Neutrino Dark Energy – Revisiting the Stability Issue

(O. Bjælde, A.W. Brookfield, C. van de Bruck, S. Hannestad, D. Mota, L. Schrempp, D. Tocchini-Valentini)

astro-ph/0705.2018

Lily Schrempp

DESY Hamburg
Outline

1 Motivation

2 Neutrino Dark Energy–The Mass Varying Neutrino (MaVaN) Scenario

3 The Stability Issue

4 Summary
What is the nature of Dark Energy?

Neutrino Dark Energy (Mass Varying Neutrinos)

[Fardon, Nelson, Weiner '03]

Idea of varying neutrino masses in other contexts

[Kawasaki, Murayama, Yanagida '92, Stephenson et al '97]

- Attractive scalar force between Big Bang relic neutrinos (the analog of the Cosmic Microwave Background (CMB) photons) → smooth background, can form a negative pressure fluid
- → acts as a form of Dark Energy → accelerated expansion
- → neutrino mass $m_\nu$ becomes a function of neutrino energy density $\rho_\nu(z)$, which evolves on cosmological time scales (here parametrized in terms of cosmic redshift $z$)

→ Neutrino mass not constant, but promoted to a dynamical quantity $m_\nu(z)$!
Mass Varying Neutrino (MaVaN) Scenario

The non-SM neutrino interaction mediated by a scalar field

- Introduce a light scalar field $\phi$ with mass $m_{\phi,0} \gg H_0 \sim 10^{-33}\text{eV}$
- Introduce a coupling between neutrinos $\nu$ and $\phi$
- Consider class of models with
  \[
  \mathcal{L} \supset \mathcal{L}_\phi + \mathcal{L}_{\nu\text{kin}} + \mathcal{L}_{\nu\text{mass}}, \quad \text{where}
  \]
  \[
  \mathcal{L}_\phi = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V_\phi(\phi)
  \]
  \[
  \mathcal{L}_{\nu\text{mass}} = -m_\nu(\phi) \bar{\nu} \nu + h.c.
  \]

- Neutrino mass $m_\nu(\phi)$ is generated from the VEV of $\phi$ and becomes linked to its dynamics
- Neutrinos interact through a new non-SM force
Dynamics of the MaVaN Scenario

Complex interplay between $\nu$ and $\phi$

- Neutrino energy density $\rho_\nu$ and pressure $p_\nu$ are functions of neutrino mass $m_\nu(\phi) \rightarrow \rho_\nu(m_\nu(\phi)), p_\nu(m_\nu(\phi))$

- Neutrinos can stabilize $\phi$ by contributing to its effective potential $V_{\text{eff}}(\phi) = [\rho_\nu(m_\nu(\phi)) - 3p_\nu(m_\nu(\phi))] + V_\phi(\phi)$

- Evolution of $\phi$ governed by modified Klein-Gordon equation

$$\ddot{\phi} + 2H\dot{\phi} + a^2 V'_\phi = -a^2 \frac{d\log m_\nu}{d\phi} (\rho_\nu - 3p_\nu), \text{ with } (\dot{\phi} = d/d\phi)$$

- Extra source term on RHS accounts for energy exchange between $\phi$ and neutrinos

- As long as neutrinos relativistic, coupling term suppressed 
  $(\rho_\nu - 3p_\nu \sim 0)$
Dynamics of the MaVaN Scenario

Adiabatic evolution in the non-relativistic regime

- Consider late-time dynamics of MaVaNs in the non-relativistic limit $m_{\nu} \gg T_{\nu} \rightarrow \rho_{\nu} \sim 0$, $\rho_{\nu} = m_{\nu} n_{\nu}$ ($n_{\nu} \equiv$ neutrino number density)

\[ \longrightarrow V_{\text{eff}}(\phi) = \rho_{\nu}(m_{\nu}(\phi)) + V_{\phi}(\phi) \]

