Theory of the dynamic response of a coplanar grid semiconductor detector

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The authors have developed a theoretical model for the response of a coplanar grid semiconductor detector to hard x- and γ-ray radiation. Carrier drift trajectories were obtained by solving the coupled dynamical equations for carriers driven by electrostatic fields of the coplanar grid configuration. The pulse spectra calculated by summing the individual contributions for all carriers are compared to experimental results for a large volume optimized cadmium zinc telluride coplanar grid detector and good agreement is obtained. © 2007 American Institute of Physics. [DOI: 10.1063/1.2755931]

Currently, there is great interest in developing large volume compound semiconductor detectors for high energy photons in the hard x- and γ-ray range. A central issue is the design of the electrodes, and a number of different configurations have been used, including small pixel arrays, Frisch grids, and coplanar grid configurations (CGDs). So far, the last of these appears to be the most promising, but previously there has been little advance in constructing an analytical model of intrinsic effects in a CGD in order to facilitate optimization of this configuration. Only the results of computer modeling of the potential performance for various electrode configurations including CGD have been described. In the present letter, we develop a theoretical model for an ideal periodic grid topology. We use our model to generate pulse spectra for comparison with results obtained on a cadmium zinc telluride (CZT) coplanar grid detector.

We consider a typical CGD detector, as shown in Ref. 7, with the cathode on the z=0 plane and the anodes at z=L. We assume that the elements of the two grid anodes are fingers of equal width a, and that the gap between the fingers of the adjacent grids is infinitesimally small. Thus, both anode grid patterns vary along the y direction with a periodicity of 2a. We calculated the electric field distribution by solving the electrostatic boundary problem with the potential at the anodes alternating periodically along the y axis between $V_0 \pm \delta V$, and the cathode plane maintained at ground potential. The solution for the electrostatic potential was obtained by Fourier series expansion, and can be expressed as a geometric progression provided that all terms of the order of $\exp(-L/a) \ll 1$ are ignored. The topology of the resulting equipotentials depends on the ratio of lateral electric field in the near field region ($E_z \sim \delta V/2a$) to the field along the y direction ($E_y \sim V_0/L$), conveniently expressed in terms of a characteristic parameter $\alpha = 4(\delta V/V_0)(L/a)$. The potential distribution is illustrated in Fig. 1 for $\alpha = 4$ and $\alpha = 1$. Saddle points $z_s$, at which $E_{z_s}=0$, exist for all $\alpha > 2$ at $z_s=L + a/\pi \ln \left[ \alpha/2 - \sqrt{\alpha^2/4-1} \right]$, but are absent for $\alpha < 2$. To obtain the electron and hole drift trajectories, we solved the following dynamic equations within the drift model for carriers moving in the first elementary cell ($-a<y<a$).

The right hand sides of these equations are the exact analytic expressions for the normal and lateral components of the gradient of electrostatic potential.

$$
\frac{dz}{dt} = \pm \frac{\exp(-z)\cos y[1 + \exp(-2z)]}{1 + 2\exp(-2\cos y + \exp(-4z)}, \\
\frac{dy}{dt} = \pm \frac{\alpha \exp(-z)\sin y[1 - \exp(-2z)]}{1 + 2\exp(-2\cos y + \exp(-4z)}. 
$$

The dimensionless coordinates $\bar{z}$ and $\bar{y}$ are measured in units of $a/\pi$, and the time t is expressed in units of either $L/v_e$ or $L/v_h$, where $v_e$ and $v_h$ are drift velocities for electron and holes, respectively. The equations are solved with initial conditions $\bar{y}(0) = \pi y_0/a$, $\bar{z}(0) = \pi (L-z_0)/a$, where $z_0$ and $y_0$ are the coordinates of the absorption site $r_p$.

The resulting solutions show that if $\alpha \geq 2$, for all photon absorption sites, all mobile electron trajectories will terminate on the higher potential grid. If $\alpha < 2$, a fraction of the electrons depending on the position of the absorption site will have trajectories terminating instead on the lower potential grid. The criterion $\alpha = 2$ also distinguishes the two differ-

![FIG. 1. General shape of equipotential surfaces in the near-field region of the anodes for the two values of the characteristic voltage parameter, $\alpha$, equal to (a) 4 and (b) 1. In each case, the higher potential grid elements extend between $\pm a/2$ within the elementary cell of size 2a on the $z/L=1$ plane.](Image)
ent classes of trajectories for holes. For \( \alpha < 2 \), regardless of the position of the absorption site, all hole trajectories will terminate on the cathode. However, for \( \alpha > 2 \) a different situation arises in the vicinity of saddle points. While all holes originating at \( z_0 < z_c \) move along trajectories terminating on the cathode, some of those produced at \( z_0 > z_c \) travel to the lower potential anode. Thus, a CGD possesses intrinsic inhomogeneity because the relative amount of charge induced on the two anode grids depends on the position of the absorption site. In addition, inhomogeneity can arise because electrons moving along longer drift trajectories will have a greater probability of being trapped.

