Electromagnetic response of a highly granular hadronic calorimeter

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ABSTRACT: The CALICE collaboration is studying the design of high performance electromagnetic and hadronic calorimeters for future International Linear Collider detectors. For the hadronic calorimeter, one option is a highly granular sampling calorimeter with steel as absorber and scintillator layers as active material. High granularity is obtained by segmenting the scintillator into small tiles individually read out via silicon photomultipliers (SiPM). A prototype has been built, consisting of thirty-eight sensitive layers, segmented into about eight thousand channels. In 2007 the prototype was exposed to positrons and hadrons using the CERN SPS beam, covering a wide range of beam energies and incidence angles. The challenge of cell equalization and calibration of such a large number of channels is best validated using electromagnetic processes. The response of the prototype steel-scintillator calorimeter, including linearity and uniformity, to electrons is investigated and described.

KEYWORDS: Calorimeter; electromagnetic shower; Silicon Photomultiplier.
1. Introduction

A new generation of calorimeters that exploit unprecedented high granularity to reach excellent jet energy resolution is one of the main R&D goals towards the future International Linear Collider (ILC) [1]. The particle flow (PFLOW [2, 3, 4]) algorithm favors single particle separability over...
single particle energy resolution in the attempt to improve the overall jet energy resolution. Typical single hadronic showers in the 10–100 GeV range are best separated in a hadronic calorimeter with cell size of the order of $3 \times 3$ cm$^2$ [4]. In addition, fine longitudinal segmentation is required for PFLOW algorithms to be effective.

The CALICE collaboration [5] is studying several calorimeter designs for experiments at the ILC. With the first generation of prototype detectors new readout technologies have been established for highly granular calorimeters and the stability of these detectors has been demonstrated. Furthermore, a unique set of data has been collected to study hadronic showers at low and medium energies in detail with high resolution longitudinal and transverse sampling.

This paper focuses on the prototype of an analog hadron calorimeter (AHCAL) consisting of 38 layers of highly-segmented scintillator plates sandwiched between 2 cm thick steel plates. Each scintillator tile is an individual calorimeter cell read out by a silicon photo-multiplier (SiPM). The active layers are referred to as modules, and the sum of active and passive material adds up to a total depth of 5.3 nuclear interaction lengths ($\lambda_i$). A more detailed description of the AHCAL prototype structure is given in [6]. The analog SiPM signal is routed to the very-front-end (VFE) electronics where a dedicated ASIC chip [7] is used for multiplexed readout of 18 SiPMs.

A sketch of one AHCAL module, as well as a picture of an open module showing the arrangement of scintillator tiles, are shown in Figure 1.

Tests using particle beams have been conducted in order to evaluate the performance of the highly granular calorimeters built by CALICE. The AHCAL has been installed and tested at the CERN Super Proton Synchrotron (SPS [8]) and at the FNAL Meson Test Beam Facility (MTBF [9]).

In 2007, the whole detector with 38 active layers was commissioned and exposed to muon, positron and pion beams in the energy range 6 GeV to 80 GeV provided by the CERN H6 beam
line. In 2008, the AHCAL together with the ECAL and TCMC were moved to Fermilab to take data in the 1–6 GeV energy range over the course of two years.

Ongoing data analyses will quantify the energy and spatial resolutions of the prototype for hadrons, and will continue the validation and further development of existing models of hadronic showers, e.g. the various GEANT4 [11] physics lists. They will also be important for the experimental validation of the PFA approach [11]. The studies in this article focus on the calibration and performance of the device when exposed to electrons and positrons.

In Section 2, the AHCAL electromagnetic calibration procedure is discussed. The CERN test beam experiment is described in Section 3. Results on calorimeter response to positrons are given in Section 4, followed by uniformity studies in Section 5. Conclusions are reported in Section 6.

2. Calibration procedure

One of the aims of the tests is to establish a reliable and robust calibration chain. This requires measurements with beam particles and with light from the LED monitoring system. The calibration chain is summarized in the following steps:

- equalization of inter-cell response;
- calibration of the SiPM pixel signal and correction for the non-linear response;
- calibration to an energy scale (in GeV) with electromagnetic showers from test beam facilities.

The equalization of all cell responses is performed using minimum ionizing particles (MIPs) as the reference signal. The ADC value for a cell is converted to a number of MIPs by dividing by \( C_i^{\text{MIP}} \), the cell equalization factor. This factor is obtained by taking the most probable value of the response of cell \( i \) to a muon beam. This procedure and its stability are described in more detail in [12]. In addition, the MIP calibration serves to fix the noise threshold; all hits with an energy below 0.5 MIP are rejected.

The number of SiPM pixels, \( A_i[\text{pix}] \), firing for a single cell \( i \) is related to the ADC value for the cell, \( A_i[\text{ADC}] \), and the corresponding SiPM gain, \( C_i^{\text{pix}}[\text{ADC}] \) by \( A_i[\text{pix}] = A_i[\text{ADC}] / C_i^{\text{pix}}[\text{ADC}] \). The procedure to obtain the gain of each individual SiPM is discussed in Section 2.1.

The limited number of SiPM pixels leads to a non-linear response for large signals. These effects are corrected for by a function, \( f_{\text{sat}}(A[\text{pix}]) \), depending on the number of fired pixels \( A[\text{pix}] \). This procedure is discussed in detail in Section 2.2.

Finally, a common calibration factor, \( w \), scales the visible energy of electrons in each cell in units of MIP to the total deposited energy in units of GeV. This factor is determined to be \( w = (42.3 \pm 0.4) \text{ MIP/GeV} \), as discussed in Section 4.