- In the limit $H^2 \ll V''_{\text{eff}}(\phi) = m_{\phi}^2$ adiabatic solution to EOM of $\phi$ apply

\[ (\text{EOM: } \ddot{\phi} + 2H \dot{\phi} + a^2 V'_{\text{eff}}(\phi, z) = 0, \text{ can safely neglect effects of kinetic energy terms}) \]

- $\phi$ instantaneously tracks the minimum of its effective potential $V_{\text{eff}}$:

\[ V'_{\text{eff}}(\phi, z) = V'_{\phi}(\phi) + \rho'_{\nu}(m_{\nu}(\phi), z) = 0 \text{ with } (\text{'} = \partial / \partial \phi) \]

\[ m'_{\nu}(\phi)n_{\nu}(z) \]

Crucial effect: $n_{\nu}(z)$ is diluted by expansion $\rightarrow \phi$ varies on cosmological time scales (slowly)
Dynamics of the MaVaN Scenario

Neutrino mass varies

- $m_\nu(\phi) = m_\nu(\phi, z)$, $\rightarrow$

  $V_{\text{eff}}(\phi, z) = V_{\text{eff}}(m_\nu(\phi), z)$

  $= \rho_\nu(m_\nu(\phi), z) + V_\phi(m_\nu(\phi))$

- $\rightarrow \frac{\partial V_{\text{eff}}(\phi)}{\partial \phi} = \frac{\partial m_\nu}{\partial \phi} \frac{\partial V_{\text{eff}}(m_\nu)}{\partial m_\nu} |_{m_\nu=m_\nu(\phi)} = 0$

- Neutrino mass variation determined from

  $\frac{\partial V_{\text{eff}}(m_\nu, z)}{\partial m_\nu} = 0 = n_\nu(z) + \frac{\partial V_\phi(m_\nu)}{\partial m_\nu}$

→ Combined scalar-neutrino fluid has dynamical Eq. of State $\omega(z) \equiv \frac{\rho_\text{DE}(z)}{\rho_\text{DE}(z)}$

$\omega(z) + 1 = -\frac{m_\nu(z) V_\phi'(m_\nu(z))}{m_\nu'(z) V_{\text{eff}}(m_\nu(z))}$
Instabilities? Formation of dense neutrino bound states?

‘In the non-relativistic neutrino regime any realistic MaVaN scenario with $m_\phi^2 \gg H^2 > 0$ is characterized by a negative sound speed squared $c_s^2 < 0$ and thus becomes unstable to hydrodynamic perturbations…with the likely outcome of the formation of non-linear structures in the neutrino density (‘neutrino nuggets’)’

[Afshordi, Kohri, Zaldarriaga ’05]

Note: Outcome of neutrino instability is an inherently non-linear process …but if ’nuggets’ really form, they redshifts similar to cold dark matter with $\omega \sim 0 \sim -1 \rightarrow$ no acceleration (Quintessence? Cosmological Constant?)

Reconsider stability issue in framework of linear perturbation theory

Aim: Identification of condition for stabilization of the neutrino density contrast
Instabilities

- Neutrino instabilities driven by attractive force mediated by $\phi$
- Phenomenon similar to gravitational instabilities of CDM
- Good observational evidence, at early times universe homogeneous and isotropic on all scales
- Apart from small primeval perturbations $\delta \rho_i$ in densities $\rho_i$ of each individual particle $i$

$$\rho_i(x, \tau) = \underbrace{\rho_i(\tau)}_{\text{mean background density}} + \underbrace{\delta \rho_i(x, \tau)}_{\text{small perturbation}}, \quad \delta_i(x, \tau) \equiv \frac{\delta \rho_i(x, \tau)}{\rho_i(\tau)}$$

- → grew by gravity into observable structure on scales of galaxies and clusters of galaxies
- Small amplitudes $|\delta \rho_i(x, \tau)| \ll \rho_i(\tau) \leftrightarrow |\delta_i(x, \tau)| \ll 1$ → growth of fluctuations can be solved from linear perturbation theory
The Stability Issue in Models of Neutrino Dark Energy

