The bootstrap approach to $4d$ superconformal field theories

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3d Ising Model

[El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi]
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→ Saturated by 3d Ising model
3d Ising Model

[El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi]

→ Saturated by 3d Ising model
→ 3d Ising lives at “kink”
3d Ising Model

[Poland Simmons-Duffin Kos, Simmons-Duffin, Poland Simmons-Duffin Kos Vichi]

Comparison to Monte Carlo
→ One $\mathbb{Z}_2$–even, one $\mathbb{Z}_2$–odd relevant scalar operator
Outline

1. The Superconformal Bootstrap Program
2. Numerical bootstrap
3. Inversion formula
4. Summary and Outlook
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2. Numerical bootstrap
3. Inversion formula
4. Summary and Outlook
What is the space of consistent $4d$ SCFTs?
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→ Maximally supersymmetric theories: well known list of theories
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→ $\mathcal{N} = 2$ theories: large known list of theories
   many lacking a Lagrangian description
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The Superconformal Bootstrap Program

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Can we bootstrap specific theories?
The Superconformal Bootstrap Program

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Can we bootstrap specific theories?

→ “Simplest” $\mathcal{N} = 2$ Argyres-Douglas theory?
Conformal field theory defined by
Set of local operators and their correlation functions
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CFT data
\{ O_{\Delta, \ell, \ldots} (x) \} and
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Operator Product Expansion
\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_k \lambda_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_k} \mathcal{O}_k(0)
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→ Finite radius of convergence
Conformal Bootstrap

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\[ O_1(x)O_2(0) = \sum_{k_{\text{prim.}}} \lambda_{O_1 O_2 O_k} c(x, \partial_x)O_k(0) \]

→ Finite radius of convergence
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\rightarrow \text{ Finite radius of convergence }

\rightarrow \text{ } n\text{-point function by recursive use of the OPE until } \langle 1 \rangle = 1
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→ Finite radius of convergence
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CFT data strongly constrained

- Unitarity
- Associativity of the operator product algebra
Conformal Bootstrap

Crossing Symmetry

\[ \langle (O_1(x_1) \, O_2(x_2)) \, O_3(x_3) \, O_4(x_4) \rangle = \]

\[ \sum_{\Delta, \ell} O_{\Delta, \ell} \]

\[ 1 \rightarrow \bullet \rightarrow 3 \rightarrow 4 \]

\[ 2 \rightarrow \bullet \rightarrow \]

\[ O_{\Delta, \ell} \]

where \( \Delta_{O_i} = \Delta_{O}, u = x_{12} x_{23} x_{24} \)

\[ v = (1 - \bar{z}) (1 - \bar{z}) \]
Crossing Symmetry

\[ \langle \mathcal{O}_1(x_1)(\mathcal{O}_2(x_2) \mathcal{O}_3(x_3))\mathcal{O}_4(x_4) \rangle = \sum_{\mathcal{O}_{\Delta, \ell}} \mathcal{O}_{\Delta, \ell} \]

\[ = \sum_{\tilde{\mathcal{O}}_{\Delta, \ell}} \tilde{\mathcal{O}}_{\Delta, \ell} \]
Crossing Symmetry

\[
\langle (\mathcal{O}_1(x_1) \mathcal{O}_2(x_2)) \mathcal{O}_3(x_3) \mathcal{O}_4(x_4) \rangle = 
\]

\[
\sum_{\Delta, \ell} \mathcal{O}_{\Delta, \ell} \cdot \mathcal{O}_{\Delta, \ell} = \sum_{\tilde{\Delta}, \ell} \tilde{\mathcal{O}}_{\Delta, \ell}
\]

\[
\frac{1}{x_{12}^{2\Delta} x_{34}^{2\Delta}} \sum_{\Delta, \ell} \lambda_{\mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_\Delta, \ell} \lambda_{\mathcal{O}_3 \mathcal{O}_4 \mathcal{O}_\Delta, \ell} g_{\Delta, \ell}(z, \bar{z}) =
\]

where \( \Delta_{\mathcal{O}_i} = \Delta_{\mathcal{O}} \), \( u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z \bar{z} \), \( v = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2} = (1 - z)(1 - \bar{z}) \)
Conformal Bootstrap

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\]

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\textbf{The Superconformal Bootstrap}

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**The Superconformal Bootstrap**

- Various conformal families related by action of supercharges
- Finite re-organization of an infinite amount of data
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A solvable subsector

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$4d \mathcal{N} \geq 2$ SCFTs $\rightarrow$ 2d chiral algebra

▶ Protected subsector of $4d$ SCFT isomorphic to $2d$ chiral algebra
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$4d \mathcal{N} \geq 2$ SCFTs $\rightarrow$ $2d$ chiral algebra

- Protected subsector of $4d$ SCFT isomorphic to $2d$ chiral algebra
- Exact results for protected operators
Landscape of $\mathcal{N} \geq 2$ SCFTs

- Stress tensor supermultiplet
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$\Rightarrow$ Saturated by $\mathcal{N} = 4$ SYM with gauge group $SU(2)$
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$c \geq \frac{11}{30}$ [Liendo Ramirez Seo]
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The \((A_1, A_2)\) Argyres-Douglas theory

Argyres-Douglas theories

→ Originally obtained on the Coulomb branch of an \(\mathcal{N} = 2\) susy gauge theory with gauge groups \(SU(3)\)
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→ \(a = \frac{43}{120}\) \(c = \frac{11}{30}\)
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\[ \Delta_\phi = \frac{6}{5} \]
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→ Chiral algebra = Lee-Yang minimal model
Beyond the protected subsector

Our tools
Beyond the protected subsector

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- Numerical bootstrap
  [Rattazzi Rychkov Tonni Vichi]
Our tools

- Numerical bootstrap
  [Rattazzi Rychkov Tonni Vichi]

- Lightcone bootstrap
  [Fitzpatrick Kaplan Poland Simmons-Duffin, Komargodski Zhiboedov]
Beyond the protected subsector

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  \[\leftrightarrow \text{Lorentizan inversion formula of } [\text{Caron-Huot}]\]
Outline

1 The Superconformal Bootstrap Program

2 Numerical bootstrap

3 Inversion formula

4 Summary and Outlook
Numerical bootstrap

- Solving crossing equations $\Rightarrow$ constraining space of solutions
  - How large can an OPE coefficient be?

\[
\sum_{\Delta, \ell} \phi_{\Delta, \ell} \phi = \sum_{\tilde{\Delta}, \tilde{\ell}} \tilde{\phi}_{\Delta, \ell} \tilde{\phi} = 1 \quad \text{for states not on the identity operator axis.}
\]
Numerical bootstrap

- Solving crossing equations $\Rightarrow$ constraining space of solutions
  $\rightarrow$ How large can an OPE coefficient be?

**Sum rule: identical scalars $\phi$**

\[
\frac{1}{x_{12}^{2\Delta_\phi} x_{34}^{2\Delta_\phi}} \sum_{\mathcal{O}_{\Delta,\ell}} \lambda^2_{\phi\phi} \mathcal{O}_{\Delta,\ell} g_{\Delta,\ell}(z, \bar{z}) = \\
\frac{1}{x_{14}^{2\Delta_\phi} x_{23}^{2\Delta_\phi}} \sum_{\tilde{\mathcal{O}}_{\Delta,\ell}} \lambda^2_{\phi\phi} \tilde{\mathcal{O}}_{\Delta,\ell} g_{\Delta,\ell}(1 - z, 1 - \bar{z})
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Numerical bootstrap

- Solving crossing equations $\Rightarrow$ constraining space of solutions
  $\leftrightarrow$ How large can an OPE coefficient be?