Therefore, in summary, the reconstructed energy of electromagnetic processes in the calorimeter is expressed as

\[
E_{\text{reco}}[\text{GeV}] = \frac{\sum_i E_i[\text{MIP}]}{w[\text{MIP/GeV}]},
\]  

(2.1)
Figure 2. Single photoelectron peak spectrum taken with a SiPM in the AHCAL detector.

where the energy of one single cell with index $i$ is $E_i$. The energy, $E_i$, given in units of MIP is calculated according to

$$E_i [\text{MIP}] = \frac{A_i [\text{ADC}]}{C_{\text{MIP}}} \cdot f_{\text{sat}}(A_i [\text{pix}]).$$  \hspace{1cm} (2.2)

### 2.1 SiPM gain and intercalibration factors

The gain of each individual SiPM is extracted from single photoelectron spectra taken in dedicated runs with low LED light intensity. The SiPM gain, $G_i^{\text{fit}}$, is the distance between two consecutive peaks in the single photoelectron spectrum. A typical gain spectrum is shown in Figure 2. A multi-Gaussian fit is performed to the single photoelectron peaks to determine their average relative distance \[17\]. The mean of each Gaussian function in the multi-Gaussian sum is left as a free parameter. Before fitting, a peak finder routine is used to set each peak mean value to the approximate location of the corresponding photoelectron peak. The width of each Gaussian function is dominated by electronic noise, allowing to neglect statistical contributions which lead to an increase of the peak width with increasing number of fired pixels. Accordingly, all widths are fixed to the electronic noise width in the fit. A fit quality flag, based on an overall chi-square test and the statistics in the first two peaks of the spectrum, is used to reject bad fit results. The uncertainty on the gain determination is mainly due to the fit and is about 2% for fits which pass the quality criteria.

SiPM gain measurements were repeated approximately every eight hours during test beam operation. The SiPM gain varies with temperature and the gain measurements can be used to stabilize the calorimeter response over time. The temperature dependence of the SiPM gain is further discussed in \[12\]. The efficiency of the gain extraction is defined as the number of successful fits in one gain run divided by the number of channels which can be calibrated. About 2% of all SiPMs are considered inactive because of initially bad soldering or subsequent broken connections to the SiPM leads. Additionally, about 0.11% of all channels are connected to a broken LED. All these channels are not accounted for in the total of channels that can be calibrated.
The efficiency of the gain extraction with one measurement run is indicative of the quality of the LED monitoring system, namely the small spread of LED light intensity. Figure 3 shows the efficiency of the gain extraction for a series of runs taken in the first three months of data taking at CERN and in the first three months at FNAL. Initial problems during the system commissioning phase led to low efficiency, but after commissioning a gain extraction efficiency of about 95% per run has been achieved. The gain efficiency was also stable after transportation and throughout the FNAL runs. Combination of several gain runs yields calibration of more than 99% of all cells. The remaining 1% of cells are calibrated with the average of the module to which they belong.

The measurement of SiPM gain is performed with a special mode of the readout chip, with a choice of high pre-amplification gain and short shaping time of 50 ns which improves the signal to noise ratio at the single pixel level. In contrast, the muon calibration and the physics data taking are performed with approximately ten times smaller electronic amplification, to optimally fit the available dynamic range, and about 200 ns shaping time to provide sufficient latency for the beam trigger. The inter-calibration factor, $I_i$, of the chip gain between the calibration mode (CM) and the physics data mode (PM) along with the SiPM gain are used to determine the overall SiPM calibration factor, $C_\text{pix}[\text{ADC}]$, used in Eq. 2.2:

$$C_\text{pix} = \frac{G_i[\text{ADC}(\text{CM})]}{I_i}.$$  \hspace{1cm} (2.3)

The extraction of the inter-calibration coefficients depends on the linear response of the chip in both modes for an overlapping range of input signals. The input signal is provided by the LED system injecting light into the tiles. The amplitude of the signal is varied within the linear range by varying the LED light intensity. The ratio between the linear responses in the two operational modes is the inter-calibration coefficient for one given readout channel. Ideally, this factor should be a simple constant between the two chip readout modes, but it turns out to depend on the SiPM

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**Figure 3.** Gain calibration efficiency (left) and electronics inter-calibration efficiency (right) over the AH-CAL data taking period at CERN in 2007 (red dots) and at FNAL in 2008 (open blue triangles). More than 85.0% of the channels could be monitored for gain and inter-calibration variation during these periods.
signal form due to the different shaping times in the two modes. For longer SiPM signals (larger quenching resistor) the inter-calibration is bigger than for shorter SiPM signals (smaller quenching resistor). The inter-calibration factors between the chip readout modes range between 4 and 13.

As with the gain, the inter-calibration extraction efficiency is influenced by the quality of the LED light distribution system. The inter-calibration coefficient extraction efficiencies during the 2007 and the 2008 data taking periods are plotted in Figure 3 (right). After commissioning was completed, all channels with the exception of the 2% inactive channels and the channels connected to a broken LED, could be inter-calibrated. For the missing inter-calibration values the average of the module to which a SiPM belongs is used instead.

The uncertainty on the inter-calibration coefficient has been estimated from the comparison of several runs and is found to be better than 1%. Temperature and voltage changes do not affect this coefficient since it is mainly driven by the properties of the readout chip.

2.2 SiPM response

Due to the limited number of pixels and the finite pixel recovery time, the SiPM is an intrinsically non-linear device. The SiPMs used in the AHCAL have a total of 1156 pixels with a recovery time between 25 ns and 1 µs, depending on the value of the poly-silicon quenching resistor of each device varying between 0.5 MΩ and 20 MΩ. More details on the SiPM working principle and its properties are given in [13, 14].

