}{d\phi} = \sqrt{Z^{-1}(\phi)} \quad \Rightarrow \quad \phi \sim \frac{1}{\sqrt{\xi}} e^{\varphi/\sqrt{6}} \]

\[ \Rightarrow \quad V_E(\varphi) \approx \frac{\lambda}{4\xi^2} \left( 1 - 2e^{-\frac{2}{\sqrt{6}} \varphi} \right) \quad \Rightarrow \quad \left\{ \begin{array}{l} n_s = 1 - \frac{2}{N} = 0.967 \\ r = 16\epsilon = \frac{12}{N^2} = 0.003 \end{array} \right. \]
Higgs inflation...

\[ V \]

\[ \phi \sim \varphi \]

\[ \phi \sim \frac{1}{\sqrt{\xi}} e^{\varphi/\sqrt{6}} \]

\[ v \ll \frac{1}{\sqrt{\xi}} \]

equivalent to Starobinsky’s \( R + R^2 \) for large \( \xi \)

[Barvinsky, Shamenshchik & Starobinsky, ’08]
COBE normalization ($\delta \rho / \rho \sim 10^{-5}$ at $N = 60$ e folds) can be satisfied for electro-weak parameters

$$\lambda = \mathcal{O}(1) \quad , \quad \nu \sim 100 \text{ GeV} \quad \text{IF} \quad \xi \sim 5 \times 10^4$$
COBE normalization ($\delta \rho / \rho \sim 10^{-5}$ at $N = 60$ efolks) can be satisfied for electro-weak parameters

$$\lambda = \mathcal{O}(1) \quad , \quad \nu \sim 100 \text{ GeV} \quad \text{IF} \quad \xi \sim 5 \times 10^4$$

quantum consistency still under discussion ...

[Lerner & McDonald ’09, Barbon & Espinosa ’09, Burgess, Lee & Trott ’10, Hertzberg ’10, Giudice & Lee ’10, more ...]

- SM Higgs has couples with would-be Goldstone bosons: gravity induces UV strong coupling scale

$$\mathcal{L}_E \supset (\partial \vec{\phi})^2 + \frac{\xi^2}{M_P^2} (\vec{\phi} \cdot \partial \vec{\phi})^2 + \ldots \quad \Rightarrow \quad \Lambda_{UV} \sim M_P / \xi \ll M_P / \sqrt{\xi} \sim \phi_{end}$$

- transformation to Einstein frame does not help: “unitary gauge” - strong coupling problem shifted into gauge field sector of the SM; exception: singlet
non-minimally coupled inflation in supergravity ...
we can port non-minimal inflation into supergravity by identifying

$$- \int d^4 x d^2 \theta \mathcal{E} \frac{3}{4} (\bar{D}^2 - 8R) e^{-\frac{1}{3} K(\bar{\Phi}, \Phi)} = \int d^4 x d^2 \theta \mathcal{E} \frac{1}{4} (\bar{D}^2 - 8R) \Omega^2 (\bar{\Phi}, \Phi)$$

$$= \int d^4 x \sqrt{-g_J} \Omega^2 (\phi) R + \ldots$$

supersymmetrized frame function

$$\Omega^2 (\bar{\Phi}, \Phi) = -3e^{-\frac{1}{3} K(\bar{\Phi}, \Phi)}$$

canonical kinetic terms in Jordan frame

$$\Rightarrow \Omega^2 (\Phi) = -3 + \bar{\Phi} \Phi + (J(\Phi) + h.c.)$$
2 cases

\[ J(\Phi) = 0 \implies \Omega^2(\phi)R \rightarrow \frac{1}{6}\phi^2 R \quad "conformal coupling" \]

\[ J(\Phi) = -\frac{3}{4} \chi \Phi^2 \implies \Omega^2(\phi)R \rightarrow (1+\xi(\text{Re} \Phi)^2) R \quad "\text{non-minimal coupling}" \]

with \( \chi = \pm \frac{2}{3}(1 + 6\xi) \)

we can rewrite

\[ \Omega^2 = -3 + \Phi\Phi + (J(\Phi) + h.c.) = -3 - \frac{1}{4} \left( 1 + \frac{3}{2} \chi \right) (\Phi - \Phi)^2 + \frac{1}{4} \left( 1 - \frac{3}{2} \chi \right) (\Phi + \Phi)^2 \]