In order to analyze the effects of inhomogeneous broadening due to both lateral and vertical effects, we need to derive an expression for the induced charge. Since the carriers move in inhomogeneous electric fields and arrive at different instants of time, it is convenient to introduce parameterization of their trajectories in terms of distance from the collecting plane.

Introducing dimensionless variables \( T(Z) \) and \( Y(Z) \), where \( T \) is the time it takes for a carrier to reach \( Z \) from the position of the absorption site along a drift trajectory and \( Y \) is its new lateral coordinate, instead of Eq. (1) we may write

\[
\frac{dT_{e,h}}{dZ} = \pm \frac{1 + 2Z^2(2Y^2 - 1) + Z^4}{Z[1 + 2Z^2(2Y^2 - 1) + Z^4 + \alpha Z(1 + Z^2)Y]},
\]

\[
\frac{dY}{dZ} = \frac{\alpha (1 - Z^2)(1 - Y^2)}{[1 + 2Z^2(2Y^2 - 1) + Z^4 + \alpha Z(1 + Z^2)Y]},
\]

\( Y(0) = \cos \bar{y}(0), T_{e,h}[\exp(-\bar{y}(0))] = 0, \) with positive sign now corresponding to electrons and negative to holes, respectively. Here, \( T_{e,h} \) is measured in units of \( \alpha/\mu e_{h,c}, Y = \cos(\bar{y}) \) and \( Z = \exp(-\bar{y}) \).

A general formula for the charge collected by each electrode can be written as

\[
Q(t) = -\int_S d\tau' \int_0^t dt' j(z)(\tau',L,t') - \int_\Omega d\tau' \Phi_{w}(\tau') \rho(\tau',t).
\]

Here, \( \rho(\tau',t) \) and \( j(\tau',t) \) are the charge and current density, respectively, both assumed to be equal to zero at \( t=0 \), \( \Phi_{w}(\tau) \) is the weighting potential for the electrode, \( S \) is its area, and \( \Omega \) is the volume of the detector. For the coplanar grid configuration, we first calculate the collected charge for both grids using this formula, and then take the difference signal weighted by the dynamic amplification factor for \( t \to \infty \). Hence, we obtain for the collected charge

\[
\frac{Q(r_\alpha)}{e} = -g \exp\left(-\frac{aT_{e,h}(r_\alpha)}{\pi l_c}\right) + \Theta(2 - \alpha) \times \exp\left(-\frac{aT_{e,h}(r_\alpha)}{\pi l_c}\right) - \Theta(\alpha - 2) \times \exp\left(-\frac{aT_{h,h}(r_\alpha)}{\pi l_h}\right) - \frac{a}{\pi l_h} \int_{Z(0)}^{Z(Z_r)} dZ' \frac{dT(h,Z,r_\alpha)}{dZ} \times \Phi_{CGD}^{w}(L + \frac{a}{\pi} \ln Z,\arcsin Y(Z,r_\alpha)) + \frac{a}{\pi l_h} \int_{Z(0)}^{Z(Z_r)} dZ' \frac{dT(h,Z,r_\alpha)}{dZ} \times \Phi_{CGD}^{w}(L + \frac{a}{\pi} \ln Z,\arcsin Y(h(Z,r_\alpha)),
\]

where \( \Theta \) is the Heaviside function. The first three terms describe contributions from carriers moving along drift trajectories and terminating at the anode grids, while the latter terms give the contributions from trapped carriers. Here, \( Q \) is the CGD difference signal, \( e \) is electron charge, \( l_{e,h} \) are the mean drift paths for electrons and holes, respectively, \( T_{e,h}(r_\alpha) \) are arrival times at the higher and lower potential anodes, respectively, of electrons and holes created at \( r_\alpha \), and \( \Phi_{CGD}^{w} = g(\Phi_{w,e} - \Phi_{w,h}) \) is the CGD weighting potential, equal to the weighted difference of \( \Phi_{w,e} \) for the higher and \( \Phi_{w,h} \) for the lower potential grids. The dynamic amplification factor \( g \) describes the relative amounts of charge collected from the two anodes. The expression for \( \Phi_{CGD}^{w} \) can be written as

\[
\Phi_{CGD}^{w}(z,y) = g(\Phi_{w,e}(z,y) - \Phi_{w,h}(z,a-y)),
\]

since the two grids are of the same geometry, related by lateral translation along the \( y \) axis by \( a \). For \( \Phi_{w,c}(z,y) \) we obtain