**Sum rule: identical scalars $\phi$**

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\frac{1}{x_{12}^{2\Delta_{\phi}} x_{34}^{2\Delta_{\phi}}} \sum_{\mathcal{O}_{\Delta,\ell}} \lambda_{\phi\phi}^2 \mathcal{O}_{\Delta,\ell} g_{\Delta,\ell}(z, \bar{z}) =
$$

$$
\frac{1}{x_{14}^{2\Delta_{\phi}} x_{23}^{2\Delta_{\phi}}} \sum_{\bar{\mathcal{O}}_{\Delta,\ell}} \lambda_{\phi\phi}^2 \bar{\mathcal{O}}_{\Delta,\ell} g_{\Delta,\ell}(1 - z, 1 - \bar{z})
$$

$\leftrightarrow$ Identity operator $\lambda_{\phi\phi} \mathbb{1} = 1$
Numerical bootstrap

- Solving crossing equations $\implies$ constraining space of solutions
  - How large can an OPE coefficient be?

**Sum rule: identical scalars $\phi$**

\[
\frac{1}{x_{12}^{2\Delta} x_{34}^{2\Delta}} \sum_{\Delta, \ell} \lambda_{\phi\phi}^2 \mathcal{O}_{\Delta, \ell} \, g_{\Delta, \ell}(z, \bar{z}) = \\
\frac{1}{x_{14}^{2\Delta} x_{23}^{2\Delta}} \sum_{\bar{\Delta}, \bar{\ell}} \lambda_{\phi\phi}^2 \mathcal{\bar{O}}_{\bar{\Delta}, \bar{\ell}} \, g_{\Delta, \ell}(1 - z, 1 - \bar{z})
\]

- Identity operator $\lambda_{\phi\phi} \mathbb{I} = 1$

\[
\sum_{\mathcal{O} \in \phi \phi \mathcal{O}, \mathcal{O}_{\Delta, \ell} \neq \mathbb{I}} \lambda_{\phi\phi}^2 \mathcal{O} \frac{u^{\Delta_{\phi}} g_{\Delta, \ell}(v, u) - v^{\Delta_{\phi}} g_{\Delta, \ell}(u, v)}{v^{\Delta_{\phi}} - u^{\Delta_{\phi}}} = 1
\]

\[
\frac{\mathcal{O}}{F_{\Delta, \ell}}
\]
Sum rule

\[ \sum_{\mathcal{O} \in \phi \phi, \mathcal{O}_{\Delta, \ell} \neq \mathbb{1}} \lambda_{\phi \phi \mathcal{O}}^2 F_{\Delta, \ell} = 1 \]
Numerical bootstrap

Sum rule

\[ \lambda_{\phi\phi}^2 \mathcal{O}_{\Delta_*, \ell_*} F_{\Delta_*, \ell_*} + \sum_{\mathcal{O} \in \phi \phi, \mathcal{O}_{\Delta_{\ell}} \neq 1} \lambda_{\phi\phi}^2 \mathcal{O} F_{\Delta, \ell} = 1 \]
Numerical bootstrap

Sum rule

\[ \lambda^{2}_{\phi \phi \mathcal{O}_{\Delta_{*}, \ell_{*}}} \psi \cdot F_{\Delta_{*}, \ell_{*}} + \sum_{\mathcal{O} \in \phi \phi, \mathcal{O}_{\Delta_{\ell}} \neq \mathbb{1}, \mathcal{O} \neq \mathcal{O}_{\Delta_{*}, \ell_{*}}} \lambda^{2}_{\phi \phi \mathcal{O}} \psi \cdot F_{\Delta, \ell} = \psi \cdot 1 \]

- Find Functional \( \psi \) such that
Numerical bootstrap

Sum rule

\[ \lambda_{\phi \phi}^{2} \psi \cdot F_{\Delta_{\ast}, \ell_{\ast}} + \sum_{\phi \not= \phi} \lambda_{\phi \phi}^{2} \psi \cdot F_{\Delta, \ell} = \psi \cdot 1 \]

- Find Functional \( \psi \) such that
  \[ \psi \cdot F_{\Delta_{\ast}, \ell_{\ast}}(u, v) = 1 \]
Numerical bootstrap

Sum rule

\[ \lambda_{\phi\phi O_{\Delta^*,\ell^*}}^2 \psi \cdot F_{\Delta^*,\ell^*} + \sum_{O \in \phi \neq O_{\Delta^*,\ell^*}} \lambda_{\phi\phi O}^2 \psi \cdot F_{\Delta,\ell} = \psi \cdot 1 \]

▶ Find Functional \( \psi \) such that

\[ \psi \cdot F_{\Delta^*,\ell^*}(u, v) = 1 \]

\[ \psi \cdot F_{\Delta,\ell}(u, v) \geq 0 \text{ for all } \{\Delta, \ell\} \text{ in spectrum} \]
Numerical bootstrap

**Sum rule**

\[ \lambda^2_{\phi\phi, O_{\Delta^*,\ell^*}} \psi \cdot F_{\Delta^*,\ell^*} + \sum_{O \in \phi, O_{\Delta,\ell} \neq 1, O \neq O_{\Delta^*,\ell^*}} \lambda^2_{\phi\phi, O} \psi \cdot F_{\Delta,\ell} = \psi \cdot 1 \]

- Find Functional \( \psi \) such that
  - \( \psi \cdot F_{\Delta^*,\ell^*}(u, v) = 1 \)
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  - Minimize \( \psi \cdot 1 \)
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\[ \lambda_{\phi\phi,\Delta_*,\ell_*}^2 \psi \cdot F_{\Delta_*,\ell_*} + \sum_{\mathcal{O} \in \phi\phi, \mathcal{O}_{\Delta,\ell} \neq 1} \lambda_{\phi\phi,\mathcal{O},\ell}^2 \psi \cdot F_{\Delta,\ell} = \psi \cdot 1 \]

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  - Minimize \( \psi \cdot 1 \)

- \[ \lambda_{\phi\phi,\Delta_*,\ell_*}^2 \leq \psi \cdot 1 \]
Numerical bootstrap

Sum rule

\[ \lambda^{2}_{\phi\phi} O_{\Delta,\ell} \psi \cdot F_{\Delta,\ell} = \sum_{O \in \phi\phi, \Delta, \ell \neq \phi\phi, \Delta, \ell} \lambda^{2}_{\phi\phi} O \psi \cdot F_{\Delta,\ell} = \psi \cdot 1 \]

- Find Functional \( \psi \) such that
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  - Minimize \( \psi \cdot 1 \)

- \( \lambda^{2}_{\phi\phi} O_{\Delta,\ell} \leq \psi \cdot 1 \)

- Truncate \( \psi = \sum_{m,n} a_{mn} \partial_{z}^{m} \partial_{\bar{z}}^{n} \bigg|_{z=\bar{z}=\frac{1}{2}} \)
Numerical bootstrap

**Sum rule**

\[
\lambda^2_{\phi\phi\mathcal{O}_{\Delta^*,\ell^*}} \psi \cdot F_{\Delta^*,\ell^*} + \sum_{\mathcal{O} \in \phi\phi, \mathcal{O}_{\Delta,\ell} \neq \mathcal{1}} \lambda^2_{\phi\phi\mathcal{O}} \psi \cdot F_{\Delta,\ell} = \psi \cdot 1
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- **Find Functional** \( \psi \) such that
  - \( \psi \cdot F_{\Delta^*,\ell^*}(u, v) = 1 \)
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  - **Minimize** \( \psi \cdot 1 \)

- \( \lambda^2_{\phi\phi\mathcal{O}_{\Delta^*,\ell^*}} \leq \psi \cdot 1 \)

- **Truncate** \( \psi = \sum_{m,n \leq \Lambda} a_{mn} \partial_z^m \partial_{\bar{z}}^n \bigg|_{z=\bar{z}=\frac{1}{2}} \)

  - Increase \( \Lambda \Rightarrow \) bounds get stronger
Numerical bootstrap

Sum rule

\[ \lambda_{\phi\phi O_{\Delta^*,\ell^*}}^2 \psi \cdot F_{\Delta^*,\ell^*} + \sum_{O \in \phi\phi, O_{\Delta\ell} \neq 1} \lambda_{\phi\phi O}^2 \psi \cdot F_{\Delta,\ell} = \psi \cdot 1 \]