Gravitational instability in Newtonian theory

- Assume static (non-expanding) universe, consider perfect fluid, density $\rho$, pressure $p$, velocity $v$ (Continuity eq. + Euler eq. + Newtonian gravity)

- Add small perturbations $\delta p$, $\delta \rho$, $\delta v$ and linearise $\rightarrow$ for $k^{th}$ Fourier component

$$\ddot{\delta} + \left( c_s^2 k^2 - 4\pi G \rho \right) \delta = 0, \text{ where } \omega = \sqrt{c_s^2 k^2 - 4\pi G \rho}$$

- Perturbations adiabatic ($c_s^2 = \frac{\dot{p}}{\dot{\rho}}$ adiabatic sound speed squared)

- $\rightarrow$ sign of $\omega^2$ (which depends on $c_s^2$) determines perturbation evolution

- change of sign of $\omega^2$ at critical value $k_{\text{Jeans}} = \sqrt{4\pi G \rho / c_s^2}$

- for $k < k_{\text{Jeans}}$: $\omega^2 < 0$ (gravity overcomes pressure) $\rightarrow$ $\delta \propto e^{\pm|\omega|t}$, growing solution

- for $k > k_{\text{Jeans}}$: $\omega^2 > 0$ $\rightarrow$ $\delta \propto e^{\pm i\omega t}$, no growth but acoustic oscillations

$\rightarrow$ sound speed squared $c_s^2$ governs evolution of density contrast $\delta_k$
Make contact with MaVaN instabilities

- MaVaNs interact through gravity and the force mediated by $\phi$ (both attractive), $4\pi G \rightarrow 4\pi G_{\text{eff}}(\beta(\phi))$

- Sound speed squared? For a general fluid $i$ (with $'c_g'$ general, $'c_s'$ adiabatic, $'\Gamma_i'$ intrinsic entropy perturbation)

$$w_i \Gamma_i = (c_{gi}^2 - c_{si}^2) \delta_i,$$

$$c_g^2 = \frac{\delta p_i}{\delta \rho_i}, \quad c_s^2 = \frac{\dot{\rho}_i}{\rho_i}, \quad \delta_i(x, \tau) \equiv \frac{\delta \rho_i(x, \tau)}{\rho_i(\tau)}$$

- Dissipative processes invoke entropy perturbations ($\Gamma_i \neq 0$) [Hu '98, Bean, Dore '03, ...]
- For MaVaNs? Depends on scales/regimes one considers!
- Relativistic neutrinos: free-streaming and relativistic pressure support $\rightarrow$ no growth (on all scales)
- Non-relativistic neutrinos: $\rho_\nu \sim 0 \rightarrow$ possible growth
- $m^{-1}_\phi$ sets physical length scales $a/k$ as of which gradient terms become unimportant ($\Gamma_\phi \sim 0$) (for small deviations away from its minimum, $\phi$ re-adjusts to new minimum on a time scale $m^{-1}_\phi \ll H^{-1}$) [Afshordi, Kohri, Zaldarriaga '05, Kaplinghat, Rajaraman '06]

On scales $m^{-1}_\phi < a/k < H^{-1}$ MaVaN perturbations adiabatic $\rightarrow$ $\nu - \phi$ system can be treated as unified fluid with $\Gamma_{DE} = 0$ and $c_{s}^2 = \frac{\dot{\rho}_{DE}}{\rho_{DE}} = \omega - \frac{\dot{\omega}}{3H(1+\omega)}$
Evolution of scalar field perturbations $\delta\phi$

where $\phi(x, \tau) = \phi(\tau) + \delta\phi(x, \tau)$

- Perturbed Klein-Gordon equation in the non-relativistic neutrino regime ($\rightarrow$ neglect terms $\propto \rho_\nu, \omega_\nu, c_\nu^2$ and $\dot{\phi}$)

$$\ddot{\delta}\phi + 2H\dot{\delta}\phi + \left[k^2 + a^2(V''_{\phi} + \beta'\rho_\nu)\right]\delta\phi = -a^2\beta\delta_\nu\rho_\nu$$

