Double parton scattering: basics and recent theory developments

M. Diehl

Deutsches Elektronen-Synchroton DESY

BNL, 7 June 2018
Hadron-hadron collisions

- standard description based on factorisation formulae
  \[ \text{cross sect} = \text{parton distributions} \times \text{parton-level cross sect} \]

- net transverse momentum \( p_T \) of hard-scattering products:
  - \( p_T \) integrated cross sect \( \sim \) collinear factorisation
  - \( p_T \ll \) hard scale of interaction \( \sim \) TMD factorisation
  - small \( x \) \( \sim \) high-energy factorisation, CGC: not discussed here

- particles resulting from interactions between spectator partons unobserved
Hadron-hadron collisions

- standard description based on factorisation formulae
  \[ \text{cross sect} = \text{parton distributions} \times \text{parton-level cross sect} \]

- net transverse momentum \( p_T \) of hard-scattering products:
  - \( p_T \) integrated cross sect \( \sim \) collinear factorisation
  - \( p_T \ll \) hard scale of interaction \( \sim \) TMD factorisation
  - small \( x \) \( \sim \) high-energy factorisation, CGC: not discussed here

- particles resulting from interactions between spectator partons unobserved

- spectator interactions can be soft \( \sim \) underlying event
  or hard \( \sim \) multiparton interactions

- here: double parton scattering with factorisation formula
  \[ \text{cross sect} = \text{double parton distributions} \times \text{parton-level cross sections} \]
Single vs. double parton scattering (SPS vs. DPS)

▶ example: prod’n of two gauge bosons, transverse momenta $q_1$ and $q_2$

single scattering:

$|q_1|$ and $|q_2| \sim$ hard scale $Q$

$|q_1 + q_2| \ll Q$

▶ for transv. momenta $\sim \Lambda \ll Q$:

$$\frac{d\sigma_{SPS}}{d^2 q_1 \, d^2 q_2} \sim \frac{d\sigma_{DPS}}{d^2 q_1 \, d^2 q_2} \sim \frac{1}{Q^4 \Lambda^2}$$

but single scattering populates larger phase space:

$$\sigma_{SPS} \sim \frac{1}{Q^2} \gg \sigma_{DPS} \sim \frac{\Lambda^2}{Q^4}$$
Single vs. double parton scattering (SPS vs. DPS)

(example: prod’n of two gauge bosons, transverse momenta $q_1$ and $q_2$

single scattering:

$|q_1|$ and $|q_2| \sim$ hard scale $Q$

$|q_1 + q_2| \ll Q$

double scattering:

both $|q_1|$ and $|q_2| \ll Q$

for small parton mom. fractions $x$

double scattering enhanced by parton luminosity

depending on process: enhancement or suppression

from parton type (quarks vs. gluons), coupling constants, etc.
A numerical example

gauge boson pair production

\[ W^+ + W^+ \]

single scattering:
\[ qq \rightarrow qq + W^+W^+ \]
suppressed by \( \alpha_s^2 \)

\[ W^+ + W^+ \]

integrated cross section

J Gaunt et al, arXiv:1003.3953
DPS cross section: collinear factorisation

\[
\frac{d\sigma_{\text{DPS}}}{dx_1 d\bar{x}_1 dx_2 d\bar{x}_2} = \frac{1}{C} \hat{\sigma}_1 \hat{\sigma}_2 \int d^2 y \ F(x_1, x_2, y) F(\bar{x}_1, \bar{x}_2, y)
\]

- \( C \) = combinatorial factor
- \( \hat{\sigma}_i \) = parton-level cross sections
- \( F(x_1, x_2, y) \) = double parton distribution (DPD)
- \( y \) = transv. distance between partons

- follows from Feynman graphs and hard-scattering approximation
  no semi-classical approximation required

- can make \( \hat{\sigma}_i \) differential in further variables (e.g. for jet pairs)

- can extend \( \hat{\sigma}_i \) to higher orders in \( \alpha_s \)
  get usual convolution integrals over \( x_i \) in \( \hat{\sigma}_i \) and \( F \)

Paver, Treleani 1982, 1984; Mekhfi 1985, . . . , MD, Ostermeier, Schäfer 2012
**DPS cross section: TMD factorisation**