- Find Functional \( \psi \) such that
  \[ \psi \cdot F_{\Delta^*,\ell^*}(u, v) = 1 \]
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  \[ \text{Minimize } \psi \cdot 1 \]

- \[ \lambda_{\phi\phi O_{\Delta^*,\ell^*}}^2 \leq \psi \cdot 1 \]

- Truncate \( \psi = \sum_{m,n \leq \Lambda} a_{mn} \partial_z^m \partial_{\bar{z}}^n \big|_{z=\bar{z}=\frac{1}{2}} \)
  \[ \text{Increase } \Lambda \Rightarrow \text{bounds get stronger} \]
  \[ \text{Always true bounds} \]
The “simplest” Argyres-Douglas theory

Approach it through Coulomb branch

- $\mathcal{N} = 2$ chiral operator $\phi$, $Q^I_{\alpha} \phi = 0$
  $\Delta_{\phi} = r$

\[ \langle \phi \bar{\phi} \phi \rangle \quad \text{Two channels: } \phi \bar{\phi} \text{ and } \bar{\phi} \phi \]

Conformal blocks $\Rightarrow$ superconformal blocks (only in $\bar{\phi} \phi$)

[Fitzpatrick, Kaplan, Khandker, Li, Poland, Simmons-Duffin]
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Approach it through Coulomb branch

- $\mathcal{N} = 2$ chiral operator $\phi$ \quad $Q^I_\alpha \phi = 0$

  $\Delta_\phi = r$ \quad $r : \quad U(1)_r$ charge
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Approach it through Coulomb branch

- $\mathcal{N} = 2$ chiral operator $\phi \quad Q^I_{\alpha}\phi = 0$
  $\Delta_\phi = r \quad r : \quad U(1)_r$ charge

- $\mathcal{N} = 2$ anti-chiral operator $\bar{\phi} \quad \tilde{Q}_I\bar{\alpha}\bar{\phi} = 0$
  $\Delta_\phi = -r$
The “simplest” Argyres-Douglas theory

Approach it through Coulomb branch

- $\mathcal{N} = 2$ chiral operator $\phi$ \quad $Q^I_{\alpha} \phi = 0$
  \[ \Delta_{\phi} = r \quad r : \quad U(1)_r \text{ charge} \]

- $\mathcal{N} = 2$ anti-chiral operator $\bar{\phi}$ \quad $\bar{Q}_I \bar{\phi} = 0$
  \[ \Delta_{\phi} = -r \]

\[ \Delta_{\phi} = \frac{6}{5} \]
The “simplest” Argyres-Douglas theory

Approach it through Coulomb branch

- \( \mathcal{N} = 2 \) chiral operator \( \phi \)
  \[ Q^I_{\alpha} \phi = 0 \]
  \[ \Delta_{\phi} = r \quad r : \quad U(1)_r \text{ charge} \]

- \( \mathcal{N} = 2 \) anti-chiral operator \( \bar{\phi} \)
  \[ \bar{Q}^{I\dot{\alpha}} \bar{\phi} = 0 \]
  \[ \Delta_{\phi} = -r \]

\[ \Delta_{\phi} = \frac{6}{5} \]
and \( c = \frac{11}{30} \) \( \langle TT \rangle \sim c \)
The “simplest” Argyres-Douglas theory

Approach it through Coulomb branch

- $\mathcal{N} = 2$ chiral operator $\phi \quad Q^I_{\alpha \bar{\alpha}} \phi = 0$
  \[ \Delta_{\phi} = r \quad r : \quad U(1)_r \text{ charge} \]
- $\mathcal{N} = 2$ anti-chiral operator $\bar{\phi} \quad \bar{Q}^I_{\alpha \bar{\alpha}} \bar{\phi} = 0$
  \[ \Delta_{\phi} = -r \]
- $\boxed{\Delta_{\phi} = \frac{6}{5}}$ and $c = \frac{11}{30} \quad (\langle TT \rangle \sim c)$
- $\langle \phi \phi \bar{\phi} \bar{\phi} \rangle$
The “simplest” Argyres-Douglas theory

Approach it through Coulomb branch

- $\mathcal{N} = 2$ chiral operator $\phi$ \quad $Q^I_{\alpha} \phi = 0$
  \[ \Delta \phi = r \quad r : \quad U(1)_r \text{ charge} \]

- $\mathcal{N} = 2$ anti-chiral operator $\bar{\phi}$ \quad $\bar{Q}^I_{\dot{\alpha}} \bar{\phi} = 0$
  \[ \Delta \phi = -r \]

- \[ \Delta \phi = \frac{6}{5} \quad \text{and} \quad c = \frac{11}{30} \quad (\langle TT \rangle \sim c) \]

- $\langle \phi \phi \bar{\phi} \bar{\phi} \rangle$

- Two channels: $\phi \phi$ and $\bar{\phi} \bar{\phi}$
The “simplest” Argyres-Douglas theory

Approach it through Coulomb branch

- $\mathcal{N} = 2$ chiral operator $\phi$ \quad $Q^I_{\alpha \phi} = 0$
  \[
  \Delta_{\phi} = r \quad r : \quad U(1)_r \text{ charge}
  \]

- $\mathcal{N} = 2$ anti-chiral operator $\bar{\phi}$ \quad $\bar{Q}^I_{\dot{\alpha} \bar{\phi}} = 0$
  \[
  \Delta_{\bar{\phi}} = -r
  \]

- $\Delta_{\phi} = \frac{6}{5}$ and $c = \frac{11}{30}$ \quad ($\langle TT \rangle \sim c$)

- $\langle \phi \phi \bar{\phi} \bar{\phi} \rangle$

- Two channels: $\phi \phi$ and $\bar{\phi} \bar{\phi}$

- Conformal blocks $\rightsquigarrow$ superconformal blocks (only in $\bar{\phi} \bar{\phi}$)

[Fitzpatrick Kaplan Khandker Li Poland Simmons-Duffin]
Minimum allowed central charge

Does $\langle \phi \bar{\phi} \phi \bar{\phi} \rangle$ know about $c \geq \frac{11}{30}$?
Does $\langle \phi \bar{\phi} \phi \bar{\phi} \rangle$ know about $c \geq \frac{11}{30}$?
Bounding OPE coefficients

\[ \phi \phi \sim \phi^2 + \ldots \]

\[ \Delta = 2\Delta_\phi \]

“\(\phi^2\)” protected dimension \(\Delta = 2\Delta_\phi\), unknown OPE coefficient
Bounding OPE coefficients

\[ \phi \phi \sim \phi^2 + \ldots \]

\[ \Delta = 2\Delta_{\phi} \]

"\( \phi^2 \)" protected dimension \( \Delta = 2\Delta_{\phi} \), unknown OPE coefficient

\[
\begin{array}{c}
\lambda_{\mathcal{E}_{12/5}}^2 \\
0.32 & 0.33 & 0.34 & 0.35 & 0.36 \\
2.10 & 2.12 & 2.14 & 2.16 & 2.18 & 2.20
\end{array}
\]

excluded
1 The Superconformal Bootstrap Program
2 Numerical bootstrap
3 Inversion formula
4 Summary and Outlook
The lightcone bootstrap

\[ \bar{z} \to 1 \Rightarrow \Delta \to 2\Delta \phi + 2n + \ell(\phi \Box n \partial_\mu_1 ... \partial_\mu_\ell \phi) \]

\[ \text{\textbackslash{}t-channel dominated by lowest twist } \tau_m \text{ operators} \]

\[ \to \text{behavior reproduced by infinite sum over } s\text{-channel spins} \]

\[ \Rightarrow \text{Large spin spectrum of CFT} \]
The lightcone bootstrap

\( \bar{z} \to 1 \) at fixed \( z \)

Diagram:
- \( s \)-channel
- \( t \)-channel
- Point \( (\frac{1}{2}, \frac{1}{2}) \)
The lightcone bootstrap

\[ \bar{z} \rightarrow 1 \text{ at fixed } z \]

\[
\lim_{\bar{z} \rightarrow 1} (1 - \bar{z})^{2\Delta} \sum_{\Delta, \ell} \lambda_{\phi \phi}^{2} O_{\Delta, \ell} \cdot g_{\Delta, \ell}(z, \bar{z})
\]

