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Accurate modeling of the hose instability in plasma wakefield accelerators

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Hosing is a major challenge for the applicability of plasma wakefield accelerators and its modeling is therefore of fundamental importance to facilitate future stable and compact plasma-based particle accelerators. In this contribution, we present a new model for the evolution of the plasma centroid, which enables the accurate investigation of the hose instability in the nonlinear blowout regime. It paves the road for more precise and comprehensive studies of hosing, e.g., with drive and witness beams, which were not possible with previous models. *Published by AIP Publishing.*

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I. INTRODUCTION

Plasma wakefield accelerators (PWFA)s^{1,2} allow for an energy gain of several GeVs within distances of only tens of centimeters for electron beams^{3,4} and therefore provide a transformative potential for the miniaturization of future particle acceleration facilities. For applications driven by such compact accelerators, the beam stability is of utmost importance. In this context, the extreme focusing fields, which can be on the same order of magnitude as the accelerating fields, entail the challenge of rapid growth rates for transverse beam-plasma instabilities. In a seminal work in the early 1990s, the *hose instability* was predicted to pose a fundamental impediment for stable acceleration of beams in PWFA)s.⁵ Hosing is also a challenge for long-beam PWFA)s.⁶

Of specific interest for short-beam PWFA)s is the nonlinear *blowout regime*,⁷ for which essentially all electrons are expelled from the vicinity of the drive beam and a co-propagating ion cavity is formed. The focusing fields in this regime are linear and the accelerating field enhanced. Electrons in the sheath, which surrounds the ion-channel, can have relativistic velocities and the blowout radius can be greater than the charge neutralization radius, assumed for the treatment in Ref. 5. As a result of these effects, the coupling between beam and plasma can be less enhanced, such that the growth rate for hosing is reduced.⁸ While hosing was still predicted to be growing exponentially in time and along the beam by previous models,^{5,8} a recent work showed that the intrinsic energy evolution and/or energy spread of the beam lead to a saturation or damping of hosing in PWFA)s in the blowout regime.⁹ This is similar to the mitigation of hosing in self-modulated PWFA)s owing to a varying betatron wavenumber along the beam¹⁰ and similar to Balakin-Novokhatsky-Smirnov damping in conventional accelerators.¹¹

The hose instability occurs when transverse phase space asymmetries of the beam coherently couple with phase space

asymmetries of the plasma. For example, a displacement of the transverse beam centroid X_b from the propagation axis induces a deviation of the plasma wake centroid X_p along the beam, which then feeds back to the temporal evolution of the trailing part of the beam. A mathematical description of the hose instability in PWFA)s is given by the coupled differential equations^{8,9}

$$\frac{\partial^2 X_p}{\partial \xi^2} + \frac{c_\psi(\xi)c_r(\xi)}{2}(X_p - X_b) = 0, \quad (1)$$

$$\frac{\partial^2 X_b}{\partial t^2} + \lambda(\xi, t)\frac{\partial X_b}{\partial t} + \Omega^2(\xi, t)(X_b - X_p) = 0, \quad (2)$$

where $\xi = t - z$ is the co-moving variable, z the propagation distance, and t the time. Length-scales within this work are normalized by the plasma wavenumber $k_p = \omega_p/c$ and timescales by the plasma frequency $\omega_p = \sqrt{4\pi n_0 e^2/m}$, where c is the speed of light. Furthermore, densities are normalized to the ambient plasma electron density n_0 , charges to the elementary charge e , masses to the electron mass m , and potentials to mc^2/e .

The parameters $c_r(\xi)$ and $c_\psi(\xi)$ in Eq. (1), introduced in Ref. 8, account for the ξ -dependence of the blowout radius and the beam current, and for relativistic velocities of electrons in the sheath. The original hosing model for the adiabatically generated ion channel with non-relativistic sheath electrons is retrieved for $c_r c_\psi = 1$. However, for $c_r c_\psi < 1$ the growth rate is reduced.⁸

The coefficients $\lambda(\xi, t)$ and $\Omega(\xi, t)$ in Eq. (2) were introduced in Ref. 9. A finite $\lambda(\xi, t)$ accounts for the damping/amplification of the beam centroid oscillations owing to a relativistic mass gain/loss of beam electrons as well as the damping of the centroid oscillations from a finite uncorrelated beam energy spread. The coefficient $\Omega(\xi, t)$ incorporates the effect of the change of the betatron frequency for a changing energy. If different slices of the beam gain or lose energy at differing rates, the beam centroid oscillations become detuned and hosing stops.⁹

The coefficients $c_r(\xi)$ and $c_\psi(\xi)$ in the plasma centroid equation (1) were derived in Ref. 8 through investigation

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of the dynamics of a plasma electron at the blowout-sheath boundary, so as to infer the perturbation of the channel centroid. This model posed a dramatic improvement in the modeling of hosing in PWFAs in the nonlinear blowout regime. However, as discussed within this work, this model is not sufficiently accurate, and in some regards, provides unphysical results. In this work, we therefore generalize the plasma centroid equation by investigating the collective dynamics of all sheath electrons subject to beam and plasma centroid deviations. As shown, the model introduced here is more accurate and can provide the basis for crucial studies of hosing of drive and witness beam pairs in PWFAs in the blowout regime.

The paper is organized as follows. In Sec. II, we derive a general moment equation and from this, we derive the plasma centroid equation. The plasma centroid equation is then applied to the blowout regime in Sec. III and compared to three-dimensional (3D) particle-in-cell (PIC) simulation results with HiPACE¹² in Sec. IV. Section V concludes this work.