- for measured transv. momenta

\[
\frac{d\sigma_{\text{DPS}}}{dx_1 d\bar{x}_1 d^2q_1 dx_2 d\bar{x}_2 d^2q_2} = \frac{1}{C} \hat{\sigma}_1 \hat{\sigma}_2 \\
\times \int \frac{d^2z_1}{(2\pi)^2} \frac{d^2z_2}{(2\pi)^2} e^{-i(z_1 q_1 + z_2 q_2)} \int d^2y F(x_i, z_i, y) F(\bar{x}_i, z_i, y)
\]

- \( F(x_i, z_i, y) = \text{double-parton TMDs} \)
- \( z_i = \text{Fourier conjugate to parton transverse mom.} \ k_i \)

- operator definition as for TMDs: **schematically have**

\[
F(x_i, z_i, y) = \mathcal{F}T_{z_i \rightarrow x_i p^+} \langle p | \bar{q} \left(-\frac{1}{2} z_2\right) \Gamma_2 q \left(\frac{1}{2} z_2\right) \bar{q} \left(y - \frac{1}{2} z_1\right) \Gamma_1 q \left(y + \frac{1}{2} z_1\right) |p\rangle
\]

- to be completed by renormalisation, Wilson lines, soft factors
- essential for studying factorisation, scale and rapidity dependence
Introduction
Parton correlations
Problems!
A solution
Numerics
Factorisation
Summary

DPS cross section: TMD factorisation

- for measured transv. momenta

\[ \frac{d\sigma_{\text{DPS}}}{dx_1 \, d\bar{x}_1 \, d^2 q_1 \, dx_2 \, d\bar{x}_2 \, d^2 q_2} = \frac{1}{C} \hat{\sigma}_1 \hat{\sigma}_2 \]

\[ \times \int \frac{d^2 z_1}{(2\pi)^2} \frac{d^2 z_2}{(2\pi)^2} e^{-i(z_1 q_1 + z_2 q_2)} \int d^2 y \, F(x_i, z_i, y) F(\bar{x}_i, z_i, y) \]

- \( F(x_i, z_i, y) = \) double-parton TMDs
  \( z_i = \) Fourier conjugate to parton transverse mom. \( k_i \)

- operator definition as for TMDs: schematically have

\[ F(x_i, z_i, y) = \mathcal{F}T \left[ \left\langle p | \bar{q} \left( -\frac{1}{2}z_2 \right) \Gamma_2 q \left( \frac{1}{2}z_2 \right) \bar{q} \right( y - \frac{1}{2}z_1 \right) \Gamma_1 q \left( y + \frac{1}{2}z_1 \right) | p \right\rangle \]

- to be completed by renormalisation, Wilson lines, soft factors
- essential for studying factorisation, scale and rapidity dependence
- analogous def for collinear distributions \( F(x_i, y) \)
  \( \Rightarrow \) not a twist-four operator but product of two twist-two operators
Pocket formula

- make simplest possible assumptions

- if two-parton density factorises as

\[ F(x_1, x_2, y) = f(x_1) f(x_2) G(y) \]

where \( f(x_i) = \) usual PDF

- if assume same \( G(y) \) for all parton types
  then cross sect. formula turns into

\[
\frac{d\sigma_{\text{double}}}{dx_1 d\bar{x}_1 dx_2 d\bar{x}_2} = \frac{1}{C} \frac{d\sigma_1}{dx_1 d\bar{x}_1} \frac{d\sigma_2}{dx_2 d\bar{x}_2} \frac{1}{\sigma_{\text{eff}}}
\]

with \( 1/\sigma_{\text{eff}} = \int d^2 y \ G(y)^2 \)

\( \sim \) scatters are completely independent

- underlies bulk of phenomenological estimates

- fails if any of the above assumptions is invalid
  or if original cross sect. formula misses important contributions

Experimental investigations (incomplete)

- CDF 4 jets (1993)
- CDF $\gamma + 3$ jets (1997)
- CDF reanalysis, Bahr et al (2013)
- D0 $\gamma + 3$ jets (2009)
- D0 $\gamma + 3$ jets (2014)
- D0 $\gamma + 2$ jets + b/c jet (2014)
- D0 $2\gamma + 2$ jets (2015)
- CMS $W + 2$ jets (2013)
- ATLAS $W + 2$ jets (2013)
- ATLAS 4 jets (2016)
- LHCb $\Upsilon + D$ (2015)
- LHCb $J/\Psi + \Upsilon$ (2011)
- D0 $J/\Psi + J/\Psi$ (2014)
- D0 $J/\Psi + \Upsilon$ (2015)
- ATLAS $J/\Psi + J/\Psi$ (2016)