\( (\frac{1}{2}, \frac{1}{2}) \)
The lightcone bootstrap

\[ \bar{z} \to 1 \text{ at fixed } z \]

\[
\lim_{\bar{z} \to 1} (1 - \bar{z})^{2\Delta} \sum_{\Delta, \ell} \lambda^2_{\phi\phi} O_{\Delta, \ell} \ g_{\Delta, \ell}(z, \bar{z})
\]

\[
= \left( f(z) + (1 - \bar{z})^{\tau_m} f_{\tau_m, \ell}(z) + \ldots \right)
\]
The lightcone bootstrap

\[ \bar{z} \rightarrow 1 \text{ at fixed } z \]

\[ \lim_{\bar{z} \rightarrow 1} (1 - \bar{z})^{2\Delta} \phi \sum_{\Delta, \ell} \lambda_{\phi \phi}^{2} O_{\Delta, \ell} g_{\Delta, \ell}(z, \bar{z}) \]

\[ = \left( f(z) + (1 - \bar{z})^{\tau m} f_{\tau m, \ell}(z) + \ldots \right) \]
The lightcone bootstrap

\[ \bar{z} \to 1 \text{ at fixed } z \]

\[ \lim_{\bar{z} \to 1} (1 - \bar{z})^{2\Delta} \sum_{\Delta, \ell} \lambda_{\phi \phi}^{2} O_{\Delta, \ell} \ g_{\Delta, \ell}(z, \bar{z}) \]

\[ = \left( f(z) + (1 - \bar{z})^{\tau_{m}} f_{\tau_{m}, \ell}(z) + \ldots \right) \]

\[ \tau_{m} = \Delta - \ell \]

\[ \rightarrow \ t-\text{channel dominated by lowest twist } \tau_{m} \text{ operators} \]
The lightcone bootstrap

\[ \bar{z} \to 1 \text{ at fixed } z \]

\[ \lim_{\bar{z} \to 1} (1 - \bar{z})^{2\Delta} \sum_{\Delta, \ell} \lambda_{\phi \phi}^2 O_{\Delta, \ell} \quad \text{with } g_{\Delta, \ell}(z, \bar{z}) \]

\[ \log(1 - \bar{z}) \text{ as } \bar{z} \to 1 \]

\[ = \left( f(z) + (1 - \bar{z})^{\tau_m} f_{\tau_m, \ell}(z) + \ldots \right) \]

\[ \tau_m = \Delta - \ell \]

\[ \tau_m = \Delta - \ell \]

\[ \to t\text{-channel dominated by lowest twist } \tau_m \text{ operators} \]
The lightcone bootstrap

$\bar{z} \to 1$ at fixed $z$

$$
\lim_{\bar{z} \to 1} (1 - \bar{z})^{2\Delta} \sum_{\Delta, \ell} \lambda_{\phi\phi}^{2} \mathcal{O}_{\Delta, \ell} \sum_{\ell} g_{\Delta, \ell}(z, \bar{z}) \log(1 - \bar{z}) \text{ as } \bar{z} \to 1
$$

$$
= \left( f(z) + (1 - \bar{z})^{\tau_{m}} f_{\tau_{m}, \ell}(z) + \ldots \right)
$$

$\rightarrow$ $t-$channel dominated by lowest twist $\tau_{m}$ operators

$\rightarrow$ behavior reproduced by infinite sum over $s-$channel spins
The lightcone bootstrap

\[ \tilde{z} \to 1 \text{ at fixed } z \]

\[ \lim_{\tilde{z} \to 1} (1 - \tilde{z})^{2\Delta} \sum_{\Delta, \ell} \lambda_{\phi\phi}^2 \mathcal{O}_{\Delta, \ell} \frac{g_{\Delta, \ell}(z, \tilde{z})}{\log(1 - \tilde{z}) \text{ as } \tilde{z} \to 1} \]

\[ = \left( \begin{array}{c}
\frac{1}{1} f(z) + (1 - \tilde{z})^{\tau_m} f_{\tau_m, \ell}(z) + \ldots \end{array} \right) \]

→ \text{\(t\)–channel dominated by lowest twist } \tau_m \text{ operators}

→ \text{behavior reproduced by infinite sum over } \text{\(s\)–channel spins}

⇝ \text{Large spin spectrum of CFT}
The lightcone bootstrap

\[ \bar{z} \to 1 \text{ at fixed } z \]

\[ \lim_{\bar{z} \to 1} (1 - \bar{z})^{2\Delta} \sum_{\Delta, \ell} \lambda_{\phi \phi}^2 \mathcal{O}_{\Delta, \ell} \frac{g_{\Delta, \ell}(z, \bar{z})}{\log(1 - \bar{z}) \text{ as } \bar{z} \to 1} \]

\[ = \left( f(z) + (1 - \bar{z})^{\tau_m} f_{\tau_m, \ell}(z) + \ldots \right) \]

\[ \Rightarrow \text{ } t-\text{channel dominated by lowest twist } \tau_m \text{ operators} \]

\[ \Rightarrow \text{ } \text{behavior reproduced by infinite sum over } s-\text{channel spins} \]

\[ \sim \text{ } \text{Large spin spectrum of CFT} \]

\[ \mathbb{1} \Rightarrow \Delta \to 2\Delta + 2n + \ell \quad (\phi \Box^n \partial_{\mu_1} \ldots \partial_{\mu_\ell} \phi) \]
**A Lorentizan inversion formula**

**Large spin perturbation theory**

→ Very successful for 3d Ising model

[Alday Zhiboedov, Simmons-Duffin]
A Lorentizan inversion formula

Large spin perturbation theory

→ Very successful for 3d Ising model
  
  [Alday Zhiboedov, Simmons-Duffin]

→ down to spin two!
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[Alday Zhiboedov, Simmons-Duffin]

→ down to spin two!

→ Invert $s$—channel OPE: Euclidean inversion formula
Large spin perturbation theory

→ Very successful for $3d$ Ising model

[Alday Zhiboedov, Simmons-Duffin]

→ down to spin two!

→ Invert $s$–channel OPE: Euclidean inversion formula

→ $c(\Delta, \ell)$ with poles where operators are, residues $\sim \lambda_{\Delta, \ell}^2$
A Lorentizan inversion formula

Large spin perturbation theory

→ Very successful for 3d Ising model
  [Alday Zhiboedov, Simmons-Duffin]
→ down to spin two!
→ Invert s—channel OPE: Euclidean inversion formula
→ $c(\Delta, \ell)$ with poles where operators are, residues $\sim \lambda^{2}_{\Delta,\ell}$
→ Need to know full correlation function to get full spectrum
Large spin perturbation theory

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[Alday Zhiboedov, Simmons-Duffin]

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→ [Caron-Huot] Inversion formula *analytic* in spin for $\ell > 1$

→ Operators organize in trajectories
A Lorentizan inversion formula

Large spin perturbation theory

→ Very successful for 3d Ising model
  [Alday Zhiboedov, Simmons-Duffin]
  down to spin two!
  Invert s–channel OPE: Euclidean inversion formula
  \( c(\Delta, \ell) \) with poles where operators are, residues \( \sim \lambda^{2}_{\Delta, \ell} \)
  Need to know full correlation function to get full spectrum
  only makes sense for integer \( \ell \)
  [Caron-Huot] Inversion formula analytic in spin
  for \( \ell > 1 \)
  Operators organize in trajectories
  large \( \ell \) dominated by low \( t \)–channel twists
Lorentzian inversion formula: Superconformal case

Invert $\phi \phi \text{ OPE}$
Lorentzian inversion formula: Superconformal case

Invert $\phi \phi \text{ OPE}$

→ Same as bosonic inversion, valid for $\ell > 1$
Lorentzian inversion formula: Superconformal case