II. DERIVATION OF THE PLASMA CENTROID EQUATION

A. Moment equation for the plasma electron phase space density

In the following, we derive a general equation for the evolution of the moments of the plasma-electron phase space density f_p along ξ , when the electrons are subject to the fields of a beam with a finite centroid displacement and to the fields of a blowout wake with a finite transverse displacement. We use the quasi-static approximation^{13–15} for the plasma-electron phase space density, $\partial f_p \ll \partial_\xi f_p$. This approximation entails a constant of motion $\gamma_e - P_z - \Psi$ for plasma electrons.¹⁶ Here, γ_e denotes the Lorentz factor of an electron, P_z the longitudinal momentum of an electron, and $\Psi = \phi - A_z$ the wakefield potential at the instantaneous location of the electron, with ϕ being the scalar potential and A_z the longitudinal component of the vector potential. Considering that electrons are initially at rest, the constant of motion allows for the introduction of the phase space variable $\psi = \gamma - p_z - 1$ which is, via integration over all momentum phase space, associated with the wakefield potential Ψ . Here, γ refers to the Lorentz factor as a function of the phase space momentum coordinates and p_z denotes the longitudinal phase space momentum coordinate. Hence, for a given time t and a comoving position ξ , the phase space distribution of the plasma electrons in an azimuthally symmetric plasma wave, $f_{p,0}$, is a function only of radial position r , radial momentum p_r , and ψ , such that $f_{p,0} = f_{p,0}(r, p_r, \psi; \xi, t)$.

From the conservation of the phase space density $df_{p,0}/dt = 0$, we obtain the Vlasov equation for $f_{p,0}$ in the quasi-static approximation¹⁷

$$\partial_\xi f_{p,0} = -\frac{\gamma}{1+\psi} \left(\frac{p_r}{\gamma} \partial_r + F_r \partial_{p_r} + F_\psi \partial_\psi \right) f_{p,0}, \quad (3)$$

with the forces $F_r = dp_r/dt$ and $F_\psi = d\psi/dt$, and where the Lorentz factor is expressed as

$$\gamma = \frac{1 + p_r^2 + (1 + \psi)^2}{2(1 + \psi)}. \quad (4)$$

When subject to hosing, the phase space distribution of the plasma electrons is not azimuthally symmetric. In leading order of a moment expansion of the phase space distribution within a slice, the perturbation owed to hosing results in a finite mean transverse position $\langle x \rangle$ and a finite mean transverse momentum $\langle p_x \rangle$. We therefore express the asymmetric plasma electron phase space distribution f_p as an expansion of $f_{p,0}(r^*, p_r^*, \psi; \xi, t)$ for small $\langle x \rangle$ and $\langle p_x \rangle$, where $r^* = \sqrt{(x - \langle x \rangle)^2 + y^2}$ and $p_r^* = (p_x - \langle p_x \rangle) \cos \theta + p_y \sin \theta$, such that

$$f_p \simeq -\cos \theta (\langle x \rangle \partial_r + \langle p_x \rangle \partial_{p_r}) f_{p,0}, \quad (5)$$

and $f_p = f_p(r, \theta, p_r, \psi; \xi, t)$. The truncation of this expansion after the first order in $\langle x \rangle$ and $\langle p_x \rangle$ accurately reproduces the plasma electron phase space distribution in the initial phase of the hose instability before beam breakup occurs.

The Vlasov equation for the perturbed plasma electron distribution is

$$\partial_\xi f_p = -\frac{\gamma}{1+\psi} \left(\frac{p_r}{\gamma} \partial_r + \dot{\theta} \partial_\theta + F_r \partial_{p_r} + F_\psi \partial_\psi \right) f_p. \quad (6)$$

We are interested in moments of the phase space density, which are defined as

$$\langle \Phi(r, \theta, p_r, \psi) \rangle(\xi, t) = \frac{1}{N} \int_0^\infty r dr \int_0^{2\pi} d\theta \int_{-\infty}^\infty dp_r \int_{-1}^\infty d\psi \Phi f_p, \quad (7)$$

where the normalization is given by

$$\begin{aligned} N &= \int_0^\infty r dr \int_0^{2\pi} d\theta \int_{-\infty}^\infty dp_r \int_{-1}^\infty d\psi f_p \\ &= 2\pi \int_0^\infty r dr \int_{-\infty}^\infty dp_r \int_{-1}^\infty d\psi f_{p,0}. \end{aligned} \quad (8)$$

Moments with respect to the unperturbed phase space distribution are denoted by $\langle \Phi \rangle_0$ and are defined as

$$\langle \Phi(r, p_r, \psi) \rangle_0(\xi, t) = \frac{2\pi}{N} \int_0^\infty r dr \int_{-\infty}^\infty dp_r \int_{-1}^\infty d\psi \Phi f_{p,0}. \quad (9)$$

The multiplication of the Vlasov equation (6) with a quantity Φ and integration by parts with the assumption that the phase space density decays to zero at the integration limits yields the general moment equation

$$\begin{aligned} \partial_\xi \langle \Phi \rangle &= \left\langle \frac{p_r}{1+\psi} \partial_r \Phi \right\rangle + \left\langle \frac{\gamma \dot{\theta}}{1+\psi} \partial_\theta \Phi \right\rangle \\ &+ \left\langle \frac{\gamma F_r}{1+\psi} \partial_{p_r} \Phi \right\rangle + \left\langle \frac{\gamma F_\psi}{1+\psi} \partial_\psi \Phi \right\rangle. \end{aligned} \quad (10)$$

This equation expresses how a macroscopic property $\langle \Phi \rangle$ of the plasma electron phase space distribution in a slice changes along the comoving coordinate.