- other channels:
  - double open charm $C + C$ with $C = D^0, D^+, D_s^+, \Lambda_c^+$ LHCb 2012
  - $\Upsilon + \Upsilon$ (estimate $\sigma_{eff} \approx 2.2 \div 6.6$ mb) CMS 2016
  - same-sign $WW$ (LHC run 2) CMS 2017
Parton correlations

- if neglect correlations between two partons
  \[ F(x_1, x_2, y) = \int d^2b \ f(x_1, b + y) f(x_2, b) \]
  where \( f(x_i, y) = \) impact parameter dependent single-parton density

- and if neglect correlations between \( x \) and \( y \) of single parton
  \[ f(x_i, y) = f(x_i) F(y) \]

then in pocket formula
\[
G(y) = \int d^2b \ F(b) F(b + y)
\]

- for Gaussian \( F(b) \) with average \( \langle b^2 \rangle \)
  \[ \sigma_{\text{eff}} = 4\pi \langle b^2 \rangle = 41 \text{ mb} \times \langle b^2 \rangle/(0.57 \text{ fm})^2 \]

determinations of \( \langle b^2 \rangle \) from form factors or GPDs: (0.57 fm – 0.67 fm)^2

- if \( F(b) \) is Fourier trf. of dipole then 41 mb \( \rightarrow \) 36 mb

is \( \gg \) \( \sigma_{\text{eff}} \sim 2 \) to 21 mb from experimental extractions
Parton correlations

at certain level of accuracy expect correlations between

- $x_1$ and $x_2$ of partons
  - most obvious: energy conservation $\Rightarrow x_1 + x_2 \leq 1$
  - significant $x_1 - x_2$ correlations found in quark models

Chang, Manohar, Waalewijn 2012; Rinaldi et al 2013–16
Broniowski et al 2013–16; Kasemets, Mukherjee 2016
Parton correlations

at certain level of accuracy expect correlations between

- $x_1, x_2$ and $y$
  even for single partons see correlations between $x$ and $b$ distribution

  - HERA results on $\gamma p \rightarrow J/\Psi p$ give
    \[
    \langle b^2 \rangle \propto \text{const} + 4\alpha' \log(1/x) \quad \text{with} \quad 4\alpha' \approx (0.16 \text{ fm})^2
    \]
    for gluons with $x \sim 10^{-3}$
  
  - lattice calculations of $x^0, x^1, x^2$ moments
    $\rightarrow$ strong decrease of $\langle b^2 \rangle$ with $x$ above $\sim 0.1$

  - JLab@12 and EIC will give $\gg$ complete picture over wide $x$ range

plausible to expect similar correlations in double parton distributions
even if two partons not uncorrelated

impact on observables: L Frankfurt, M Strikman, C Weiss 2003
  
  R Corke, T Sjöstrand 2011; B Blok, P Gunnellini 2015
Mind the difference

- single parton imaging in $ep$
  impact parameter distributions $f(x, b)$:
  $b$ is Fourier conjugate to measurable momentum transfer $p - p'$

- double parton distributions $F(x_i, y)$:
  distance $y$ is unobservable
  cross section $\propto \int d^2 y \, F(x_i, y) \, F(\bar{x}_i, y)$

- $f(x, b)$: no direct information on two-parton correlations
  but provides baseline prediction $F(x_1, x_2, y) = \int d^2 b \, f(x_1, b + y) \, f(x_2, b)$
  if correlations are absent
Double parton scattering: ultraviolet problem

\[
\frac{d\sigma_{\text{DPS}}}{dx_1 dx_1 d\bar{x}_2 d\bar{x}_2} = \frac{1}{C} \hat{\sigma}_1 \hat{\sigma}_2 \int d^2 y \ F(x_1, x_2, y) F(\bar{x}_1, \bar{x}_2, y)
\]

- for \( y \ll 1/\Lambda \) can compute

\[
F(x_1, x_2, y) \sim \frac{1}{y^2} \text{ splitting fct} \otimes \text{usual PDF}
\]

gives strong correlations between two partons
Double parton scattering: ultraviolet problem

\[ \frac{d\sigma_{\text{DPS}}}{dx_1 d\bar{x}_1 dx_2 d\bar{x}_2} = \frac{1}{C} \hat{\sigma}_1 \hat{\sigma}_2 \int d^2 y \ F(x_1, x_2, y) F(\bar{x}_1, \bar{x}_2, y) \]