**Invert** \(\phi\phi\) OPE

→ Same as bosonic inversion, valid for \(\ell > 1\)
→ Feed in \(\bar{\phi}\phi \sim 1 + \text{Stress tensor multiplet} + \ldots\)
Lorentzian inversion formula: Superconformal case

**Invert \( \phi \phi \) OPE**

\[ \rightarrow \text{Same as bosonic inversion, valid for } \ell > 1 \]
\[ \rightarrow \text{Feed in } \bar{\phi} \phi \sim 1 + \text{Stress tensor multiplet } + \ldots \]

**Invert \( \bar{\phi} \phi \) OPE**

\[ \rightarrow \text{Supersymmetric inversion: valid for } \ell \geq 0 \]
Lorentzian inversion formula: Superconformal case

**Invert $\phi\phi$ OPE**

→ Same as bosonic inversion, valid for $\ell > 1$
→ Feed in $\bar{\phi}\phi \sim 1 + \text{Stress tensor multiplet} + \ldots$

**Invert $\bar{\phi}\phi$ OPE**

→ Supersymmetric inversion: valid for $\ell \geq 0$
→ Feed in $\bar{\phi}\phi \sim 1 + \text{Stress tensor multiplet} + \ldots$
→ and $\phi\phi \sim \phi^2 + \ldots$
Lorentzian inversion formula: Superconformal case

**Invert $\phi \phi$ OPE**

→ Same as bosonic inversion, valid for $\ell > 1$
→ Feed in $\bar{\phi} \phi \sim 1 + \text{Stress tensor multiplet} + \ldots$

**Invert $\bar{\phi} \phi$ OPE**

→ Supersymmetric inversion: valid for $\ell \geq 0$
→ Feed in $\bar{\phi} \phi \sim 1 + \text{Stress tensor multiplet} + \ldots$
→ and $\phi \phi \sim \phi^2 + \ldots$

**Combine the two?**
A Lorentizan inversion formula

**Inverting the \(\phi\phi\) OPE**

→ Same as bosonic inversion, valid for \(\ell > 1\)
→ Only input: \(\bar{\phi}\phi \sim 1 + \text{Stress tensor multiplet}\)
A Lorentizan inversion formula

Inverting the \( \phi \phi \) OPE

\[ \phi \phi \sim \phi^2 + \lambda^2 \frac{C_\ell}{\Delta = 2\Delta_\phi} + \lambda^2 \frac{C_{\ell>0}}{\Delta = 2\Delta_\phi + \ell} + \ldots \]

→ Same as bosonic inversion, valid for \( \ell > 1 \)
→ Only input: \( \bar{\phi} \phi \sim 1 \) + Stress tensor multiplet
A Lorentizan inversion formula

Inverting the $\phi \phi$ OPE

→ Same as bosonic inversion, valid for $\ell > 1$
→ Only input: $\bar{\phi} \phi \sim 1 + \text{Stress tensor multiplet}$

$$\phi \phi \sim \phi^2 + \lambda^2_{C_{\ell}} + C_{\ell>0} + \ldots$$

$\Delta = 2\Delta_\phi$
$\Delta = 2\Delta_\phi + \ell$

![Graphs showing the behavior of different terms with respect to $\ell$ and $\lambda_{C_{\ell}}$](image-url)
1 The Superconformal Bootstrap Program
2 Numerical bootstrap
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4 Summary and Outlook
Constraints on the “simplest” Argyres-Douglas theory
Summary and Outlook

Constraints on the “simplest” Argyres-Douglas theory

→ Hybrid numeric-analytical approach?
Summary and Outlook

Constraints on the “simplest” Argyres-Douglas theory

→ Hybrid numeric-analytical approach?

Zooming in to other strongly coupled $\mathcal{N} = 2$ SCFTs

→ how should we approach them?
Constraining the space of $4d \mathcal{N} = 2$ SCFTs

$SU(2)$ flavor symmetry

- stress tensor and flavor current supermultiplets
Constraining the space of 4d $\mathcal{N} = 2$ SCFTs

**SU(2) flavor symmetry**

- stress tensor and flavor current supermultiplets

![Graph showing analytically ruled out regions](image)

[Beem ML Liendo Peelaers Rastelli van Rees, ML Liendo]
Constraining the space of $4d \mathcal{N} = 2$ SCFTs

**SU(2) flavor symmetry**
(similar for SU(3), SO(8), $E_6$, $E_7$, $E_8$, $G_2$, $F_4$)

- stress tensor and flavor current supermultiplets

[Beem ML Liendo Peelaers Rastelli van Rees, ML Liendo]
Summary and Outlook

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(at corners of $SU(2)$, $SU(3)$, $E_6$, $E_7$, $E_8$ exclusion curves)
Summary and Outlook

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→ Mixed system:

stress tensor - flavor current supermultiplet
Summary and Outlook

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(at corners of $SU(2)$, $SU(3)$, $E_6$, $E_7$, $E_8$ exclusion curves)

→ Mixed system:
  stress tensor - flavor current supermultiplet

→ Required superblocks not known!
Constraints on the “simplest” Argyres-Douglas theory

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Zooming in to other strongly coupled $\mathcal{N} = 2$ SCFTs
(at corners of $SU(2)$, $SU(3)$, $E_6$, $E_7$, $E_8$ exclusion curves)

→ Mixed system:
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→ Required superblocks not known!

→ Stronger numerical constraints on the space of theories?
  → Is $c_{4d}/k_{4d} \geq \ldots$?
Summary and Outlook

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Zooming in to other strongly coupled $\mathcal{N} = 2$ SCFTs
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→ Mixed system:
  stress tensor - flavor current supermultiplet

→ Required superblocks not known!

→ Stronger numerical constraints on the space of theories?
  ↔ Is $c_{4d}/k_{4d} \geq ...$?

→ What about $G_2$ and $F_4$?
Thank you!
5 A solvable subsector

6 $4d \mathcal{N} = 3$ SCFTs

7 Lorentizan inversion formula

8 Constraining the space of $\mathcal{N} = 2$ SCFTs
Chiral algebra

Organize operators in representations of superconformal algebra

\( \{ O_{\Delta,(j_1,j_2)}, \} \)
Organize operators in representations of superconformal algebra

\[ \{ \mathcal{O}_{\Delta,(j_1,j_2)}, R^\mathbb{R}, r^\mathbb{R}, f \} \]
Chiral algebra

Organize operators in representations of superconformal algebra

\{ O_{\Delta, (j_1, j_2)}, R, r, f \}

Claim

→ Pick a plane \( \mathbb{R}^2 \in \mathbb{R}^4 \),
Chiral algebra

Organize operators in representations of superconformal algebra
\[ \{ \mathcal{O}_{\Delta,(j_1,j_2)}, R \_{SU(2)_R \ U(1)_r}, r, f \} \]

Claim

\[ \rightarrow \text{ Pick a plane } \mathbb{R}^2 \in \mathbb{R}^4, \ (z, \bar{z}) \in \mathbb{R}^2 \]
Chiral algebra

Organize operators in representations of superconformal algebra

\[ \{ \mathcal{O}_{\Delta,(j_1,j_2)}, R, r, f \} \]

Claim

\[ \rightarrow \text{ Pick a plane } \mathbb{R}^2 \in \mathbb{R}^4, \ (z, \bar{z}) \in \mathbb{R}^2 \]

\[ \langle \mathcal{O}_1^{l_1}(z_1, \bar{z}_1) \ldots \mathcal{O}_n^{l_n}(z_n, \bar{z}_n) \rangle \]
Chiral algebra

Organize operators in representations of superconformal algebra

\[ \{ \mathcal{O}_{\Delta,(j_1,j_2)}, R, r, f \} \]

Claim

\[ \rightarrow \text{ Pick a plane } \mathbb{R}^2 \in \mathbb{R}^4, \ (z, \bar{z}) \in \mathbb{R}^2 \]
\[ \rightarrow \text{ Restrict to operators with } \Delta = 2R + j_1 + j_2 \]

\[ \langle \mathcal{O}_{1}^{l_1}(z_1, \bar{z}_1) \ldots \mathcal{O}_{n}^{l_n}(z_n, \bar{z}_n) \rangle \]
Chiral algebra