Figure 4 (left) shows an ideal response function of a SiPM which correlates the observed number of pixels fired, \( N_{\text{pix}} \), to the true number of photoelectrons generated\(^1\), \( N_{\text{pe}} \). The response of a SiPM can be approximated by the function \( N_{\text{pix}} = N_{\text{tot}} \cdot (1 - e^{-N_{\text{pe}}/N_{\text{tot}}}) \), with \( N_{\text{tot}} \) the maximum number of fired pixels, in this case set to 1156.

Figure 4 (right) sketches the correction function applied in the calibration chain (Eq. 2.2). This function is the inverted residual to linearity of the SiPM response function in Figure 4 (left).

The correction factor is unity for signals of about 30 pixels or 2 MIPs, and increases exponentially up to infinity for signals in saturation. A check is made to ensure that no unreasonable correction factor is applied. In practice, the largest correction factor is \( \sim 5 \).

The saturation correction can be either taken as a common curve for all channels or parametrized from real measurements of each SiPM response. As a first approach the AHCAL data have been corrected with a linear interpolation of measurement points stored in a database during the calorimeter construction phase. Studies to improve this procedure using a single common function are ongoing.

The response curves of each SiPM has been determined on a test bench setup illuminating each SiPM with LED light of variable intensity. For these measurements, the SiPMs were not mounted on a tile. Therefore, all the pixels have been illuminated with light in a homogeneous way. The measurement results for all SiPMs installed in the AHCAL are given in [6]. The spread (RMS) in the maximum number of fired pixels between all the curves is about 20%. SiPMs with a total number of fired pixels larger than 900 have been pre-selected. This ensures not too large variations

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\(^1\)The number of photoelectrons is the true number of photons reaching the SiPM front face multiplied by the SiPM photo-detection efficiency.
in the non-linear response function of each device. The response functions of all SiPMs are stored in a database and are used to linearize the response of each calorimeter cell individually.

Since the AHCAL was the first detector to employ such a large number of SiPMs, a specialized system for monitoring the long-term stability and performance of the photodetectors was required. In order to monitor the SiPM response function in-situ, a versatile UV LED light distribution system was installed in the calorimeter [15]. This system is capable of delivering light to all tiles with

\[ N_{\text{tot}} \] measured with SiPM mounted on a tile to \( N_{\text{tot}}\text{(bare)} \) measured directly with bare SiPMs.

**Figure 4.** The SiPM response function (left) and the saturation correction function, \( f_{\text{sat}} \), applied in the data calibration chain (right).

**Figure 5.** Ratio of maximum number of fired pixels, \( N_{\text{tot}}\text{(mounted)} \), measured with SiPM mounted on a tile to \( N_{\text{tot}}\text{(bare)} \) measured directly with bare SiPMs.
an intensity ranging from a few photoelectrons to the level that saturates the SiPM. Furthermore, the LED system monitors variations of SiPM gain and signal response, both sensitive to temperature and voltage fluctuations. The LED light is distributed to the individual tiles by means of a clear fiber. From that point on, the light follows the same path as light from the scintillation process created by particles interacting in the calorimeter, which includes collection and wavelength shifting in the WLS fiber embedded in the tile before final collection by the SiPM. This enables monitoring of the entire readout chain with the LED system.

Figure 5 shows the ratio of the maximum number of pixels, \(N_{\text{tot}}\), measured with SiPM mounted on a tile, \(N_{\text{tot}}(\text{mounted})\), to that measured with bare SiPMs \(N_{\text{tot}}(\text{bare})\). The individual values are obtained from an exponential fit to the corresponding SiPM curves. The plot shows that the maximum number of pixels in the in-situ setup is on average 80.5% of the value determined in the laboratory setup [16] with a wide distribution (RMS=9%). This is an effect of a geometric mismatch between the WLS fiber and the photodetector. The fiber has a 1 mm diameter while the SiPM active surface area is \(1 \times 1 \text{ mm}^2\); the geometric ratio between areas is 79%, in agreement with the measured value. Therefore, only a fraction of the SiPM surface is illuminated and the laboratory curves are re-scaled by the measured value of 80.5% to correct for this effect before they are used to correct for the SiPM saturation.

The uncertainty of the determination of the saturation point for a single channel is lower than 3%, if the LED light range properly covers the SiPM saturation region, and if this region is measured well below the ADC saturation. Unfortunately, these conditions are true only for a sub-sample of about 73% channels. Due to this, an average correction factor instead of a channel-by-channel correction was used. Furthermore, the SiPM response function, applied as a correction of non-linear detector response, is affected by the SiPM gain uncertainty of 2%, which is discussed in the following section.

3. The test beam experiment

3.1 The experimental setup at CERN

The data discussed in the following were collected in July 2007 at the CERN SPS test beam facility H6b. A sketch of the experimental setup is shown in Figure 6. Apart from the fully equipped AHCAL and a prototype of a tail-catcher and muon tracker (TCMT [18]), the beam installation consists of various trigger and beam monitoring devices. A threshold Čerenkov counter was used to discriminate between electrons and pions. The beam trigger was defined by the coincidence signal of two scintillator counters with \(10 \times 10 \text{ cm}^2\) area, referred to as Sc1 and Sc2 in Figure 6. One scintillator trigger (V1), with an area of \(20 \times 20 \text{ cm}^2\) and analog read out, tagged multi-particle events. Another scintillator with a \(100 \times 100 \text{ cm}^2\) surface and a \(20 \times 20 \text{ cm}^2\) hole in the center (V2), was used to reject the beam halo. Three drift chambers (DC1, DC2 and DC3) were used to monitor the beam and reconstruct tracks. Muons were identified by a scintillator with \(100 \times 100 \text{ cm}^2\) area (Mc1), placed behind the TCMT.