B. Plasma centroid equation

The plasma centroid in the context of the hose instability can be defined by the symmetry axis of the wakefield potential Ψ , towards which the highly relativistic beam electrons are attracted. For a cylindrically symmetric case, the wakefield exerts a force $\propto \partial_r \Psi_0$ on beam electrons towards the axis, where $\Psi_0 = \Psi_0(r)$ is the cylindrically symmetric wakefield potential. We now expand the wakefield potential around a small centroid deviation X_p

$$\Psi(r, \theta) \simeq -\cos \theta X_p \partial_r \Psi_0(r), \quad (11)$$

such that the beam electrons are attracted towards the point X_p . In order to investigate how X_p is connected with $\langle x \rangle$ in Eq. (5), we consider Maxwell equations in the quasi-static approximation using the Lorenz gauge (see, e.g., Ref. 5 or 18)

$$\nabla_{\perp}^2 \begin{bmatrix} \Psi \\ A_z \end{bmatrix} = - \begin{bmatrix} \rho - J_z \\ J_z \end{bmatrix}, \quad (12)$$

where ρ is the charge density, J_z the longitudinal current density, A_z the longitudinal component of the vector potential, and Ψ the wakefield potential, as defined before. The Lorenz gauge condition is given by¹⁸

$$\nabla_{\perp} \cdot \mathbf{A}_{\perp} = -\frac{\partial \Psi}{\partial \xi}. \quad (13)$$

Using Eqs. (11) and (12), the source term of the wakefield potential, $S = -(\rho - J_z)$, is

$$S(r, \theta) \simeq -\cos \theta X_p \partial_r S_0(r) \quad (14)$$

for $r \gg |X_p|$, where $S_0 = S_0(r)$ refers to the cylindrically symmetric charge and current distribution, which gives rise to the symmetric wakefield potential according to $\nabla_{\perp}^2 \Psi_0 = S_0$. The source term S is given by the plasma charge and current distribution and connected to the plasma phase space distribution via

$$S = -1 + \int_{-\infty}^{\infty} dp_r \int_{-1}^1 d\psi \frac{1+\psi}{\gamma} f_p, \quad (15)$$

where the first term corresponds to the background ion contribution and where the identity $1 - p_z/\gamma = (1 + \psi)/\gamma$ was used. Inserting the phase identity expansion according to Eq. (5) in Eq. (15) and performing integration by parts show that the wakefield centroid X_p is equivalent to the plasma electron centroid $\langle x \rangle$ when assuming that the transverse momentum deviation $\langle p_x \rangle$ does not cause a significant relativistic mass change of plasma electrons, i.e., if $\langle p_x \rangle \partial_{p_r} \gamma \approx 0$.

Hence, in the following, we investigate the evolution of the plasma centroid $\langle x \rangle \simeq X_p$ along the co-moving variable ξ for small perturbations $\langle x \rangle$ and $\langle p_x \rangle$. Using the moment equation (10), the change of the plasma centroid with respect to the co-moving variable is given by

$$\partial_{\xi} \langle x \rangle = \partial_{\xi} \langle r \cos \theta \rangle = \left\langle \frac{p_r \cos \theta}{1 + \psi} \right\rangle. \quad (16)$$

The moment term containing $\partial_{\theta}(r \cos \theta)$ vanishes. Expanding $1/(1 + \psi)$ around $\langle \psi \rangle$ yields

$$\left\langle \frac{p_r \cos \theta}{1 + \psi} \right\rangle = \left\langle \frac{p_r \cos \theta}{\langle 1 + \psi \rangle} \sum_{k=0}^{\infty} (-1)^k \left(\frac{\psi - \langle \psi \rangle}{\langle 1 + \psi \rangle} \right)^k \right\rangle. \quad (17)$$

In the linear regime, $|\psi| \simeq |p_z| \ll 1$, the above series expansion converges to

$$\left\langle \frac{p_r \cos \theta}{1 + \psi} \right\rangle \simeq \frac{\langle p_r \cos \theta \rangle}{\langle 1 + \psi \rangle}. \quad (18)$$

This is also a valid approximation for the blowout regime as discussed in Appendix A. Using the moment equation (10), the change of the electron-plasma centroid along the comoving variable is then given by

$$\begin{aligned} \partial_{\xi}^2 \langle x \rangle &= \frac{\left\langle \frac{\gamma}{1 + \psi} F_r \cos \theta \right\rangle}{\langle 1 + \psi \rangle} - \frac{\langle p_r \cos \theta \rangle}{\langle 1 + \psi \rangle^2} \left\langle \frac{\gamma F_{\psi}}{1 + \psi} \right\rangle \\ &= \frac{\left\langle \frac{\gamma}{1 + \psi} F_r \cos \theta \right\rangle}{\langle 1 + \psi \rangle} - \frac{\partial_{\xi} \langle x \rangle}{\langle 1 + \psi \rangle} \left\langle \frac{\gamma F_{\psi}}{1 + \psi} \right\rangle. \end{aligned} \quad (19)$$

The radial force F_r acting on the plasma electrons is composed of the force exerted by the fields originating from the perturbed plasma charge and current densities $F_{r,p}(r, \theta, \xi)$, and a force exerted by the fields originating from the beam charge and longitudinal current densities $F_{r,b}(r, \theta, \xi)$, such that $F_r = F_{r,p} + F_{r,b}$. These forces are expanded as

$$F_{r,b} \simeq -\cos \theta X_b \partial_r F_{r,b,0}, \quad (20a)$$

$$F_{r,p} \simeq -\cos \theta \langle x \rangle \partial_r F_{r,p,0}, \quad (20b)$$

where $F_{r,p,0}(r, \xi)$ and $F_{r,b,0}(r, \xi)$ refer to the respective unperturbed, azimuthally symmetric forces.