Organize operators in representations of superconformal algebra

\[ \{ \mathcal{O}_{\Delta,(j_1,j_2)}, \underbrace{R}_{SU(2)_R}, \underbrace{r}_{U(1)_R}, f \} \]

Claim

\[ \rightarrow \text{ Pick a plane } \mathbb{R}^2 \in \mathbb{R}^4, \ (z, \bar{z}) \in \mathbb{R}^2 \]
\[ \rightarrow \text{ Restrict to operators with } \Delta = 2R + j_1 + j_2 \]

\[ u_{l_1}(\bar{z}_1) \ldots u_{l_n}(\bar{z}_n) \langle \mathcal{O}_{1}^{l_1}(z_1, \bar{z}_1) \ldots \mathcal{O}_{n}^{l_n}(z_n, \bar{z}_n) \rangle \]
Chiral algebra

Organize operators in representations of superconformal algebra
\[ \{ \mathcal{O}_{\Delta,(j_1,j_2)}, \underbrace{R}_{SU(2)_R}, \underbrace{r}_{U(1)_{\Delta}}, f \} \]

Claim

\[ \rightarrow \text{ Pick a plane } \mathbb{R}^2 \in \mathbb{R}^4, \ (z, \bar{z}) \in \mathbb{R}^2 \]
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\[ u_{l_1}(\bar{z}_1) \ldots u_{l_n}(\bar{z}_n) \langle \mathcal{O}_{1}^{l_1}(z_1, \bar{z}_1) \ldots \mathcal{O}_{n}^{l_n}(z_n, \bar{z}_n) \rangle = f(z_i) \]

\[ \rightarrow \text{ Meromorphic!} \]
Why?

- Subsector = Cohomology of nilpotent $Q$
Why?

- Subsector = Cohomology of nilpotent $\mathbb{Q} \sim \mathbb{Q} + \mathcal{S}$
Chiral algebra

Why?

- Subsector = Cohomology of nilpotent \( Q \sim Q + S \)
- Cohomology at the origin \( \Rightarrow \) non-empty classes
Why?

- Subsector = Cohomology of nilpotent $\mathbb{Q} \sim \mathcal{Q} + \mathcal{S}$
- Cohomology at the origin $\Rightarrow$ non-empty classes
  \[ \Delta = 2R + j_1 + j_2 \]
Chiral algebra

Why?

- Subsector = Cohomology of nilpotent $\mathbb{Q} \sim \mathcal{Q} + S$
- Cohomology at the origin $\Rightarrow$ non-empty classes
- $\Delta = 2R + j_1 + j_2$
- On plane $\mathfrak{sl}_2 \times \mathfrak{sl}_2$
Chiral algebra

Why?

- Subsector = Cohomology of nilpotent $\mathcal{Q} \sim \mathcal{Q} + \mathcal{S}$
- Cohomology at the origin $\Rightarrow$ non-empty classes
  \[ \Delta = 2R + j_1 + j_2 \]
- On plane $\mathfrak{sl}_2 \times \mathfrak{sl}_2$
  \[ \text{commutes with } \mathcal{Q} \]
Chiral algebra

Why?

- Subsector = Cohomology of nilpotent $\mathbb{Q} \sim \mathcal{Q} + \mathcal{S}$
- Cohomology at the origin $\Rightarrow$ non-empty classes
  \[ \Delta = 2R + j_1 + j_2 \]
- On plane $\mathfrak{sl}_2 \times \mathfrak{sl}_2$
  - commutes with $\mathcal{Q}$
  - does not

$\mathfrak{su}(2) \mathbb{R}$ is $\mathbb{Q}$ exact $\Rightarrow$ anti-holomorphic dependence drops out
Why?

- Subsector = Cohomology of nilpotent $\mathbb{Q} \sim \mathbb{Q} + S$
- Cohomology at the origin $\Rightarrow$ non-empty classes
- $\Delta = 2R + j_1 + j_2$
- On plane $\mathfrak{sl}_2 \times \mathfrak{sl}_2$
  - commutes with $\mathbb{Q}$
  - does not
- twisted translations $u_I(\bar{z})$
Chiral algebra

Why?

- Subsector = Cohomology of nilpotent \( Q \sim Q + S \)
  
  → Cohomology at the origin ⇒ non-empty classes
  
  \[ \Delta = 2R + j_1 + j_2 \]

- On plane \( \mathfrak{sl}_2 \times \mathfrak{sl}_2 \)

  commutes with \( Q \)

  does not

  → twisted translations \( u_I(\bar{z}) \)

  → diagonal subalgebra \( \mathfrak{sl}_2 \times \mathfrak{su}(2)_R \) is \( Q \) exact
Chiral algebra

Why?

- Subsector = Cohomology of nilpotent $Q \sim Q + S$
- Cohomology at the origin $\Rightarrow$ non-empty classes
  $\Delta = 2R + j_1 + j_2$
- On plane $\mathfrak{sl}_2 \times \mathfrak{sl}_2$
  commutes with $Q$
  does not
- Twisted translations $u_1(\bar{z})$
- Diagonal subalgebra $\mathfrak{sl}_2 \times \mathfrak{su}(2)_R$ is $Q$ exact
- Anti-holomorphic dependence drops out
Chiral algebra

Example: free hypermultiplet

Complex scalars in hypermultiplet are in the cohomology
Chiral algebra

Example: free hypermultiplet

Complex scalars in hypermultiplet are in the cohomology

\[ Q' = \begin{bmatrix} Q \\ \tilde{Q}^* \end{bmatrix}, \quad \tilde{Q}' = \begin{bmatrix} \tilde{Q} \\ -Q^* \end{bmatrix} \]
Chiral algebra

Example: free hypermultiplet

Complex scalars in hypermultiplet are in the cohomology

\[ Q' = \begin{bmatrix} Q \\ \tilde{Q}^* \end{bmatrix}, \quad \tilde{Q}' = \begin{bmatrix} \tilde{Q} \\ -Q^* \end{bmatrix} \]

\[ u_I = (1, \bar{z}) \]
Example: free hypermultiplet

Complex scalars in hypermultiplet are in the cohomology

\[
Q' = \begin{bmatrix} Q \\ \tilde{Q}^* \end{bmatrix}, \quad \tilde{Q}' = \begin{bmatrix} \tilde{Q} \\ -Q^* \end{bmatrix}
\]

\[
u_I = (1, \bar{z})
\]

\[
q(z, \bar{z}) = u_I Q'
\]
Chiral algebra

Example: free hypermultiplet

Complex scalars in hypermultiplet are in the cohomology

\[ Q' = \begin{bmatrix} Q \\ \tilde{Q}^* \end{bmatrix}, \quad \tilde{Q}' = \begin{bmatrix} \tilde{Q} \\ -Q^* \end{bmatrix} \]

\[ u_I = (1, \bar{z}) \]

\[ q(z, \bar{z}) = u_I Q' = Q(z, \bar{z}) + \bar{z}\tilde{Q}^*(z, \bar{z}) \]
Chiral algebra

Example: free hypermultiplet

Complex scalars in hypermultiplet are in the cohomology

\[
Q' = \begin{bmatrix} Q \\ \tilde{Q}^* \end{bmatrix}, \quad \tilde{Q}' = \begin{bmatrix} \tilde{Q} \\ -Q^* \end{bmatrix}
\]

\[
u_I = (1, \bar{z})
\]

\[
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\]

\[
\tilde{q}(z, \bar{z}) = u_I \tilde{Q}'
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Example: free hypermultiplet

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\[ \to q(z, \bar{z})\tilde{q}(0) \sim \]
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\[ \rightarrow q(z, \bar{z}) \tilde{q}(0) \sim \bar{z} \tilde{Q}^*(z, \bar{z}) \tilde{Q}(0) \sim \]
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Complex scalars in hypermultiplet are in the cohomology

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\[ \rightarrow q(z, \bar{z})\tilde{q}(0) \sim \bar{z} \tilde{Q}^*(z, \bar{z})\tilde{Q}(0) \sim \frac{\bar{z}}{z\bar{z}} \]
Example: free hypermultiplet

Complex scalars in hypermultiplet are in the cohomology

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\[ \tilde{q}(z, \bar{z}) = u_I \tilde{Q}' = \tilde{Q}(z, \bar{z}) - \bar{z} Q^*(z, \bar{z}) \]

\[ q(z, \bar{z}) \tilde{q}(0) \sim \bar{z} \tilde{Q}^*(z, \bar{z}) \tilde{Q}(0) \sim \frac{\bar{z}}{z \bar{z}} = \frac{1}{z} \]
Which operators are in the cohomology?