During most of the tests, a silicon tungsten electromagnetic calorimeter [13] was placed in front of the AHCAL, but this was not the case for the results reported here. The AHCAL was placed on a movable stage, which could shift the detector vertically and horizontally. In addition,
Figure 6. Top view of the CERN beam test setup. The plot shows the instrumentation in 2007. The beam enters from the left side. See text for explanations of the components.

the detector can be rotated with respect to the beam direction from an angle of 90° (beam normal to the detector plane) to approximately 60°.

3.2 Monte Carlo simulation

The test beam setup as shown in Figure 6 is simulated with Mokka [20], a GEANT4-based [10] Monte Carlo program, followed by a digitization package simulating the response of the detector and electronics. The particle gun of the simulation is positioned upstream of the Čerenkov detector. The beam position and spread are chosen to match the beam shapes measured in data by the Drift Chamber, DC3. The beam particles are parallel to the beam axis, according to the measurements in the three Drift Chamber detectors. The material upstream of the AHCAL is simulated. The sub-detectors are simulated with different levels of detail, depending on their impact on the physics analysis: material simulation only for the Čerenkov counter, raw energy depositions stored for the trigger counters, and partial electronics simulation for the tracking detectors. For the AHCAL, the simulation gives the raw energy depositions in a virtual scintillator grid of 1 × 1 cm² tile size. The simulation is followed by a digitization procedure, which takes into account

- the realistic detector granularity,
- light cross-talk between neighboring tiles,
- non-linearity and statistical fluctuations on the pixel scale,
- SiPM and readout electronics noise.

The actual geometry of the AHCAL is simulated by summing up the signal yield of 9 (36, 144) virtual cells to obtain those of the actual geometry 3 × 3 (6 × 6, 12 × 12) cm² cells.

Light cross-talk between neighboring cells, due to the imperfect reflective coating of the tile edges, is simulated assuming that from each 3 cm-long tile edge 2.5 % of the scintillator light leaks homogeneously to the neighboring tile. This value is scaled to take into account the fraction of edge shared with the neighbors for cells of different size. The amount of light cross-talk was checked experimentally only for two tiles. The leakage from one tile edge was quantified to be about 2.5 %. No information on the spread of this value between all tiles is given. This value is expected to influence the energy reconstructed and the transverse shower profile. From the comparison of the energy reconstructed in simulation and data, the value of 2.5 % for the light cross-talk on each tile
edge is found to be adequate. A light cross-talk of 1.25\% or 3.75\% leads to a difference in the energy scale between data and Monte Carlo larger than 5\%.

To simulate the non-linear behavior of the photodetectors, the energy deposition is translated from GeV to the number of fired SiPM pixels. For this, an intermediate step converts the response simulated in units of GeV to MIP equivalents. The conversion factor is estimated from the simulation of an 80 GeV muon beam in the AHCAL and is found to be 816 keV/MIP, corresponding to the energy lost by a minimum ionizing particle in the scintillator. The amplitude in units of MIPs is then converted into pixels, using the measured light yield for each individual channel. With this scale the measured SiPM response curves from the test bench are used to simulate the SiPM non-linearity. Where not available, the curve of the next neighboring tile is applied.

If \( N_{\text{pix}} \) is the amplitude in pixels obtained this way and \( N_{\text{max}} \) is the saturation level of the individual channel, statistical effects are accounted for by generating a binomial random number with \( N_{\text{max}} \) repetitions and a probability of \( N_{\text{pix}}/N_{\text{max}} \). The result is treated as the number of pixels firing for this specific event, and is translated back to the MIP scale with the channel-specific light yield.

At this stage, the Monte Carlo signal simulated the response of the AHCAL to the energy deposited by particles in an event. However, both the electronic components and the SiPM dark current induce noise. This noise component is assumed to be completely independent of the physics signal amplitude in each channel and thus is taken into account by adding to each Monte Carlo event a MIP-calibrated random-trigger event from data.

After addition of noise, simulated events are assumed to be equivalent to MIP-calibrated data and are treated the same way for all successive steps. A cell that could not be calibrated in the real detector, either due to an inactive photodetector or to missing calibration values, is also ignored in the simulation. This is about 2\% of the total number of cells in the calorimeter.

4. Calorimeter response to positrons

4.1 Selection of positron events

The analysis presented here is based on positron runs between 10 and 50 GeV. Each energy point has more than 150k recorded beam triggers. All positron runs have been simulated with similar statistics to the corresponding data runs.

Single positron showers are selected for analysis using the beam instrumentation. Although the beam configurations are set to deliver a positron enriched beam, some contamination, mainly from muons, exists. The pion contamination is expected to be negligible, since the tertiary positron beam is produced from a higher-energy mixed beam impinging on a thin (2 \( X_0 \)) lead target which does not result in the production of lower-energy tertiary pions. The muon contamination originates from the in-flight decay of hadrons upstream of the production target, which results in a muon component that passes the momentum selection.

Cells with a signal above threshold are called hits and \( E_{\text{hit}} > 0.5 \text{ MIP} \) is required. To reject empty events that can occur due to random triggers or scattered particles, the number of hits has to be \( N_{\text{hit}} > 65 \). Furthermore, the energy weighted center-of-gravity in the beam direction (\( z \)), defined as \( \langle z \rangle = \sum_i z_i E_i / \sum_i E_i \), has to be \( \langle z \rangle < 390 \text{ mm} \) (about half of the calorimeter depth).
This requirement reduces muons, which deposit their energy equally distributed over the entire calorimeter depth, as opposed to electrons which have a short shower contained in the first half of the calorimeter. It was found that this muon rejection was more efficient than the selection based on the Čerenkov counter, which does not provide electron-muon separation for 30 GeV and above. Particles which interact in the material upstream of the AHCAL are removed by requiring a good track in the drift chambers ($\chi^2$/dof $< 6$), and a MIP-like energy deposition in the multiplicity counter (V1). With these selection criteria, 45% of all recorded events at 10 GeV are accepted. According to Monte Carlo studies 99.9% of all electron events pass the selection criteria, whereas 99.8% of all muon events are rejected. The typical fraction of muons in a run is about 5-10%.