We now use the expansions (5), (20a), and (20b) together with $F_r = F_{r,p} + F_{r,b}$ in the plasma centroid equation (19). Truncation after $\mathcal{O}(X_b)$, $\mathcal{O}(\langle x \rangle)$, and $\mathcal{O}(\langle p_x \rangle)$ yields

$$\begin{aligned} \partial_{\xi}^2 \langle x \rangle &= -\frac{\langle x \rangle}{2\langle 1 + \psi \rangle_0} \left\langle \frac{\gamma}{1 + \psi} \partial_r F_{r,p,0} \right\rangle_0 \\ &\quad - \frac{X_b}{2\langle 1 + \psi \rangle_0} \left\langle \frac{\gamma}{1 + \psi} \partial_r F_{r,b,0} \right\rangle_0 \\ &\quad + \frac{\langle x \rangle}{2\langle 1 + \psi \rangle_0} \left\langle \frac{\gamma}{1 + \psi} (\partial_r + r^{-1}) F_{r,p,0} \right\rangle_0 \\ &\quad + \frac{\langle x \rangle}{2\langle 1 + \psi \rangle_0} \left\langle \frac{\gamma}{1 + \psi} (\partial_r + r^{-1}) F_{r,b,0} \right\rangle_0 \\ &\quad + \partial_{\xi} \langle x \rangle \left\langle \frac{p_r F_{r,0}}{(1 + \psi)^2} \right\rangle_0 \\ &\quad - \frac{\partial_{\xi} \langle x \rangle}{\langle 1 + \psi \rangle_0} \left\langle \frac{\gamma F_{\psi}}{1 + \psi} \right\rangle_0, \end{aligned} \quad (21)$$

where integration by parts was used to obtain the third, fourth, and fifth terms and where the identities $\int_0^{2\pi} \cos^2 \theta f_{p,0} d\theta = \pi f_{p,0}$ and $\langle p_x \rangle / 2 = \langle p_r \cos \theta \rangle = \langle 1 + \psi \rangle \partial_{\xi} \langle x \rangle$ were used.

Note that all moments, apart from $\langle x \rangle$, are now defined in terms of the unperturbed phase space density $f_{p,0}$. From this point on, $\langle x \rangle$ is denoted by X_p , and Eq. (21) is rearranged to form of the second-order differential *plasma centroid equation*

$$\frac{\partial^2 X_p}{\partial \xi^2} + C_d(\xi) \frac{\partial X_p}{\partial \xi} + \frac{C_p(\xi)}{2} X_p = \frac{C_b(\xi)}{2} X_b. \quad (22)$$

This equation is equivalent to a driven, damped harmonic oscillator for the plasma centroid $X_p(\xi, t)$ along ξ . The second term thereby acts as a damping/amplifying term for $C_d \neq 0$ and the coefficient $C_p/2$ is the square of the undamped oscillation wavenumber of the system. The oscillator is driven by the beam centroid X_b for a finite C_b . The coefficients $C_b(\xi)$, $C_p(\xi)$ and $C_d(\xi)$ are given by the respective moments in Eq. (21)

$$C_p = -\frac{1}{\langle 1 + \psi \rangle_0} \left(\left\langle \frac{\gamma}{1 + \psi} r^{-1} F_{r,p,0} \right\rangle_0 + \left\langle \frac{\gamma}{1 + \psi} (\partial_r + r^{-1}) F_{r,b,0} \right\rangle_0 \right), \quad (23a)$$

$$C_b = -\frac{1}{\langle 1 + \psi \rangle_0} \left\langle \frac{\gamma}{1 + \psi} \partial_r F_{r,b,0} \right\rangle_0, \quad (23b)$$

$$C_d = \frac{1}{\langle 1 + \psi \rangle_0} \left\langle \frac{\gamma F_\psi}{1 + \psi} \right\rangle_0 - \left\langle \frac{p_r F_{r,0}}{(1 + \psi)^2} \right\rangle_0. \quad (23c)$$

For the calculation of C_p , we used the fact that the first term in Eq. (21) cancels the part of the third term which includes the radial derivative of $F_{r,p,0}$. The coefficient C_b determines the amount a local beam centroid deviation contributes to the curvature of the plasma centroid along ξ . The respective contributions from the plasma centroid and the slope of the plasma centroid are incorporated by the coefficients C_p and C_d , respectively. A nonzero C_d results in a damping/amplification owing to relativistic mass gain/loss of plasma electrons along the plasma wave. To simplify C_d , we apply the Lorentz force to $\psi = \gamma - p_z - 1$ to find that

$$\frac{\gamma F_\psi}{1 + \psi} = \frac{p_r F_{r,0}}{1 + \psi} - F_{z,0}. \quad (24)$$

Hence, expanding the second term in Eq. (23c) in a similar way to that in Eq. (18) gives

$$C_d = \frac{1}{\langle 1 + \psi \rangle_0} \left\langle \frac{p_r F_{r,0}}{1 + \psi} \right\rangle_0 - \left\langle \frac{p_r F_{r,0}}{(1 + \psi)^2} \right\rangle_0 - \frac{\langle F_{z,0} \rangle_0}{\langle 1 + \psi \rangle_0} \simeq -\frac{\langle F_{z,0} \rangle_0}{\langle 1 + \psi \rangle_0}. \quad (25)$$