- Solution of homogenous equation is oscillating with decaying amplitude

- Particular solution given by forcing term on RHS

$$\delta\phi = -\frac{a^2\beta\rho_\nu\delta_\nu}{a^2(V''_{\phi} + \beta'\rho_\nu) + k^2}$$

[cf. eg. Amendola' 03, Koivisto' 05]
Equation of motion of the neutrino density contrast

\[ \delta_\nu = \frac{\delta \rho_\nu}{\rho_\nu} \]

- Energy-momentum conservation equations for the coupled neutrinos

\[ T^{\mu}_{\gamma;\mu} = \frac{d \log m_\nu}{d \phi} \phi, \gamma \left( T^\alpha_{\beta}, \right) \]

where \( T_{\mu\gamma} \) is the energy-momentum tensor

- → consider perturbed part in the non-relativistic neutrino regime

(where instabilities can possibly grow)

- → use

\[ \delta \phi = -\frac{a^2 \beta \rho_\nu \delta_\nu}{a^2(V''_\phi + \beta' \rho_\nu) + k^2} \]
The Stability Issue in Models of Neutrino Dark Energy

**Equation of motion of the neutrino density contrast**

$$\delta_\nu = \frac{\delta \rho_\nu}{\rho_\nu}$$

In the non-relativistic neutrino regime on length scales $m_\phi^{-1} < a/k < H^{-1}$ with negligible neutrino shear and $\rho_\nu \sim \omega_\nu \sim 0$, $c_\nu^2 \ll 1$

$$\ddot{\delta} + (c_s^2 k^2 - 4\pi G \rho) \delta = 0$$

$$\delta_b \simeq \delta_{CDM}$$  deep in matter-dom. regime

Compare: Newtonian theory, static universe, perfect fluid

$$\ddot{\delta}_\nu + H \dot{\delta}_\nu + [c_\nu^2 k^2 - 4\pi a^2 G_{\text{eff}} \rho_\nu] \delta_\nu = 4\pi a^2 G [\rho_{CDM} \delta_{CDM} + \rho_b \delta_b]$$

$$G_{\text{eff}} = G \left[ 1 + \frac{2\beta^2 M_{pl}^2}{1 + a^2 (V'' + \beta' \rho_\nu) / k^2} \right]$$

$$G [1 + 2\beta^2 M_{pl}^2] \gtrsim G_{\text{eff}} \gtrsim G$$
Equation of motion of the neutrino density contrast $\delta_\nu$

In the non-relativistic neutrino regime on length scales $m_\phi^{-1} < a/k < H^{-1}$ with negligible neutrino shear and $p_\nu \sim \omega_\nu \sim 0$, $c_s^2 \ll 1$

\[ \Omega_i = \frac{8\pi G a^2}{3H^2} \rho_i \]

\[ G_{\text{eff}} = G \left[ 1 + \frac{2\beta^2 M_{\text{pl}}^2}{1 + a^2 (V''_\phi + \beta' \rho_\nu)/k^2} \right] \]

\[ \delta_b \simeq \delta_{\text{CDM}} \] deep in matter-dominated regime

\[ \ddot{\delta}_\nu + H \dot{\delta}_\nu + \left[ c_s^2 k^2 - \frac{3}{2} H^2 \frac{G_{\text{eff}}}{G} \Omega_\nu \right] \delta_\nu \simeq \frac{3}{2} H^2 \left[ \Omega_{\text{CDM}} + \Omega_b \right] \delta_{\text{CDM}} \]

\[ \sim 10^{-4} \ldots 0.02 \]

\[ \sim 0.22 \quad \sim 0.04 \]
The Stability Issue in Models of Neutrino Dark Energy