→ Stress tensor $T_{\mu\nu}$
Which operators are in the cohomology?

→ Stress tensor $T_{\mu\nu} \leadsto$ superdescendant
Which operators are in the cohomology?

→ Stress tensor $T_{\mu\nu} \rightsquigarrow$ superdescendant
→ Stress tensor supermultiplet
Which operators are in the cohomology?

→ Stress tensor $T_{\mu\nu}$ \leadsto superdescendant
→ Stress tensor supermultiplet

\[
T(z)T(0) \sim -12 \frac{c_{4d}/2}{z^4} + 2 \frac{T(0)}{z^2} + \frac{\partial T(0)}{z} + \ldots ,
\]
4d $\mathcal{N} \geq 2$ SCFT $\rightarrow$ chiral algebra

Which operators are in the cohomology?

$\rightarrow$ Stress tensor $T_{\mu\nu} \rightsquigarrow$ superdescendant

$\rightarrow$ Stress tensor supermultiplet $\Rightarrow$ 2d stress tensor

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4d $\mathcal{N} \geq 2$ SCFT $\longrightarrow$ chiral algebra

Which operators are in the cohomology?

→ Theory with flavor symmetry
4d $\mathcal{N} \geq 2$ SCFT $\rightarrow$ chiral algebra

Which operators are in the cohomology?

$\rightarrow$ Theory with flavor symmetry
$\rightarrow$ Multiplet containing flavor current
4d $\mathcal{N} \geq 2$ SCFT $\rightarrow$ chiral algebra

Which operators are in the cohomology?

$\rightarrow$ Theory with flavor symmetry
$\rightarrow$ Multiplet containing flavor current
$\leftarrow$ Affine Kac Moody current algebra

\[ J^a(z)J^b(0) \sim -\frac{k_{4d}/2\delta^{ab}}{z^2} + if^{abc} \frac{J^c(0)}{z} + \ldots , \]
Which operators are in the cohomology?

→ Theory with flavor symmetry
→ Multiplet containing flavor current
← Affine Kac Moody current algebra

\[ J^a(z)J^b(0) \sim -\frac{k_{4d}/2\delta^{ab}}{z^2} + if^{abc}\frac{J^c(0)}{z} + \ldots, \]

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→ \[ \ldots \]
5 A solvable subsector

6 4d $\mathcal{N} = 3$ SCFTs

7 Lorentizan inversion formula

8 Constraining the space of $\mathcal{N} = 2$ SCFTs
$\mathcal{N} = 3$ Chiral algebra

- $4d \mathcal{N} \geq 3$: some of the extra supercharges commute with $Q$
$\mathcal{N} = 3$ Chiral algebra

- $4d \mathcal{N} \geqslant 3$: some of the extra supercharges commute with $\mathbb{Q}$
  \[\rightarrow 4d \mathcal{N} = 4 \Rightarrow 2d \text{ “small” } \mathcal{N} = 4 \text{ chiral algebra}\]
$\mathcal{N} = 3$ Chiral algebra

- $4d \, \mathcal{N} \geq 3$: some of the extra supercharges commute with $\mathbb{Q}$
  - $4d \, \mathcal{N} = 4 \Rightarrow 2d$ “small” $\mathcal{N} = 4$ chiral algebra
  - $4d \, \mathcal{N} = 3 \Rightarrow 2d \, \mathcal{N} = 2$ chiral algebra [Nishinaka, Tachikawa]
\[ \mathcal{N} = 3 \] Chiral algebra

- 4d \( \mathcal{N} \geq 3 \): some of the extra supercharges commute with \( Q \)
  - \( 4d \mathcal{N} = 4 \) \( \Rightarrow \) 2d “small” \( \mathcal{N} = 4 \) chiral algebra
  - \( 4d \mathcal{N} = 3 \) \( \Rightarrow \) 2d \( \mathcal{N} = 2 \) chiral algebra [Nishinaka, Tachikawa]
- 2d stress tensor promoted to supermultiplet

![Diagram]

- \( J(z) \)
- \( Q \)
- \( D(z) \)
- \( T(z) \)
$\mathcal{N} = 3$ Chiral algebra

- $4d \mathcal{N} \geq 3$: some of the extra supercharges commute with $Q$
  - $4d \mathcal{N} = 4 \Rightarrow 2d$ “small” $\mathcal{N} = 4$ chiral algebra
  - $4d \mathcal{N} = 3 \Rightarrow 2d \mathcal{N} = 2$ chiral algebra [Nishinaka, Tachikawa]

- $2d$ stress tensor promoted to supermultiplet

$2d \mathcal{N} = 2$ Stress tensor $\mathcal{J}$
\( \mathcal{N} = 3 \) Chiral algebra

- 4d \( \mathcal{N} \geq 3 \): some of the extra supercharges commute with \( Q \)
  - \( 4d \mathcal{N} = 4 \Rightarrow 2d \) “small” \( \mathcal{N} = 4 \) chiral algebra
  - \( 4d \mathcal{N} = 3 \Rightarrow 2d \mathcal{N} = 2 \) chiral algebra [Nishinaka, Tachikawa]

- 2d stress tensor promoted to supermultiplet

2d \( \mathcal{N} = 2 \) Stress tensor \( \mathcal{J} \)

- Present in any local \( \mathcal{N} = 3 \) SCFT
\( \mathcal{N} = 3 \) Chiral algebra

- 4d \( \mathcal{N} \geq 3 \): some of the extra supercharges commute with \( Q \)
  - 4d \( \mathcal{N} = 4 \) \( \Rightarrow \) 2d “small” \( \mathcal{N} = 4 \) chiral algebra
  - 4d \( \mathcal{N} = 3 \) \( \Rightarrow \) 2d \( \mathcal{N} = 2 \) chiral algebra [Nishinaka, Tachikawa]

- 2d stress tensor promoted to supermultiplet

2d \( \mathcal{N} = 2 \) Stress tensor \( J \)

- Present in any local \( \mathcal{N} = 3 \) SCFT
- A trivial statement in 2d:
  \( \langle J J J J \rangle \) is fixed in terms of \( c_{2d} \)
2d $\mathcal{N} = 2$ Stress tensor $\mathcal{J}$

$\langle \mathcal{J} \mathcal{J} \mathcal{J} \mathcal{J} \rangle$ is fixed in terms of $c_{2d}$
Space of $\mathcal{N} = 3$ SCFTs

$2d \, \mathcal{N} = 2$ Stress tensor $\mathcal{J}$

$\langle \mathcal{J} \mathcal{J} \mathcal{J} \mathcal{J} \rangle$ is fixed in terms of $c_{2d}$

- $2d$ Superblock decomposition:

\[
\sum_{\mathcal{O}_{2d}} \lambda_{\mathcal{O}_{2d}}^2 \mathcal{O}_{2d}
\]
Space of $\mathcal{N} = 3$ SCFTs

2d $\mathcal{N} = 2$ Stress tensor $\mathcal{J}$

$\langle \mathcal{J} \mathcal{J} \mathcal{J} \mathcal{J} \rangle$ is fixed in terms of $c_{2d}$