The uncertainty on the mean energy of the beam is reported in [21] to be

$$\frac{\Delta E_{\text{beam}}}{E_{\text{beam}}} = \frac{0.12}{E_{\text{beam}} \text{[GeV]}} \oplus 0.1\%.$$  

(4.1)

The first term is related to hysteresis in the bending magnets while the calibration and the uncertainties on the collimator geometry give the constant term. Since this uncertainty is negligible compared to the detector uncertainties, we assume the beam energy to be fixed in the following. The dispersion of the beam energy can be calculated according to [22] from the settings of the momentum selecting collimators on the beam line and is for all the runs in this analysis below 0.24%.

### 4.2 Linearity

The linearity of the calorimeter response for a large range of incident particle energies is a key feature, which allows for an important test of the calibration chain. Electromagnetic showers offer the most rigorous test since the energy deposited per single tile in an electromagnetic shower is larger than that in a hadronic shower for the same particle energy. Figure 7 shows the hit energy spectrum of a 40 GeV positron shower compared to the spectra of 40 GeV and 80 GeV pion showers. The positron shower clearly has more hits with high energy deposition, even when the total particle energy is only half that of the pion.

A set of runs of positrons with normal incidence to the calorimeter and impinging at the center of calorimeter front face is analyzed. To minimize the influence of noise, the energy is summed up in a cylinder around the shower axis. This cylinder, sketched in Figure 8 (left), has a radius of 5 Molière radii ($r = 5 R_M$, with $R_M = 2.47 \text{ cm} [6]$), which ensures a lateral containment of more than 99% of the shower energy.

The length $L$ of the cylinder is chosen to contain the whole shower energy. From the simulation of a 50 GeV electron shower $L$ is fixed to 20 layers. Figure 8 (right) shows the final reconstructed spectra for positron runs in the energy range 10 to 50 GeV. The positrons are normally incident on the calorimeter front face, with a distribution centered in the same calorimeter cell for each run. The distribution is fit with a Gaussian function in the range $\pm 2\sigma$. The position of the peak is taken as the mean energy response, $E_{\text{mean}}$, measured in units of MIP.

The reconstructed energy of a 10 GeV positron shower is compared to the digitized energy from a Monte Carlo simulation in Figure 8. The agreement between data and simulation is satisfactory.
Figure 7. Hit energy spectrum for 40 GeV positron showers compared to that of 40 GeV and 80 GeV pion showers from a GEANT4 simulation.

Figure 8. The shower energy is summed up in a cylinder (left); see text for details. Spectra of the energy sum for positron data with energy between 10 GeV and 45 GeV (right). For each spectrum the mean energy response in units of MIP, $E_{\text{mean}}$, is obtained with a Gaussian fit in the range $\pm 2\sigma$.

The statistical uncertainties on the mean energy deposition are negligible. The main source of systematic uncertainties is 2% on the MIP scale calibration. The uncertainty of 2% on the SiPM gain determination, resulting from the fit stability and the uncertainty on the determination of the SiPM saturation level both affect the correction of the SiPM non-linear response. For the saturation level a common re-scaling factor is applied to all SiPM curves determined in the laboratory setup. The rescaling is needed to account for the partial illumination of the SiPMs from the WLS fiber as discussed in Section 2.2. As shown in Figure 5, the ratio between the in-situ measured SiPM saturation level and the test-bench determined value has a wide distribution. Since a common factor of 80.5% is used to rescale all SiPM response curves, an uncertainty of 11.3% on
Figure 9. Reconstructed energy of a 10 GeV positrons for data (black dots) and for Monte Carlo (filled yellow histogram), as well as a Gaussian fit to data (blue line).

this value is assumed, which represents the spread of all measured values as taken from Figure 5. To account for this uncertainty in an uncorrelated way for all SiPMs, 100 experiments have been performed assigning different rescaling coefficients for each channel, generated randomly with a Gaussian distribution centered at 0.80 and with a sigma of 0.09. For each experiment the energy in the calorimeter is reconstructed, using the set of curves rescaled by these randomly generated coefficients to correct the non-linear SiPM response. Finally, the one standard deviation spread of the 100 reconstructed energies from these simulated experiments is taken as the systematic uncertainty for the reconstructed energy. All of the above listed systematic uncertainties are uncorrelated and thus added in quadrature. The total systematic error ranges from 0.2 GeV (2%) at 10 GeV to 1.7 GeV (3.4%) at 50 GeV.

The reconstructed energy in GeV is obtained as $E_{\text{reco}} = E_{\text{mean}}/w$, where $w$ is the electromagnetic energy scale factor (MIP-to-GeV). The scale factor is determined with a linear fit from zero to 50 GeV to the distribution $E_{\text{mean}}[\text{MIP}]$ versus $E_{\text{beam}}[\text{GeV}]$. The resulting values for data and Monte Carlo are $w_{\text{data}} = (42.3 \pm 0.4) \text{ MIP/GeV}$ and $w_{\text{MC}} = (42.0 \pm 0.4) \text{ MIP/GeV}$, respectively. Within the uncertainties, the scale factors are in good agreement.

The linearity of the AHCAL response to positrons is shown in Figure 10. A comparison of the data before and after correction for the SiPM non-linear response indicates the magnitude of this correction, which does not exceed 10% even at 50 GeV positron energy. The values shown in Figure 11 are reported in Table 1.