Equation of motion of the neutrino density contrast $\delta_\nu$

In the non-relativistic neutrino regime on length scales $m_\phi^{-1} < a/k < H^{-1}$ with negligible neutrino shear and $p_\nu \sim \omega_\nu \sim 0$, $c_\nu^2 \ll 1$

\[
\Omega_i = \frac{8\pi G a^2}{3H^2} \rho_i
\]

\[
G_{\text{eff}} = G \left[ 1 + \frac{2\beta^2 M_{\text{pl}}^2}{1+a^2(V''_\phi + \beta' \rho_\nu)/k^2} \right]
\]

$\delta_b \simeq \delta_{\text{CDM}}$ deep in matter-dominated regime

\[
\ddot{\delta}_\nu + H \dot{\delta}_\nu + \left[ c_\nu^2 k^2 - \frac{3}{2} H^2 \frac{G_{\text{eff}}}{G} \Omega_\nu \right] \delta_\nu = \frac{3}{2} H^2 \left[ \Omega_{\text{CDM}} + \Omega_b \right] \delta_{\text{CDM}}
\]

\[
\sim 10^{-4} \ldots 0.02
\]

\[
\frac{G_{\text{eff}}}{G} \Omega_\nu \ll \left[ \Omega_{\text{CDM}} + \Omega_b \right]
\]

Dynamics of $\delta_\nu$ governed by CDM → moderate growth like ordinary gravitational instabilities ('neutrinos follow CDM') → up to the present time $\delta_\nu \ll 1$ (≡ stability) possible

$\frac{G_{\text{eff}}}{G} \Omega_\nu \gg \left[ \Omega_{\text{CDM}} + \Omega_b \right]$

$\Omega_i = \frac{8\pi G a^2}{3H^2} \rho_i$
The Stability Issue in Models of Neutrino Dark Energy

Equation of motion of the neutrino density contrast $\delta_\nu$

In the non-relativistic neutrino regime on length scales $m^{-1}_\phi < a/k < H^{-1}$ with negligible neutrino shear and $p_\nu \sim \omega_\nu \sim 0, \ c^2_\nu \ll 1$

$$\Omega_i = \frac{8\pi G a^2}{3H^2} \rho_i$$

$$G_{\text{eff}} = G \left[ 1 + \frac{2\beta^2 M^2_{\text{pl}}}{1 + a^2 (V''_\phi + \beta' p_\nu) / k^2} \right]$$

$$\delta_b \simeq \delta_{\text{CDM}}$$

deep in matter-dominated regime

$$\ddot{\delta}_\nu + H \dot{\delta}_\nu + \left[ c^2_\nu k^2 - \frac{3}{2} H^2 \frac{G_{\text{eff}}}{G} \Omega_\nu \right] \delta_\nu = \frac{3}{2} H^2 \left[ \Omega_{\text{CDM}} + \Omega_b \right] \delta_{\text{CDM}}$$

$$\sim 10^{-4} \ldots 0.02$$

$$\sim 0.22 \quad \sim 0.04$$

<table>
<thead>
<tr>
<th>$\frac{G_{\text{eff}}}{G} \Omega_\nu \ll [\Omega_{\text{CDM}} + \Omega_b]$</th>
<th>$\frac{G_{\text{eff}}}{G} \Omega_\nu \gg [\Omega_{\text{CDM}} + \Omega_b]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamics of $\delta_\nu$ governed by CDM $\rightarrow$ moderate growth like ordinary gravitational instabilities ('neutrinos follow CDM') $\rightarrow$ up to the present time $\delta_\nu \ll 1$ (≡ stability) possible</td>
<td>Dynamics of $\delta_\nu$ governed by the strong coupling $\rightarrow$ depending on coupling function $\beta(\phi(z))$ (faster than) exponential growth $\rightarrow \delta_\nu \gg 1$ (≡ instability)</td>
</tr>
</tbody>
</table>
Any realistic MaVaN scenario $c_s^2 < 0$?