- for \( y \ll 1/\Lambda \) can compute

\[ F(x_1, x_2, y) \sim \frac{1}{y^2} \] splitting fct \( \otimes \) usual PDF

\[ \text{gives UV divergent cross section } \propto \int d^2 y / y^4 \]

in fact, formula not valid for \(|y| \sim 1/Q\)

- problem also for two-parton TMDs

UV divergences logarithmic instead of quadratic
...and more problems

- **Double counting** problem between double scattering with splitting \((1v1)\) and single scattering at loop level

  MD, Ostermeier, Schäfer 2011; Gaunt, Stirling 2011; Gaunt 2012
  Blok, Dokshitzer, Frankfurt, Strikman 2011; Ryskin, Snigirev 2011, 2012
  already noted by Cacciari, Salam, Sapeta 2009

- Also have graphs with splitting in one proton only: “2v1”

  \[
  \sim \int \frac{d^2 y}{y^2} \times F_{\text{int}}(x_1, x_2, y)
  \]

  B Blok et al 2011-13
  J Gaunt 2012
  B Blok, P Gunnellini 2015
A consistent solution

MD, J. Gaunt, K. Schönwald 2017

\[ \sigma_{\text{DPS}} \propto \int d^2 y \, \Phi^2(\nu y) \, F(x_1, x_2, y) \, F(\bar{x}_1, \bar{x}_2, y) \]

- \( \Phi \to 0 \) for \( u \to 0 \) and \( \Phi \to 1 \) for \( u \to \infty \), e.g. \( \Phi(u) = \theta(u - 1) \)
- cutoff scale \( \nu \sim Q \)
- \( F(x_1, x_2, y) \) has both splitting and ‘intrinsic’ contributions

- analogous regulator for transverse-momentum dependent DPDs

- keep definition of DPDs as operator matrix elements
- cutoff in \( y \) does not break symmetries that haven’t already been broken
A consistent solution

MD, J. Gaunt, K. Schönwald 2017

- regulate DPS: \( \sigma_{\text{DPS}} \propto \int d^2y \ \Phi^2(\nu y) \ F(x_1, x_2, y) \ F(\bar{x}_1, \bar{x}_2, y) \)
  - \( \Phi \to 0 \) for \( u \to 0 \) and \( \Phi \to 1 \) for \( u \to \infty \), e.g. \( \Phi(u) = \theta(u - 1) \)
  - cutoff scale \( \nu \sim Q \)
  - \( F(x_1, x_2, y) \) has both splitting and 'intrinsic' contributions

- analogous regulator for transverse-momentum dependent DPDs

- full cross section: \( \sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub}} + \sigma_{\text{SPS}} \)
  - subtraction \( \sigma_{\text{sub}} \) to avoid double counting:
    - \( = \sigma_{\text{DPS}} \) with \( F \) computed for small \( y \) in fixed order perturb. theory
    - much simpler computation than \( \sigma_{\text{SPS}} \) at given order
A consistent solution

MD, J. Gaunt, K. Schönwald 2017

regulate DPS: \( \sigma_{\text{DPS}} \propto \int d^2 y \ \Phi^2(\nu y) \ F(x_1, x_2, y) \ F(\bar{x}_1, \bar{x}_2, y) \)

- \( \Phi \to 0 \) for \( u \to 0 \) and \( \Phi \to 1 \) for \( u \to \infty \), e.g. \( \Phi(u) = \theta(u - 1) \)
- cutoff scale \( \nu \sim Q \)
- \( F(x_1, x_2, y) \) has both splitting and 'intrinsic' contributions
  analogous regulator for transverse-momentum dependent DPDs

full cross section: \( \sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub}}(1v_1 + 2v_1) + \sigma_{\text{SPS}} + \sigma_{\text{tw2} \times \text{tw4}} \)

- subtraction \( \sigma_{\text{sub}} \) to avoid double counting:
  = \( \sigma_{\text{DPS}} \) with \( F \) computed for small \( y \) in fixed order perturb. theory
  much simpler computation than \( \sigma_{\text{SPS}} \) at given order
- can also include twist 2 \( \times \) twist 4 contribution
  and double counting subtraction for 2v1 term
Subtraction formalism at work

\[ \sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub}} + \sigma_{\text{SPS}} \]