The forces can be expressed in terms of electromagnetic potentials

$$F_{r,b,0} = \frac{1 + \psi}{\gamma} \partial_r A_{z,b} = -\frac{1 + \psi}{\gamma} \frac{1}{r} \int_0^r r' J_{z,b}(r') dr', \quad (26a)$$

$$F_{r,p,0} = \partial_r \Psi + \frac{1 + \psi}{\gamma} (\partial_\xi A_r + \partial_r A_{z,p}), \quad (26b)$$

$$F_{z,0} = -\partial_\xi \Psi, \quad (26c)$$

where A_r is the radial component of the vector potential. From here on, all potentials refer to the azimuthally symmetric potentials and the zero-subscripts are dropped for the potentials. Note that the beam only gives rise to the longitudinal wakefield potential $A_{z,b}$. The wakefield potential Ψ and radial vector potential A_r are generated by the plasma charge and current densities.

Using Eq. (26a), the radial derivative of $F_{r,b,0}$ can be written as

$$\partial_r F_{r,b,0}(r) = -\frac{1 + \psi}{\gamma} \left[\frac{\partial_r A_{z,b}(r)}{r} + J_{z,b}(r) \right]. \quad (27)$$

Expressing the coefficients C_p , C_b , and C_d in terms of the potentials therefore gives

$$C_p = -\frac{1}{\langle 1 + \psi \rangle_0} \left(\left\langle \frac{\gamma}{1 + \psi} \frac{\partial_r \Psi}{r} \right\rangle_0 + \left\langle \frac{\partial_\xi A_r}{r} \right\rangle_0 + \left\langle \frac{\partial_r A_{z,p}}{r} \right\rangle_0 - \langle J_{z,b} \rangle_0 \right), \quad (28a)$$

$$C_b = \frac{1}{\langle 1 + \psi \rangle_0} \left\langle \frac{\partial_r A_{z,b}}{r} + J_{z,b} \right\rangle_0, \quad (28b)$$

$$C_d = \frac{\langle \partial_\xi \Psi \rangle_0}{\langle 1 + \psi \rangle_0}. \quad (28c)$$

In the following, we assume that at a given radial position, the phase space distribution $f_{p,0}(r, p_r, \psi)$ features a cold fluid distribution, such that

$$f_{p,0}(r, p_r, \psi) = \tilde{n}(r) \delta(p_r - P_r(r)) \delta(\psi - \Psi(r)), \quad (29)$$

where $\tilde{n}(r) = n(r) - n_0$ is the excess electron density. Note that this cold fluid assumption is in good agreement with observations from PIC simulations for regions ahead of the blowout vertex (electron density spike) as shown in previous works.^{18–20}

In the following, we express the averaged quantities in Eqs. (28a), (28b), and (28c) as functions of the radius r and comoving position ξ . This can be done by use of Eq. (29) which allows to form fluid or momentum moments such as

$$\iint \frac{\gamma}{1 + \psi} f_{p,0} d\psi dp_r = \iint \frac{\gamma}{\gamma - p_z} f_{p,0} d\psi dp_r = \frac{\tilde{n}}{\tilde{n} - J_{z,p}} = 1 + \frac{J_{z,p}}{\tilde{n} - J_{z,p}}. \quad (30)$$

The coefficients C_b , C_p , and C_d can now be fully expressed by radial moments of potentials and charge and current densities

$$C_p = -\frac{1}{\langle 1 + \Psi \rangle_r} \left[\left\langle \left(1 + \frac{J_{z,p}}{\tilde{n} - J_{z,p}} \right) \frac{\partial_r \Psi}{r} \right\rangle_r + \left\langle \frac{\partial_\xi A_r}{r} \right\rangle_r + \left\langle \frac{\partial_r A_{z,p}}{r} \right\rangle_r - \langle J_{z,b} \rangle_r \right], \quad (31a)$$

$$C_b = \frac{1}{\langle 1 + \Psi \rangle_r} \left[\left\langle \frac{\partial_r A_{z,b}}{r} \right\rangle_r + \langle J_{z,b} \rangle_r \right], \quad (31b)$$

$$C_d = \frac{\langle \partial_\xi \Psi \rangle_r}{\langle 1 + \Psi \rangle_r}, \quad (31c)$$

where the radial moments are defined by

$$\langle \Phi(r) \rangle_r = \frac{1}{N} \int_0^\infty \tilde{n}(r) \Phi(r) r dr. \quad (32)$$

Hence, if current densities and potentials are known for the azimuthally symmetric case, the above coefficients can be computed, allowing for the investigation of the plasma centroid evolution along the beam by means of Eq. (22).

An electron beam implies $C_b \geq 0$, such that a positive beam centroid deviation X_b exerts a “kick” in the positive x -direction onto the plasma centroid X_p . For $C_p > 0$, the plasma centroid is restored by the self-consistent plasma electron and ion fields; however, if $C_p < 0$, the self-consistent plasma and ion fields enhance the plasma centroid deviation X_p . The term in Eq. (22) results in a damping of the plasma centroid for $C_d > 0$, while the plasma centroid is amplified for $C_d < 0$.

III. BLOWOUT REGIME AND NARROW BEAMS

A. Plasma wake centroid equation

As an example, we consider the nonlinear blowout regime.⁷ In Ref. 8, the evolution of the blowout centroid subject to a narrow beam with asymmetric charge distribution was derived by studying the perturbed dynamics of a plasma electron at the boundary of the blowout and sheath. In the present work, we study the collective response of the electron sheath to beam centroid and plasma centroid displacements and show that by considering a finite sheath thickness, we obtain a different blowout channel equation with different and/or reduced coupling coefficients.