The residuals for data and Monte Carlo are presented in Figure 11. Here, the green band indicates the quadratic sum of the energy dependent systematic uncertainties. In Table 1 the con-
Figure 10. Linearity of the AHCAL response to positrons in the range 10–50 GeV. The blue dotted line shows the exact linearity. Black dots correspond to data corrected for SiPM non-linear response, blue triangles show the data before this correction, and the open red triangles show the simulation. The green band indicates the systematic uncertainty as quoted in Table 1, $\Delta E^{\text{tot}}$ [GeV].

Table 1. AHCAL energy reconstructed in data and MC (in units of GeV) for various positron beam energies. The table reports the values plotted in Figure 10. The systematic uncertainties for data are detailed in their percentage values. The total absolute error $\Delta E^{\text{tot}}$ is the sum in quadrature of the uncertainties on the MIP, on the SiPM gain and on the saturation point determination.
Figure 11. Residual to a fit of the data and Monte Carlo points presented in Figure 10 using, a linear function \( y = ax \) (left), a line fit \( y = ax + b \) (right), in the range 10–50 GeV. Black dots correspond to data, and open red triangles to simulation. The green band indicates the sum in quadrature of the energy dependent systematic uncertainties, \( \delta_{E_{\text{gain}}}^{\text{Gain}} \) and \( \delta_{E_{\text{sat}}}^{\text{sat}} \) in Table 1.

Data in the range 10–50 GeV. A value of \( b = -10.3 \pm 7.4 \) MeV is found for the Monte Carlo offset. Once this offset is removed the Monte Carlo linearity is better than 0.5% over the whole range, as shown in the right plot of Figure 11.

The deviation from linearity (Fig. 11 left) in data is less than 1% in the range 10 to 30 GeV and the maximum deviation is about 3% at 50 GeV. The remaining non-linearity at high energies hints at problems with the rescaling of the saturation curves, as described in Section 2.2. This behavior is not sufficiently reproduced in the Monte Carlo digitization, where the same curve is used to simulate saturation as is used to correct for it.

The impact of the saturation correction is better seen in Figure 12 where the energy per hit is shown with and without the correction factor \( f_{\text{sat}} \) applied, for 30 GeV electromagnetic showers. Whereas the correction is negligible for low signal amplitudes, it becomes significant at larger amplitudes, resulting in a strong correction for the tail of the distribution. The maximum energy deposited in one cell for a 30 GeV electromagnetic shower is \( \sim 230 \) MIPs corresponding to about 3450 pixels (assuming a light yield, \( \text{LY} = 15 \) pixel/MIP). For this amplitude the correction factor is \( f_{\text{sat}}(A_i) \sim 3.1 \). The remaining miss-match between data and Monte Carlo around 100–200 MIPs is an effect of the non-perfect correction of the non-linear SiPM response. This imperfect correction affects only a small fraction of the total energy; the hits above 50 MIPs contribute only 0.5% (4%) of the total energy at 10 GeV (40 GeV).

4.3 Electromagnetic energy resolution

The energy resolution is a principal figure of merit in calorimetry and is estimated as the width divided by the mean of a Gaussian fit to the energy sum within \( \pm 2\sigma \) of the mean of an initial fit over the full range. The resolution achieved with the AHCAL is plotted as a function of the beam
Figure 12. Hit energy spectrum for 30 GeV positron showers in the AHCAL. Open circles (black dots) show the data before (after) correction for the non-linear response of the SiPM. The insert shows the hit distribution in a linear scale.

energy in Figure 13. The values shown in this figure are reported in Table B. Fitting the AHCAL energy resolution in a range of $\pm 2\sigma$, with

$$ \frac{\sigma_E}{E} = \frac{a}{\sqrt{E}} \oplus b \oplus \frac{c}{E} \quad (4.2) $$

results in a stochastic term of $a = (21.9 \pm 1.4)\%$, whereas the constant term is $b = (1.0 \pm 1.0)\%$. The noise term of $c = 58.0\ MeV$ is extracted from the spread (RMS) of the random trigger event distribution and kept constant during the fitting procedure. The energy resolution agrees well with that of an earlier prototype (Minical) with 108 channels and of the same sampling [24], that was tested in the energy range between 1 and 6 GeV and reached a resolution with a stochastic term $a = (20.7 \pm 0.7)\%$ and a constant term $b = (2.6 \pm 1.3)\%$.

The energy resolution of the simulation is found to have a stochastic term of $a = (21.5 \pm 1.4)\%$, a constant term of $b = (0.7 \pm 1.5)\%$ and again a fixed noise term of $c = 58.0\ MeV$. Within the fit uncertainty, the stochastic terms of data and simulation are in good agreement. The noise term is fixed to the same value as for data since the noise in the simulation is artificially added from random trigger data events. The constant term $b$, representing calibration uncertainties and non-linearities, should be zero in the simulation, since the same curves are both in the simulation of the non-linear SiPM response and in its correction.

4.4 Shower profiles

The longitudinal profile of a shower induced by a particle with incident energy $E$ in GeV traversing
Figure 13. Energy resolution of the AHCAL for positrons (black circles). The resolution agrees with that of a previous prototype (black triangles) with the same sampling structure. The errors are the quadratic sum of statistics and systematic uncertainties.