- for \( y \sim 1/Q \) have \( \sigma_{\text{DPS}} \approx \sigma_{\text{sub}} \)
  because pert. computation of \( F \) gives good approx. at considered order
  \( \Rightarrow \sigma \approx \sigma_{\text{SPS}} \)
  dependence on \( \Phi(\nu y) \) cancels between \( \sigma_{\text{DPS}} \) and \( \sigma_{\text{sub}} \)

- for \( y \gg 1/Q \) have \( \sigma_{\text{sub}} \approx \sigma_{\text{SPS}} \)
  because DPS approximations work well in box graph
  \( \Rightarrow \sigma \approx \sigma_{\text{DPS}} \)
  with regulator fct. \( \Phi(\nu y) \approx 1 \)

- same argument for 2v1 term and \( \sigma_{\text{tw2} \times \text{tw4}} \) (were neglected above)

- subtraction formalism works order by order in perturb. theory

J. Collins, Foundations of Perturbative QCD, Chapt. 10
Double counting in TMD factorisation for DPS

- left and right box can independently be collinear or hard:
  - \( \sim \) DPS, DPS/SPS interference and SPS
- get nested double counting subtractions

M Buffing, MD, T Kasemets 2017
DGLAP evolution

- Define DPDs as matrix elements of renormalised twist-two operators:

\[ F(x_1, x_2, y; \mu_1, \mu_2) \sim \langle p | O_1(0; \mu_1) O_2(y; \mu_2) | p \rangle \quad f(x; \mu) \sim \langle p | O(0; \mu) | p \rangle \]

\[ \Rightarrow \text{separate DGLAP evolution for partons 1 and 2:} \]

\[ \frac{\partial}{\partial \log \mu_i^2} F(x_i, y; \mu_i) = P \otimes x_i F \quad \text{for } i = 1, 2 \]

- DGLAP logarithm from strongly ordered region \(|q_1| \ll |k| \sim |q_2| \ll Q_2\) repeats itself at higher orders (ladder graphs)

- Resummed by DPD evolution in \(\sigma_{DPS}\) if take \(\nu \sim \mu_1 \sim Q_1, \mu_2 \sim Q_2\) and appropriate initial conditions (→ next slide)

- Can enhance DPS region over SPS region \(|q_1| \sim |q_2| \sim Q_{1,2}\) which dominates by power counting
A model study

- take DPD model with \( F = F_{\text{spl}} + F_{\text{int}} \)

\[
F_{\text{spl}}(x_1, x_2, y; 1/y^*, 1/y^*) = F_{\text{perturb.}}(y^*) e^{-y^2 \Lambda^2} \quad \text{with} \quad y^* = \frac{y}{\sqrt{1 + y^2/y_{\text{max}}^2}}
\]

inspired by \( b^* \) of Collins, Soper, Sterman

\[
F_{\text{int}}(x_1, x_2, y; \mu_0, \mu_0) = f(x_1; \mu_0) f(x_2; \mu_0) \Lambda^2 e^{-y^2 \Lambda^2 / \pi}
\]

description simplified, actual model slightly refined

- \( F_{\text{perturb.}}(y) \) ensures correct perturbative behaviour at small \( y \)

DGLAP logarithms built up between splitting scale \( \sim 1/y^* \) and \( \sim Q \)

- in SPS subtraction term take instead

\[
F_{\text{spl}}(x_1, x_2, y; Q, Q) = F_{\text{perturb.}}(y)
\]

hard scattering at fixed order, no resummation here

- following plots: show double parton luminosity

\[
\mathcal{L} = \int d^2 y \Phi^2(\nu y) F(x_1, x_2, y) F(\bar{x}_1, \bar{x}_2, \bar{y})
\]

with separate contributions from 1v1, 2v1, 2v2
DPS parton luminosities for illustration, model parameters not tuned

- plot $\mathcal{L}$ vs. rapidity $Y$ of $q_1$ with $q_2$ central (left) or at $-Y$ (right) with $\mu_{1,2} = Q_{1,2} = M_W$ at $\sqrt{s} = 14$ TeV

- blue band: vary $\nu$ from $0.5 M_W \ldots 2 M_W$
- yellow band: naive scale variation for $\sigma_{1v1} \propto \nu^2$ from $\int dy^2 \left(1/y^2\right)^2$ from $b_0^2/\nu^2$

if $\nu$ variation large then need $-\sigma_{sub} (1v1) + \sigma_{SPS}$

$\sim$ use $1v1$ as estimate for importance of SPS at high orders
DPS parton luminosities for illustration, model parameters not tuned