In this context, “narrow beam” refers to the case when the overlap of beam current density and electron sheath is negligible, $\langle J_{z,b} \rangle_r \simeq 0$. Hence, for the narrow beam case, the coefficients (31a), (31b), and (31c) simplify to

$$C_p = -\frac{1}{\langle 1 + \Psi \rangle_r} \left[\left\langle \left(1 + \frac{J_{z,p}}{\tilde{n} - J_{z,p}} \right) \frac{\partial_r \Psi}{r} \right\rangle_r + \left\langle \frac{\partial_\xi A_r}{r} \right\rangle_r + \left\langle \frac{\partial_r A_{z,p}}{r} \right\rangle_r \right], \quad (33a)$$

$$C_b = \frac{1}{\langle 1 + \Psi \rangle_r} \left[\left\langle \frac{\partial_r A_{z,b}}{r} \right\rangle_r \right], \quad (33b)$$

$$C_d = \frac{\langle \partial_\xi \Psi \rangle_r}{\langle 1 + \Psi \rangle_r}. \quad (33c)$$

The explicit computation of the radial moments in (33a), (33b), and (33c) requires the knowledge of the sheath current and charge density distributions and the potentials within the sheath. For this purpose we employ the blowout model in Ref. 18 in which the spatial distribution of the sheath plasma electron density is assumed to be given by

$$\tilde{n} = \tilde{n}_s \exp \left(-\frac{r-R}{\Delta_\rho} \right) \Theta(r-R), \quad (34)$$

with the blowout radius $R = R(\xi)$, the sheath thickness Δ_ρ , the peak sheath electron density $\tilde{n}_s = \tilde{n}_s(\xi)$, and $\Theta(x)$ is the Heaviside step function (compare illustration in Fig. 1). Following Ref. 18, we also assume that the quantity $S = -(\tilde{n} - J_{z,p})$ follows the same radial dependence as $\tilde{n}$.

The plasma electron current is assumed to be given by the functional dependence

$$J_{z,p} = J_s \exp \left(-\frac{r-R}{\Delta_J} \right) \Theta(r-R), \quad (35)$$

where $J_s = J_s(\xi)$ is the peak return current and Δ_J is the current layer thickness. We use the results obtained in Ref. 18 for the electromagnetic potentials $\Psi(\xi, r)$, $A_r(\xi, r)$, $A_{z,b}(\xi, r)$, and $A_{z,p}(\xi, r)$, with the approximation that the normalized sheath thickness is much smaller than the local blowout radius $\Delta_\rho/R \ll 1$. This thin sheath approximation is consistent with results from PIC simulations (see, e.g., Refs. 19 and 20). The plasma return-current layer thickness is assumed to spread over a plasma skin depth, $\Delta_J \simeq 1$, as assumed in Ref. 18.

Using these assumptions, the coefficient (33a), which determines the magnitude of the plasma response to a plasma centroid deviation in Eq. (22), yields

$$C_p = \frac{1 - R'^2}{4} - \frac{\Delta_\rho}{4} \left[\frac{1 + \Lambda}{R} + \frac{R(R'^2 - R^2 - 1)}{4} - \frac{R^2 R'}{2} \right] + \mathcal{O} \left(\frac{\Delta_\rho^2}{R^2} \right), \quad (36)$$

where $R'(\xi) = \partial_\xi R(\xi)$ is the slope of the blowout radius and $\Lambda(\xi) = 4I_b/I_A$ is the normalized beam current, where the beam current is denoted by $I_b(\xi)$ and the Alfvén current by $I_A \simeq 17$ kA. Equation (36) implies a restoring effect of the plasma wakefields onto the plasma centroid if $R'^2 \lesssim 1$ and an amplifying effect if $R'^2 > 1$. The latter can be explained by the fact that the wakefield exerts a force in the positive r -direction onto sheath electrons at the rear of the blowout (see, for example, Refs. 21 and 22). Hence, at the rear of the

FIG. 1. Illustration of a blowout channel and the relevant beam and plasma quantities.

blowout, where R^2 is large, wakefield perturbations may be enhanced towards the vertex of the blowout.

The coefficient (33b), defining the magnitude of the expelling force exerted onto the plasma centroid by the beam centroid, becomes

$$C_b = \frac{\Lambda}{R^2} \left[1 - \Delta_p \left(\frac{2}{R} + \frac{R}{4} \right) \right] + \mathcal{O} \left(\frac{\Delta_p^2}{R^2} \right). \quad (37)$$

The centroid deviation of an electron beam acts as a force onto the plasma centroid, where the magnitude of the effect scales with the beam current and inversely with the square of the blowout radius. A finite sheath thickness Δ_p reduces this effect. Analogously, the damping-coefficient (33c) reads

$$C_d = \frac{R'}{4} \left[R - \frac{\Delta_p}{2} \left(\frac{R^2}{2} - 1 \right) \right] + \mathcal{O} \left(\frac{\Delta_p^2}{R^2} \right). \quad (38)$$

The coefficient C_d therefore implies a damping of the plasma centroid due to a relativistic mass gain of electrons in the sheath if $R' > 0$ and an amplification owing to a relativistic mass loss for $R' < 0$.