\( \chi = \pm \frac{2}{3} \leftrightarrow \xi = 0 \) recovers minimal coupling and shift symmetry!
we can now go to Einstein frame & forget about Jordan frame by inverting the relation between $\Omega^2$ and $K$

$$K = -3 \ln \left( -\frac{\Omega^2}{3} \right)$$

if we now add back $S$ and the quartic terms in $K$

$$K = -3 \ln \left[ 1 - \frac{1}{3} \Phi \phi - \frac{1}{3} \bar{S} \bar{S} + \frac{1}{4} \chi \left( \Phi^2 + \Phi^2 \right) + c_1 (\bar{S} \bar{S})^2 \right]$$

then with $W = S f(\Phi)$ we are back with generalized non-minimal chaotic inflation in supergravity!
look at 2 examples, get $n_S$ and $r$ as function of $v$, $\xi$

\[
f(\Phi) = m\Phi \quad \rightarrow \quad V_E(\phi) = \frac{m^2 \phi^2}{(1 + \xi \phi^2)^2}
\]

\[
f(\Phi) = \frac{\lambda}{4}(\Phi^2 - v^2) \quad \rightarrow \quad V_E(\phi) = \frac{\lambda^2}{4} \frac{(\phi^2 - v^2)^2}{(1 + \xi \phi^2)^2}
\]

quadratic case:

\[\xi < 0, \quad |\xi| \phi^2 < 1\]

singularity at $\phi = 1/\sqrt{|\xi|}$
quadratic case:

\[ \frac{\xi}{2} \phi^2 R + \frac{m^2}{2} \phi^2 \]

[\text{Linde, Noorbala \\ \\ & AW '11}]

\[ \xi < 0 \]

\[ \xi > 0 \]

\[ \xi = 0 \]
quartic case I, II: $\phi < v$ (I) and $\phi > v$ (II), with $\xi > 0$, like Higgs inflation, but we scan both $\xi$ and $v$
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\( v > 1 \), inflation at \( \phi < v \)

(hill-top, for \( v = O(1) \),

or at \( \phi \leq v \) for \( v \gg 1 \)),

and inflation at \( \phi > v \)

\( v < 1 \), inflation only at large \( \phi > v \)
quartic case 1: $\xi > 0$, $\phi < v$

\[ \frac{\xi}{2} \phi^2 R + \frac{\lambda}{4} (\phi^2 - v^2)^2 \]

$\xi > 0$, $\phi < v \rightarrow \infty$ $\xi = 0$, $v \rightarrow 1$
quartic case II: $\xi > 0, \phi > v$
quartic case III: $\phi < v$ and $v < \phi < 1/\sqrt{|\xi|}$, with

$\xi < 0$, again singularity at $\phi = 1/\sqrt{|\xi|}$

unexpected coincidence - take: $v^2|\xi| < 1$, but $v^2|\xi| \to 1$

then in terms of the correct canonical inflaton:

$$V \simeq \frac{\lambda}{4\xi^2} \left(1 - 2e^{-\frac{2}{\sqrt{6}\tilde{\varphi}}}\right)$$

where $\tilde{\varphi} = \varphi_0(v) - \varphi$ and again

$$\begin{cases} 
    n_s = 1 - \frac{2}{N} = 0.967 \\
    r = 16\epsilon = \frac{12}{N^2} = 0.003 
\end{cases}$$

for $|\xi|v^2 \to 1$
quartic case III: \( \phi < v \) and \( v < \phi < 1/\sqrt{|\xi|} \), with \( \xi < 0 \)
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WMAP 7yr + BAO + \( H_0 \)

\[
\frac{\xi}{2} \phi^2 R + \frac{\lambda}{4} (\phi^2 - v^2) \quad N = 50, 60
\]

\( \xi < 0 \)

\( \xi = 0 \)

\( \phi < v \)
non-minimal chaotic inflation in supergravity ...

[Linde, Noorbala & AW '11]

quartic case III: \( \phi < v \) and \( v < \phi < \frac{1}{\sqrt{|\xi|}} \), with \( \xi < 0 \)

WMAP 7yr + BAO + \( H_0 \)

\[
\frac{\xi}{2} \phi^2 R + \frac{\lambda}{4} (\phi^2 - v^2)^2
\]
\[\xi < 0\]

size of error contours in ~ 3-5 yrs

e.g. Planck

[Image of a graph showing the relationships between various physical parameters]
some results - constraints on $\xi$:

$-7 \times 10^{-4} \lesssim \xi \lesssim 7 \times 10^{-3}$ at 95% confidence level for $m^2 \phi^2$

$\xi \gtrsim 3 \times 10^{-3}$ at 67% confidence level for $\lambda(\phi^2 - v^2)^2$ if $\phi > v$

$\xi \lesssim 6 \times 10^{-3}$ at 95% confidence level for $\lambda(\phi^2 - v^2)^2$ if $\phi < v$
summary ...

• non-minimally coupled inflation (a la Higgs inflation) can be embedded in a general class of chaotic inflation models in supergravity based on a shift symmetry; this includes large-field inflation with the observational boon of gravity waves

• a non-minimal coupling provides a modification to the scalar potential, without explicitly adding terms

• scanning the non-minimal coupling can render a quartic potential consistent with observational data, and cover significant parts of the patch of the \((n_s,r)\)-plane allowed by the WMAP 7-year data