- plot $\mathcal{L}$ vs. rapidity $Y$ of $q_1$ with $q_2$ central (left) or at $-Y$ (right) with $\mu_{1,2} = Q_{1,2} = M_W$ at $\sqrt{s} = 14$ TeV
- blue band: vary $\nu$ from $0.5 M_W \ldots 2 M_W$
- yellow band: naive scale variation for $\sigma_{1v1} \propto \nu^2$ from $\int dy^2 \left(1/y^2\right)^2$

- large rapidity separation $\sim x_1$ and $x_2$ asymmetric
- region $y \gg 1/\nu$ in 1v1 enhanced by DPD evolution
- evolved $F_{\text{spl}}$ less steep than fixed-order $1/y^2$
DPS parton luminosities for illustration, model parameters not tuned

- plot $\mathcal{L}$ vs. rapidity $Y$ of $q_1$ with $q_2$ central (left) or at $-Y$ (right) with $\mu_{1,2} = Q_{1,2} = M_W$ at $\sqrt{s} = 14$ TeV

- blue band: vary $\nu$ from $0.5 M_W \ldots 2 M_W$
  yellow band: naive scale variation for $\sigma_{1v1} \propto \nu^2$ from $\int dy^2 \left(1/y^2\right)^2$

$gg$

- gluons: prominent evolution effects at all $Y$
DPS parton luminosities for illustration, model parameters not tuned

- plot $\mathcal{L}$ vs. rapidity $Y$ of $q_1$ with $q_2$ central (left) or at $-Y$ (right) with $\mu_{1,2} = Q_{1,2} = M_W$ at $\sqrt{s} = 14$ TeV

- blue band: vary $\nu$ from $0.5 M_W \ldots 2 M_W$
  yellow band: naive scale variation for $\sigma_{1v1} \propto \nu^2$

  \[ \int_{b_0^2/\nu^2} dy^2 \left(\frac{1}{y^2}\right)^2 \]

- $u\bar{d}$ induced by splitting at $\mathcal{O}(\alpha_s^2)$, e.g. by $u \rightarrow ug \rightarrow udd\bar{d}$
DPS parton luminosities for illustration, model parameters not tuned

- plot $L$ vs. $x = x_1 = x_2 = \bar{x}_1 = \bar{x}_2$ at fixed $\sqrt{s}$
  $\mu_{1,2} = Q_{1,2} = x\sqrt{s}$

$gg$

$\sqrt{s} = 14$ TeV

$\sqrt{s} = 100$ TeV

- DPS region enhanced for small $x$ by evolution
Factorisation: single Drell-Yan

- fast-moving longitudinal gluons coupling to hard scattering
  - include in Wilson lines in parton density
- soft gluon exchange between left- and right-moving partons
  - include in soft factors = vevs of Wilson lines
    - needs: eikonal approximation, Ward identities, Glauber cancellation
  - essential for establishing factorisation
  - permits resummation of Sudakov logarithms
TMD factorisation

Collins, Soper, Sterman 1980s; Collins 2011
Double parton scattering

- can generalise previous treatment from single to double Drell-Yan and other DPS processes

- basic steps can be repeated:
  - collinear gluons \(\sim\) Wilson lines in DPDs
  - soft gluons \(\sim\) soft factor, Glauber gluons cancel

- colour structure \(\gg\) complicated
  matrices in colour space, final result reasonably simple

MD, D Ostermeier, A Schäfer 2011; MD, J Gaunt, P Plößl, A Schäfer 2015

A Vladimirov 2016–17; M Buffing, T Kasemets, MD 2017
Summary

- double parton scattering important in specific kinematics/for specific processes
- transverse distance between partons $\sim$ aspects of imaging
- significant progress towards a systematic formulation of factorisation in QCD
- solution for UV problem of DPS $\leftrightarrow$ double counting with SPS
  - simple UV regulator for DPS using distance $y$ between partons
  - simple subtraction term to avoid double counting
  - naturally includes DGLAP logarithms in DPS
  - DPS can dominate for small $x_1$ and/or $x_2$ thanks to evolution
- factorisation complicated by colour structure but tractable
  - have full generalisation of CSS formula to double Drell-Yan