B. Discussion and comparison to previous work

We compare these results to the plasma centroid equations obtained in previous works. In the seminal work on the hose instability in plasma wakefield accelerators, Whittum *et al.*⁵ assumed an adiabatically generated ion channel at the charge neutralization radius and with non-relativistic sheath electrons. Hence, the channel centroid equation used by Whittum *et al.* corresponds to Eq. (22) with $C_p = C_b = 1$ and $C_d = 0$.

In a more recent work, it was shown that this assumption greatly overestimates the growth rate of the hose instability in the nonlinear blowout regime.⁸ In Ref. 8, the plasma centroid equation was generalized by investigation of the dynamics of an electron at the blowout-sheath boundary so as to furnish a hosing model which includes the effects of a varying beam current, changing blowout radius, and relativistic velocities of electrons in the sheath. The channel centroid equation in Ref. 8 is given by Eq. (1), where $c_r = \Lambda/R^2$ and $c_\psi = [1 + \Psi(R)]^{-1}$. This is equivalent to Eq. (22) with coefficients $C_p = C_b = c_r c_\psi \simeq \Lambda/R^2 (1 - \Delta_p R/2) + \mathcal{O}(\Delta_p^2/R^2)$ and $C_d = 0$.

In the limit $\Delta_p \rightarrow 0$, the model in Ref. 8 and the coefficient C_b derived in this work, Eq. (37), recover the same result $C_b \rightarrow \Lambda/R^2$. That is, for $\Delta_p = 0$, the impact of the beam centroid deviation onto the plasma centroid deviation scales linearly with the beam current and inversely with the square of the blowout radius. However, for a finite sheath thickness, $\Delta_p > 0$ we find a reduced coefficient C_b for blowout radii $R \sim 1$. This is shown in Fig. 2, which depicts a comparison of C_b/Λ for varying blowout radii as predicted by the model derived within the present work and from the model in Ref. 8. It can be seen that the model derived within this work predicts a smaller value of C_b/Λ for moderate radii and finite Δ_p .

In addition, and in contrast to previous models^{5,8} we find a coefficient corresponding to the restoring force of the plasma centroid which differs from the coefficient for the driving

FIG. 2. Depiction of the coefficient C_b , relative to the beam current Λ as a function of the blowout radius R for differing sheath thicknesses Δ_p . Solid lines refer to C_b derived within the present work and dashed lines to $C_b (= C_p)$ in Ref. 8.

force ($C_p \neq C_b$). It should be noted that the plasma centroid equation (1) used in previous models^{5,8} has the unphysical property that $\partial_\xi X_p = \text{const}$ if the beam current drops to zero ($\Lambda = 0$, $c_r c_\psi = 0$). This means that previous models predict that the plasma centroid to maintain the slope it has at the beam tail. In contrast, for the model derived within this work, $C_p \neq 0$ for $\Lambda = 0$. Hence, the plasma centroid in this model experiences a restoring force and oscillates after the beam passed according to Eq. (22). The correct reproduction of this physical behavior is especially important for the study of hosing of a driver-witness pair, where the witness beam is subject to the plasma centroid deviations generated by the drive beam.

Furthermore, our model includes the effect of damping or amplification of the channel centroid deviation ($C_d \neq 0$) owing to a relativistic mass gain or loss of the sheath electrons as they propagate along the blowout. This effect is expected to be significant at the rear of the blowout where E_z and the slope of the blowout radius can be substantial. Hence, a nonzero damping can be of particular relevance for the study of hosing of a witness beam which might be positioned near the end of the blowout cavity.

IV. COMPARISON TO SIMULATION RESULTS

In this section, we compare the new model to results from 3D PIC simulations using the code HiPACE.¹² We investigate the hosing of a drive beam with a triangular current profile. The beam parameters are the same as the ones used in Ref. 8. The beam has an energy of $\gamma_0 = 55\,773$, and a transverse Gaussian density distribution with transverse dimensions $\sigma_x = \sigma_y = 13.0 \times 10^{-3}$. The current profile is triangular with the current rising from zero at $\xi = -0.8$ to the peak value of $\hat{I}_b = 6.65$ kA at $\xi = -0.53$. From this position, the current decreases linearly to zero at $\xi = 3.19$. The initial beam centroid is given by $X_{b,0}(\xi) = 4.17 \times 10^{-3} \times \xi \Theta(\xi)$, such that a linear tilt is introduced for the beam from position $\xi = 0$. The beam propagates through a flat-top plasma target with density n_0 and drives a plasma wave in the blowout regime.

We investigate how the channel/plasma centroid evolves along ξ if excited by the initial centroid deviation of the beam. In order to be less dependent on the model for the blowout used, we evaluate the blowout radius $R(\xi)$ from the

FIG. 3. Plasma centroid deviation along ξ relative to initial beam centroid deviation at $t=0$ with $\Delta\rho = 0.2$.

PIC simulation. The blowout radius $R(\xi)$ and the change of the blowout radius $R'(\xi)$ are then used to compute the coefficients C_p , C_b , and C_d in the present model and the coefficients c_r and c_ψ in the model from Ref. 8. We then solve numerically Eq. (22) and compare this to the numerical solution of Eq. (1) of the model in Ref. 8. The structure of the sheath in the simulation is compatible with Eq. (34) with $\Delta\rho = 0.2$. The results for $X_{p,0}/X_{b,0}$, where $X_{p,0} = X_p(\xi, t=0)$ and $X_{b,0} = X_b(\xi, t=0)$, are depicted in Fig. 3. The relative plasma centroid deviation obtained from the PIC simulation by interpolation to the zero crossing of the transverse fields $E_x - B_y$ is also shown. It can be seen that the new model (red line) is in good qualitative agreement with the PIC result (gray line), while the previous model (blue line) overestimates the channel centroid deviation.