- Require $c_s^2 = \frac{\dot{p}_{DE}}{\rho_{DE}} = \omega - \frac{\dot{\omega}}{3H(1+\omega)} \geq 0$ for $m_\nu(z) \gg T_\nu(z)$ (take into account finite temperature effects)

$$\rightarrow \sum_{i=1}^{3} \frac{\partial m_\nu_i(z)}{\partial z} \left( 1 - \frac{5\alpha T_{\nu,0}^2(1+z)^2}{3m_{\nu_i}^2(z)} \right) + \sum_{i=1}^{3} \frac{25\alpha T_{\nu,0}^2(1+z)}{3m_{\nu_i}(z)} \geq 0, \text{ with } \alpha \equiv \frac{\int_0^\infty \frac{dy}{y+1} y^4}{\int_0^\infty \frac{dy}{y+1} y^2}$$

- Assume degenerate mass spectrum with $m_\nu_i(0) \sim m_\nu(0) = 0.312$ eV, $i = 1, 2, 3$
  - determine maximally allowed neutrino mass variation

Requirement of $c_s^2 \geq 0$ strongly restricts the allowed mass variation at late times.
The Stability Issue in Models of Neutrino Dark Energy

A concrete model

Consider model proposed in the context of 'Chameleon cosmologies' [Khoury, Weltman '03, Brax, van de Bruck, Davis, Khoury, Weltman '04, ...]

- Recall: evolution of $\phi$ determined by $V'_{\text{eff}}(\phi) = 0 = V'_\phi(\phi) + \rho'_\nu(m_\nu(\phi))$

- Exponential potential

$$V_\phi(\phi) = M^4 e^{M^2 \phi^2}$$

- Exponential dependence of $m_\nu$ on $\phi$

$$m_\nu(\phi) = \tilde{m} e^{\beta \phi}, \quad \text{where} \quad \beta = \frac{d\log m_\nu}{d\phi} = \text{const.}$$

- Typically, $\beta \phi \ll 1 \rightarrow m_\nu$ very weakly depends on changes in the neutrino energy density $\rightarrow m_\nu$ hardly evolves with time

- $\rightarrow$ attractive force between neutrinos essentially time independent
A stable model

- Normalization?
- \( \Delta^2(k) = \frac{k^3 P(k)}{2\pi^2} \propto \delta_{\text{CDM}}^2 \ll 1 \)
  \( \rightarrow \text{linear} \) [Percival et al.'06]
- Since \( \tilde{\delta}^2_\nu < \tilde{\delta}^2_{\text{CDM}} \rightarrow \) neutrino density contrast linear, \( \delta^2_\nu \ll 1 = \) no 'neutrino nuggets'!
- \( \rightarrow \) Adiabatic model of Neutrino Dark Energy stable also in the highly non-relativistic regime \( \rightarrow \) viable dark energy candidate.
The Stability Issue in Models of Neutrino Dark Energy

A stable model

- Normalization?

- For \( k = 0.11h \text{ Mpc}^{-1} \)
  \[
  \Delta^2(k) = \frac{k^3 P(k)}{2 \pi^2} \propto \delta^2_{\text{CDM}} \ll 1
  \]
  \( \rightarrow \) linear

  [Percival et al.’06]

- Since \( \tilde{\delta}^2_{\nu} < \tilde{\delta}^2_{\text{CDM}} \rightarrow \) neutrino density contrast linear, \( \delta^2_{\nu} \ll 1 = \) no ’neutrino nuggets’!

- Adiabatic model of Neutrino Dark Energy stable also in the highly non-relativistic regime \( \rightarrow \) viable dark energy candidate

\[
\begin{align*}
  k &= 0.11h \text{ Mpc}^{-1}, \quad \beta = 0, \quad m_{\nu} (z = 0) = 0.312 \text{ eV}
\end{align*}
\]
Another model

Proposed by Fardon, Nelson, Weiner '05

- Logarithmic scalar potential \((V_0 \text{ fixed by requirement of } \Omega_{DE} \sim 0.7)\)

\[
V_\phi(\phi) = V_0 \log(1 + \kappa \phi), \text{ with } V_0, \kappa = \text{const.}
\]

- Mass dependence on \(\phi\) as preferred in the MaVaN literature [Fardon, Nelson, Weiner '05, '06, Afshordi, Kohri, Zaldarriaga '05, Spitzer '06...]