Table 2. AHCAL energy resolution in data and MC for various positron beam energies. The table reports the values plotted in Figure 13. The listed uncertainties include statistical uncertainties and systematic uncertainties added in quadrature.

\[
\begin{array}{c|cc|cc}
E_{\text{beam}} [\text{GeV}] & \sigma_{E}/E [%] & \text{Uncertainty} [%] & \sigma_{E}/E [%] & \text{Uncertainty} [%] \\
\hline
10 & 7.11 & 0.47 & 6.90 & 0.49 \\
15 & 5.83 & 0.36 & 5.45 & 0.38 \\
20 & 4.95 & 0.32 & 4.90 & 0.34 \\
30 & 3.97 & 0.29 & 4.00 & 0.31 \\
40 & 3.54 & 0.26 & 3.51 & 0.27 \\
50 & 3.41 & 0.25 & 3.07 & 0.26 \\
\end{array}
\]

A matter depth \( t \) can be described as

\[
f(t) = \frac{dE}{dt} = at^\omega \cdot e^{-bt},
\]

where the parameter \( a \) is an overall normalization, and the parameters \( \omega \) and \( b \) are energy and material-dependent. The first term represents the fast shower rise, in which particle multiplication is ongoing, and the second term parametrizes the exponential shower decay. Given this parametriza-
Figure 14. Longitudinal profile of a 10 GeV positron shower in units of $X_0$ (left) and scaling of the shower maximum as a function of the incident energy (right). The reconstructed energy (left plot) is shown for data (solid points), simulation (yellow-shaded area) and a fit to the data using Eq. 4.2 (black line). The bottom insert shows the data/Monte Carlo comparison. The shower maximum (right plot) is shown for data (solid points), simulation (red open triangles) and the theory expectation given in Eq. 4.3 (blue solid line).

The mean longitudinal profile of a 10 GeV positron shower is shown in the left plot of Figure 14. Due to the high longitudinal segmentation of the AHCAL, the shower rise, maximum and decay are clearly visible. Data and simulation are in qualitatively good agreement. To quantify this agreement, the profiles at each recorded beam energy are fitted with Eq. 4.3 and the maximum shower depth calculated as $t_{\text{max}} = \omega / b$. The development of the shower maximum as a function of the beam energy is shown in the right plot of Figure 14. The error bars show the uncertainty from the fits. The extracted shower maxima of both data and simulation are in good agreement with the theoretical behavior for a pure Fe calorimeter with a critical energy, from [23], of $\varepsilon_c = 21.04 \text{ MeV}$, given in Eq. 4.4.

The transverse shower profile of a 15 GeV positron shower is shown in Figure 15 together with a simulation. The radius, $\rho$, is calculated with respect to the track of the incoming particle extrapolated from the tracking system to the AHCAL front face. Therefore, the radius is defined as $\rho_i^2 = (x_i - x_{\text{track}})^2 + (y_i - y_{\text{track}})^2$, where $(x_i, y_i)$ are the coordinates of the calorimeter cell with signal above threshold. The energy deposited in a calorimeter cell is normally assigned to the center of the cell. For the radial profile studies it is redistributed uniformly in bins of $1 \text{ mm}^2$. 

\[ t_{\text{max}} = \left[ \ln \frac{E}{\varepsilon_c} - 0.5 \right] \]
Figure 15. Transverse profile of a 15 GeV positron shower. The energy density is shown in 10 mm wide concentric rings centered around the shower axis.

before being assigned to one annular bin of inner radius $\rho$. In this way the energy deposited in one calorimeter cell can be shared between two adjacent annular bins. Proper normalization accounts for the fraction of the calorimeter cell area covered by each annular bin. The data indicate a broader shower than expected from simulation. The calculated mean shower radius ($\langle R \rangle = \frac{\sum E_i r_i}{\sum E_i}$) for 15 GeV showers in data is about 9% larger than the simulated one. The energy dependence of $\langle R \rangle$ is shown on the right plot of Figure 16 (left). The difference is almost energy independent. For completeness also the comparison of the RMS ($\sqrt{\langle R^2 \rangle - \langle R \rangle^2}$) of the shower radius distribution is shown in Figure 16 (right). An energy dependent disagreement of data and Monte Carlo is observed for this variable which increases to a maximum of 7% for 50 GeV. Several studies have been performed to find the cause of this effect including effects of noisy and inactive cells, different beam shape, influence of the light cross-talk between tiles, misalignment between calorimeter and tracking system and of calorimeter layers. The broader shower in data is still not

Figure 16. Mean (left) and RMS (right) of the transverse shower distribution as a function of beam energy. Black dots correspond to data and red open triangles correspond to Monte Carlo.
Figure 17. Schematic view of tile positions in an AHCAL scintillator plane used for the uniformity test (left) and uniformity of the calorimeter response for various positions of incident beam with respect to the detector (right). Tile position eight is approximately in the center of each calorimeter layer. The dashed lines show the systematic uncertainties. Statistical errors are negligible.

understood and further studies of asymmetric light collection on the tile, influence of varying dead space between tiles due to varying thickness of reflector coating, etc., will follow to investigate the discrepancy. For the purpose of the validation of the calibration procedure the current level of agreement is acceptable, though this mismatch will have to be taken into account when comparing hadronic shower shapes. Furthermore, hadronic showers have a much smaller energy density than electromagnetic showers; therefore, any local effects, (i.e. the impact of dead areas or misalignment between layers), are strongly amplified in electromagnetic showers, while the influence is expected to be much less pronounced in hadronic showers.

5. Uniformity studies

5.1 Uniformity of the calorimeter response

The uniformity of the AHCAL response is explored by shifting the AHCAL to different positions with respect to the beam axis, at normal incidence angle. This procedure is visualized in the left part of Figure 17. Each square in the sketch represents one scintillating tile of $3 \times 3 \text{ cm}^2$ and beam events with a track pointing to a $1 \times 1 \text{ cm}^2$ region centered on each tile in turn were selected. The uniformity of the calorimeter response at the 15 different positions has been tested. For this study 10 GeV positron runs are analyzed, where the movable stage was used to displace the calorimeter in the $x$-$y$ position with respect to the beam-line ($z$-axis).