\[
\Phi_{w,c}(z,y) = \sum_{m=-\infty}^{\infty} \sum_{m'=-\infty}^{\infty} \left[ \arctan \left( \frac{(2m' + 1)L-z\alpha}{(2m' + 1)L - y - 2ma)^2 - \alpha^2/4} \right) - \arctan \left( \frac{(2m' + 1)L+z\alpha}{(2m' + 1)L + y - 2ma)^2 - \alpha^2/4} \right) + \pi \Theta(\alpha^2/4 - ((2m' + 1)L-z)^2 - (y-2ma)^2) \right].
\]
directly opposite one of the elements of the collecting grid. The pulse spectrum \( S(Q) \) of collected charge \( Q \) is the superposition of Gaussian curves, so that
\[
S(Q) = \int_0^L dz_0 \frac{P(z_0,E)}{\sqrt{2\pi \sigma(z_0)}} \exp\left[ -\frac{(Q - Q(z_0))^2}{2\sigma^2(z_0)} \right]. \tag{7}
\]
The expression for \( Q(z_0) \) is given by Eq. (4), \( P(z_0,E) = \Gamma(1 - \exp(-\Gamma L))^{(1)} \exp(-\Gamma(L-z_0)) \) is the normalized probability density of absorption for the geometry of illumination through the surface \( z=L \), and \( \Gamma \) is the photon absorption coefficient varying with photon energy \( E \). The charge variance \( \sigma(z_0) \) is the sum of statistically independent noise sources including Fano, carrier trapping, and electronic noise, \( \sigma_e \). The last, due to leakage current between collecting and noncollecting grids, does not depend on \( E \). The trapping noise can be ignored for \( E \equiv 10 \) keV leaving \( \sigma_e \) independent of \( z_0 \).

Figure 2 shows the comparison between theory and experiment for \( \gamma \)-ray absorption at 662 keV obtained with a large \( 1.5 \times 1.5 \times 1 \) cm\(^3\) CGD detector fabricated by Baltic Scientific Instruments (BSI) from CZT material grown by Yinnel Tech. Inc. The detector was fabricated to allow illumination through the anode plane and was operated at a higher anode voltage of 2000 V and an intergrid bias of 90 V, giving a value of 1.92 for the parameter \( \alpha \). Experimental data were taken with a focused beam of 20 \( \mu \)m diameter, much less than \( a=480 \) \( \mu \)m, and positioned directly opposite the center of a higher potential grid element, which consequently collected all the electrons. The reported figure of merit for electrons was \( \mu_e \tau_e = 6.5 \times 10^{-3} \) cm\(^2\)/V, but no measured figure was available for holes. The measured line shape (shaded area in Fig. 2) depended critically on the dynamic amplification factor \( g \), which was directly measured in the collecting circuit during the detector tuning at BSI as \( g = 1.28 \pm 0.6 \). A figure of \( \sigma_e = 10 \) keV was determined from x-ray data at 30 keV, for which absorption occurred so close to the collecting electrodes that inhomogeneous broadening was negligible. The only unknown fitting parameter used in our model was \( \mu_h \tau_h \) with the best fit value of \( 1.0 \times 10^{-4} \) cm\(^2\)/V. It is seen that excellent agreement between the best fit curve (heavy line) and the experimental spectrum was obtained. Experiments were also carried out for energies of 30–100 keV. In this energy range the 1/e absorption depth lies between 91 and 142 \( \mu \)m, so that practically all absorption sites lie in the near field region of the grids. Thus, due to the rapid variation of the weighting potential, the signal shape is much less sensitive to the dynamic amplification factor. We obtained good agreement between theory and experiment for 30–100 keV data with the same best fit parameters as for 662 keV.

The signal shape was sensitive both to carrier transport parameters and to the dynamic amplification factor. Any change in the fitting value for \( \mu_h \tau_h \) resulted in distortion of the theoretical line shape. Changing this parameter affected the pulse amplitude, position, and symmetry, as illustrated in Fig. 2, curves (a) and (b). In Fig. 2, curves (c) and (d), we show the sensitivity of detector response to the dynamic amplification by keeping best fit values for \( \mu_e \tau_e \) and \( \mu_h \tau_h \) and changing only \( g \). Once again the line becomes shifted and distorted. Finally, the inset figure shows the best fit spectrum at 662 keV calculated for \( \sigma_e = 0 \), together with the observed line shape (shaded area). The intrinsic shape is inherently non-Gaussian, because complete compensation of carrier trapping by dynamic amplification is not possible. The line is less narrow than that modeled by Luke, but in order to determine the ultimate theoretical resolution it would be necessary to obtain a more accurate optimization of \( g \) within the experimental uncertainty of the quoted value. It is clear that, while the signal shape is very sensitive to transport parameters, the experimental width is largely determined by noise due to leakage currents, \( \sigma_e \). This is currently the major factor preventing the CGD configuration from showing resolution close to intrinsic.

In summary, we have developed a theoretical model for predicting the response of a CGD detector. We obtain pulse shapes in good agreement with experiment, demonstrating that our method provides a firm basis for future optimization of this device.