\[
m_\nu(\phi) = \frac{\tilde{m}^2}{\phi}, \text{ where } \beta = \frac{d \log m_\nu}{d\phi} = -\frac{1}{\phi} \neq \text{const.}
\]

- dependence \(m_\nu(\phi)\) naturally arises from integrating out a heavier sterile state, whose mass varies linearly with the value of \(\phi\) (MaVaN seesaw')

- \(m_\nu\) strongly depends on changes in the scalar field VEV

- since \(\phi\) decreases, \(|\beta|\) increases with time \(\rightarrow\) attractive force between neutrinos increases with time
The Stability Issue in Models of Neutrino Dark Energy

An unstable model

- Rapid evolution of $m_\nu(z)$
- $m_\nu(z) \ll T_\nu(z)$ (non-relativistic regime): pressure support diminishes → $c_s^2$ driven to negative values

Recall:

$$\ddot{\delta}_\nu + H\dot{\delta}_\nu + \left[c_\nu^2 k^2 - \frac{3}{2} H^2 \frac{G_m}{c^2} \Omega_\nu \right] \delta_\nu = \frac{3}{2} H^2 \left[\Omega_{CDM} + \Omega_b \right] \delta_{CDM}$$

- As soon as coupling is large enough to compensate for small neutrino mass (and thus $\Omega_\nu$) →
- $\delta_\nu \gg 1$ → model unstable before today → excluded as DE candidate

$k = 0.11 h \text{Mpc}^{-1}$, $\beta \neq \text{const.}$, $m_\nu(z = 0) = 0.312 \text{eV}$
An unstable model

- Rapid evolution of \( m_\nu (z) \)
- \( m_\nu (z) \ll T_\nu (z) \) (non-relativistic regime): pressure support diminishes \( \rightarrow c_s^2 \) driven to negative values

Recall:

\[
\ddot{\delta}_\nu + H \dot{\delta}_\nu + \left[ \frac{c_s^2 k^2}{2} - \frac{3}{2} H^2 \frac{G_{\text{eff}}}{G} \Omega_\nu \right] \delta_\nu = \frac{3}{2} H^2 \left[ \Omega_{\text{CDM}} + \Omega_b \right] \delta_{\text{CDM}}
\]

- As soon as coupling is large enough to compensate for small neutrino mass (and thus \( \Omega_\nu \)) \( \rightarrow \)
- \( \delta_\nu \gg 1 \) \( \rightarrow \) model unstable before today \( \rightarrow \) excluded as DE candidate

\( k = 0.11 h \text{Mpc}^{-1} \), \( \beta \neq \text{const.}, m_\nu (z = 0) = 0.312 \text{eV} \)
Summary

• Reconsideration of the **stability issue** in models of adiabatic neutrino dark energy

• Other cosmic components (CDM and baryons) can have stabilizing effect on MaVaN perturbations

• If \( \frac{G_{\text{eff}}}{G} \Omega_\nu \ll [\Omega_{\text{CDM}} + \Omega_b] \rightarrow \text{moderate growth} \) of perturbations as in general relativity \( \rightarrow \delta_\nu \ll 1 \) (≡ **stability**) possible

• If strong coupling compensates for relative smallness of \( \Omega_\nu \rightarrow \delta_\nu \gg 1 \) (≡ **instability**) in the non-relativistic regime

• Viable model of neutrino dark energy found with \( c_s^2 > 0 \rightarrow \text{allowed mass variation strongly restricted} \) at late times

• Note: non-adiabatic models of neutrino dark energy with \( m_\phi \sim H \) are stable

  [Brookfield, van de Bruck, Mota, Tocchini-Valentini ’06, Afshordi, Kohri, Zaldarriaga ’05]

• Note: ’Hybrid’ models involving two light scalar fields can be stable until the present time even in the presence of unstable neutrino component

  [Fardon, Nelson, Weiner ’06, Spitzer ’06]