As shown in the right plot of Figure 17, when excluding position 10, the uniformity of the calorimeter response is better than 2.1%. The 10% deviation between reconstructed and beam energy at position 10 is due to an inactive cell at the shower maximum, which is not corrected in the calibration.
Figure 18. Schematic view of the AHCAL rotated with respect to the beam (left) and reconstructed energy of 10 GeV positrons normalized to the average versus angle of incidence (right). To improve legibility, the data (solid points) and the simulation (red triangles) are slightly shifted in opposite directions on the abscissa. The systematic uncertainty is shown by dash-dotted lines. Additionally, the spread of all measurements performed at one inclination angle are shown as an error for each point.

5.2 Angular dependence of the calorimeter response

The movable stage carrying the AHCAL was used to collect positron data at incident angles of $90^\circ$, $80^\circ$, $70^\circ$ and $60^\circ$. The rotation and staggering of the AHCAL are sketched in the left plot of Figure 18, where the beam is entering from the top. In the rotated configuration, the modules were staggered to ensure the highly granular core of $3 \times 3 \text{ cm}^2$ was aligned with, and hence sampled, the shower core.

An average of the normalized reconstructed energy for several 10 GeV positron runs with different impact points on the calorimeter front surface is plotted in Figure 18 as a function of the incidence angle $\theta$. The spread (RMS) between the various analyzed runs per inclination angle is used as the systematic uncertainty. This spread is smaller than the calibration systematic uncertainty in the calorimeter, shown in the plot as an error band around the ratio of one. Showers at various inclination angles only partially share the same calorimeter cells, therefore the full systematic uncertainty from calibration is an overestimate of the real error, but the spread between measurements performed at one inclination angle is an underestimate. Taking this into account, the increase in the reconstructed energy of data between $60^\circ$ and $90^\circ$ is not significant. A more precise analysis would require more data at different angles which are not available at present.

5.3 Influence of cell structure

The scintillating tiles used in the AHCAL have a WLS fiber embedded in a groove, a SiPM inserted into a small groove on one end of this fiber, and a mirror in a groove on the other end. This structure is visible in the picture of a $3 \times 3 \text{ cm}^2$ tile in Figure 19 where the SiPM is located in the lower left-
corner, the mirror is placed in the diagonal corner, and the WLS fiber is embedded in a quarter circle.

Since electromagnetic showers have a short transverse extension, the impact of the cell structure slightly reduces the energy resolution of the calorimeter. To study this effect, we take advantage of the drift chambers that were present in the beam line. They are used to reconstruct the track of the incoming particle. This track is then extrapolated to the front face of the AHCAL. The shower energy (energy summed over the entire calorimeter) for 10 GeV positrons with this impact position, normalized to the shower energy averaged over all impact positions, is plotted in Figure 19.

As shown in the figure, the measured energy drops slightly over the area of the WLS fiber. A particle with a trajectory intersecting the SiPM (in the lower left corner of the plot) or the reflecting mirror at the end of the WLS fiber (in the upper right corner of the plot) shows a significant loss of response with respect to the tile average by about 8% and 4% respectively. At the position of the WLS fiber the tile response is about 2% lower than average. The drop at the other two corners of the tile in this study reflects the energy loss associated with the SiPMs located in the neighboring tiles as the observable used is the energy summed over the entire calorimeter. Measurements of single tile uniformity using a collimated source have been performed and are reported in [25, 26]. These measurements confirm a lower response of electromagnetic showers hitting the SiPMs or the reflecting mirrors. Though this large degradation (8% at the locations of the SiPMs) is quite unrealistic in a collider detector, where the particles are always traversing the calorimeter under an angle. In this case the tile response non-uniformity averages out with no influence on the energy resolution. Furthermore, electromagnetic showers have a short lateral extension. For pion showers, which are much wider, the effect has not been observed in data. The effects of gaps between the calorimeter tiles, as well as the non-uniform response of the tiles, in view of the impact on the

Figure 19. Picture of the scintillating tile (left). Effect of the AHCAL scintillator tile structure on the energy measurement (summed over the entire calorimeter) for 10 GeV electromagnetic showers (right).
energy resolution, have been studied using Monte Carlo events. The results are reported in [27] and show that these type of effects do not have a significant influence on the measurement of hadron showers.

6. Conclusions

The response of the CALICE analog hadron calorimeter to positrons was measured for energies between 10 and 50 GeV, using data recorded at CERN in summer 2007. The calorimeter response is linear to better than 3%. A better saturation correction would improve the linearity, and for future developments a larger dynamic range is desirable. This study is ongoing, but the effect on pion energy reconstruction will be negligible due to the much smaller energy per hit in a hadronic shower compared to an electromagnetic shower. The energy resolution for positrons is found to have a stochastic term of \((21.9 \pm 1.4) \%/\sqrt{E \text{[GeV]}}\), and a constant term of about 1%. Good agreement between data and simulation validates the simulation of the various detector characteristics.

Systematic studies are performed to investigate the quality of the calibration in as many calorimeter cells as possible. The uniformity of the calorimeter response to electromagnetic showers is studied with beams at different impact points and different incident angles. The results are consistent with no angular and spatial dependence within the quoted systematic uncertainty on the calibration procedure.

The high segmentation of the AHCAL is well-suited for studying the longitudinal shower development with high accuracy and for determining the shower maximum. The point of maximum energy deposition along the shower propagation axis is located between \(5.5 X_0\) and \(7 X_0\) for the range of particle energies used, consistent with simulation and theoretical prediction.

The transverse shower spread is more difficult to measure because it is strongly affected by uncertainties in the beam profile, in the variation of light cross-talk between tiles, and in the misalignment of calorimeter layers. Currently, the data indicate a broader shower than expected from simulation. However, the level of agreement is acceptable for the validation of the calibration procedure if one considers that the effect on hadronic showers will be less important due to the lower energy density of hadronic showers.

This analysis provided confidence that the detector performance and simulation are sufficiently understood to pursue the investigation of hadronic showers.

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