In addition, the temporal beam centroid evolution is investigated. The beam is defined by the parameters above and the centroid is probed at the tail ($\xi = 3$). We numerically evaluate the model in Refs. 8 and 9 with the coupled differential equations for the plasma centroid,⁸ Eq. (1), and the beam centroid,⁹ Eq. (2). For the new model, we accordingly evaluate the new channel centroid equation (22) and the beam centroid equation (2) from Ref. 9. Furthermore, the longitudinal field $E_z(\xi)$ is extracted from the PIC simulation in order to avoid dependencies on the used blowout model.

Figure 4 depicts the comparison of the beam centroid at $k_p\xi = 3$ as resulting from the model in Refs. 8 and 9 (blue line) and from the model for the plasma centroid derived within this work using the beam centroid from Ref. 9 (red line). Additionally, the beam centroid obtained from a PIC

FIG. 4. Beam centroid evolution in time (normalized to the initial betatron frequency of the beam, $\omega_{\beta,0}$) at the tail ($\xi = 3$). For the curves obtained from the models, $\Delta\rho = 0.2$ is used in agreement with the sheath thickness observed in the PIC simulation.

simulation with HiPACE is shown (gray line). It can be seen that the beam centroid evolution expected by the present model is in good agreement with the PIC result, while the previous model overestimates the beam centroid deviation.

V. CONCLUSION

In this work, a generalized plasma centroid equation for the description of hosing in PWFAs in the blowout regime is derived. The differential equation for the plasma centroid is derived starting from the Vlasov equation and by considering the collective dynamics of the plasma electron sheath. This approach yields a mathematical model that predicts a reduced coupling between beam and plasma sheath and hence, accordingly reduced hosing growth rates for a finite sheath thickness compared to previous models. We also obtain a term associated with the damping/amplification of the plasma centroid, originating from the relativistic mass gain/loss of electrons in the sheath. Furthermore, a different coefficient for the restoring force of the plasma centroid is obtained which accounts for the fact that the plasma centroid is oscillating after the beam has passed. The inclusion of these effects yields results that are in better agreement with PIC simulations. The presented model is more accurate and general in the sense that it allows for the study of hosing of drive and witness beam pairs in PWFAs in the blowout regime, which was not possible with previous models. Hence, the presented model provides an important basis for the accurate modeling of hosing in PWFAs and for the possibility to study the stability of compact plasma-based accelerators.

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APPENDIX A: CONVERGENCE OF THE EXPANSION FOR THE BLOWOUT REGIME

In this Appendix, we evaluate Eq. (17) in Sec. II B

$$\begin{aligned} \left\langle \frac{p_r \cos \theta}{1 + \psi} \right\rangle &= \left\langle \frac{p_r \cos \theta}{\langle 1 + \psi \rangle} \sum_{k=0}^{\infty} (-1)^k \left(\frac{\psi - \langle \psi \rangle}{\langle 1 + \psi \rangle} \right)^k \right\rangle \\ &= \left\langle \frac{p_r \cos \theta - \langle p_r \cos \theta \rangle}{\langle 1 + \psi \rangle} \sum_{k=0}^{\infty} (-1)^k \left(\frac{\psi - \langle \psi \rangle}{\langle 1 + \psi \rangle} \right)^k \right\rangle \\ &\quad + \frac{\langle p_r \cos \theta \rangle}{\langle 1 + \psi \rangle} \left\langle \sum_{k=0}^{\infty} (-1)^k \left(\frac{\psi - \langle \psi \rangle}{\langle 1 + \psi \rangle} \right)^k \right\rangle, \quad (\text{A1}) \end{aligned}$$

for the blowout regime. Applying the expansion (5) to the covariance term (second line) of Eq. (A1) shows that this term vanishes by construction of f_p as an expansion with respect to $f_{p,0}$. We then note that $\langle \psi \rangle$ is dominated by the large fraction of electrons near the peak of the sheath density

where the density decay along the radial position is much greater than the decay of the wakefield potential $|\partial_r n/n| \gg |\partial_r \Psi/\Psi|$. Hence, $|\langle \psi \rangle| \leq |\Psi(R)| \sim \mathcal{O}(\Delta_\rho R)$ [compare $\Psi(R)$ computed in Ref. 18], where $\Delta_\rho \sim 0.1$ is the sheath thickness and $R \sim 1$ is the blowout radius. The first ($k=1$) central moment is zero and the higher order central moments ($k>1$) scale as $|\langle (\psi - \langle \psi \rangle)^k \rangle| \sim \mathcal{O}[(\Delta_\rho R)^k] \ll 1$. This is consistent with observations from PIC simulations. The second term (last line) of Eq. (A1) converges as

$$\begin{aligned} & \frac{\langle p_r \cos \theta \rangle}{\langle 1 + \psi \rangle} \left\langle \sum_{k=0}^{\infty} (-1)^k \left(\frac{\psi - \langle \psi \rangle}{\langle 1 + \psi \rangle} \right)^k \right\rangle \\ &= \frac{\langle p_r \cos \theta \rangle}{\langle 1 + \psi \rangle} + \mathcal{O}[(\Delta_\rho R)^2]. \end{aligned} \quad (\text{A2})$$

Hence, the series expansion in Sec. II B can be approximated as

$$\left\langle \frac{p_r \cos \theta}{1 + \psi} \right\rangle \simeq \frac{\langle p_r \cos \theta \rangle}{\langle 1 + \psi \rangle}. \quad (\text{A3})$$

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