- 2d Superblock decomposition:

$$\sum_{\mathcal{O}_{2d}} \lambda_{\mathcal{O}_{2d}}^2 \mathcal{O}_{2d}$$

$\rightarrow \lambda_{\mathcal{O}_{2d}}^2$
Space of $\mathcal{N} = 3$ SCFTs

2d $\mathcal{N} = 2$ Stress tensor $\mathcal{J}$

$\langle \mathcal{J} \mathcal{J} \mathcal{J} \mathcal{J} \rangle$ is fixed in terms of $c_{2d}$

- 2d Superblock decomposition:

  $$\sum_{\mathcal{O}_{2d}} \lambda_{\mathcal{O}_{2d}}^2 \rightarrow \mathcal{O}_{2d}$$

  $$\rightarrow \lambda_{\mathcal{O}_{2d}}^2 \sim \lambda_{\mathcal{O}_{4d}}^2$$

assumptions: interacting theory, unique stress tensor
Space of $\mathcal{N} = 3$ SCFTs

$2d \ \mathcal{N} = 2$ Stress tensor $\mathcal{J}$

$\langle \mathcal{J} \mathcal{J} \mathcal{J} \mathcal{J} \rangle$ is fixed in terms of $c_{2d}$

- 2d Superblock decomposition:

$$\sum_{\mathcal{O}_{2d}} \lambda_{\mathcal{O}_{2d}}^2 \mathcal{O}_{2d} \rightarrow \lambda_{\mathcal{O}_{4d}}^2 \geq 0$$

4d unitarity

assumptions: interacting theory, unique stress tensor
Space of $\mathcal{N} = 3$ SCFTs

2d $\mathcal{N} = 2$ Stress tensor $\mathcal{J}$

$\langle \mathcal{J} \mathcal{J} \mathcal{J} \mathcal{J} \rangle$ is fixed in terms of $c_{2d}$

- 2d Superblock decomposition:

$$\sum_{\mathcal{O}_{2d}} \lambda_{\mathcal{O}_{2d}}^2 \rightarrow\lambda_{\mathcal{O}_{2d}}^2 \sim \lambda_{\mathcal{O}_{4d}}^2 \geq 0 \Rightarrow \text{New unitarity bound}$$

assumptions: interacting theory, unique stress tensor
Space of $\mathcal{N} = 3$ SCFTs

2d $\mathcal{N} = 2$ Stress tensor $\mathcal{J}$

$\langle \mathcal{J} \mathcal{J} \mathcal{J} \mathcal{J} \rangle$ is fixed in terms of $c_{2d}$

- 2d Superblock decomposition:

$$\sum_{\mathcal{O}_{2d}} \lambda_{\mathcal{O}_{2d}}^2 \mathcal{O}_{2d}$$

$$\rightarrow \lambda_{\mathcal{O}_{2d}}^2 \sim \lambda_{\mathcal{O}_{4d}}^2 \quad \geq 0 \quad \Rightarrow \text{New unitarity bound}$$

4d unitarity

assumptions: interacting theory, unique stress tensor

$$c_{4d} \geq \frac{13}{24}$$

[Cornagliotto, ML, Schomerus]
Space of $\mathcal{N} = 3$ SCFTs

2d $\mathcal{N} = 2$ Stress tensor $\mathcal{J}$

\[ c_{4d} \geq \frac{13}{24} \quad [\text{Cornagliotto, ML, Schomerus}] \]

$\mapsto$ Not saturated by any known SCFT
Space of $\mathcal{N} = 3$ SCFTs

2d $\mathcal{N} = 2$ Stress tensor $\mathcal{J}$

\[
c_{4d} \geq \frac{13}{24}
\]  
[Cornagliotto, ML, Schomerus]

$\rightarrow$ Not saturated by any known SCFT

smallest interacting known theory: $c_{4d} = \frac{15}{12}$
Space of $\mathcal{N} = 3$ SCFTs

2d $\mathcal{N} = 2$ Stress tensor $\mathcal{J}$

$C_{4d} \geq \frac{13}{24}$ \[\text{[Cornagliootto, ML, Schomerus]}\]

$\rightarrow$ Not saturated by any known SCFT

smallest interacting known theory: $c_{4d} = \frac{15}{12}$

$\rightarrow$ Similar bounds in $\mathcal{N} = 4$ and $\mathcal{N} = 2$ saturated by known SCFTs \[\text{[Beem, Rastelli, van Rees]} \ [\text{Liendo, Ramirez, Seo}]\]
Space of $\mathcal{N} = 3$ SCFTs

2d $\mathcal{N} = 2$ Stress tensor $\mathcal{J}$

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c_{4d} \geq \frac{13}{24} \quad \text{[Cornagliotto, ML, Schomerus]}
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$\hookrightarrow$ Similar bounds in $\mathcal{N} = 4$ and $\mathcal{N} = 2$ saturated by known SCFTs [Beem, Rastelli, van Rees] [Liendo, Ramirez, Seo]

$\rightarrow$ $c_{4d} = \frac{13}{24} \Rightarrow$ reconstruct 4d operators appearing in $\mathcal{J} \mathcal{J}$
Space of $\mathcal{N} = 3$ SCFTs

2d $\mathcal{N} = 2$ Stress tensor $\mathcal{J}$

$\begin{array}{c}
c_{4d} \geq \frac{13}{24} \\
\text{[Cornagliotto, ML, Schomerus]}
\end{array}$

$\leftarrow$ Not saturated by any known SCFT
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$\leftarrow$ Similar bounds in $\mathcal{N} = 4$ and $\mathcal{N} = 2$ saturated by known SCFTs [Beem, Rastelli, van Rees] [Liendo, Ramirez, Seo]

$\rightarrow$ $c_{4d} = \frac{13}{24} \Rightarrow$ reconstruct 4d operators appearing in $\mathcal{J} \mathcal{J}$

$\rightarrow$ Inconsistent with an *interacting* 4d SCFT
2d $\mathcal{N} = 2$ Stress tensor $\mathcal{J}$

$c_{4d} > \frac{13}{24}$ \[\text{[Cornaglione, ML, Schomerus]}\]

$\leftrightarrow$ Not saturated by any known SCFT

smallest interacting known theory: $c_{4d} = \frac{15}{12}$

$\leftrightarrow$ Similar bounds in $\mathcal{N} = 4$ and $\mathcal{N} = 2$ saturated by known SCFTs \[\text{[Beem, Rastelli, van Rees]} \text{[Liendio, Ramirez, Seo]}\]

$\rightarrow$ $c_{4d} = \frac{13}{24} \Rightarrow$ reconstruct 4d operators appearing in $\mathcal{J} \mathcal{J}$

$\rightarrow$ Inconsistent with an interacting 4d SCFT
5 A solvable subsector

6 4d $\mathcal{N} = 3$ SCFTs

7 Lorentizan inversion formula

8 Constraining the space of $\mathcal{N} = 2$ SCFTs
Inverting the $\phi \bar{\phi}$ OPE

$\rightarrow$ Supersymmetric inversion: valid for $\ell \geq 0$

$\rightarrow$ Only input: $\bar{\phi}\phi \sim 1 + \text{Stress tensor multiplet}$
A Lorentzian inversion formula

Inverting the $\phi \bar{\phi}$ OPE

- Supersymmetric inversion: valid for $\ell \geq 0$
- Only input: $\bar{\phi} \phi \sim 1 + $ Stress tensor multiplet

$$\phi \bar{\phi} \sim \left[ \phi \bar{\phi} \right]_{\Delta \to 2\Delta + \ell, \ell \to \infty} + \cdots$$
5 A solvable subsector

6 $4d \mathcal{N} = 3$ SCFTs

7 Lorentizan inversion formula

8 Constraining the space of $\mathcal{N} = 2$ SCFTs
Space of $\mathcal{N} = 2$ SCFTs

$4d$ $\mathcal{N} = 2$ SCFT with flavor symmetry

$\rightarrow$ Stress tensor multiplet
Space of $\mathcal{N} = 2$ SCFTs

$4d \, \mathcal{N} = 2$ SCFT with flavor symmetry

→ Stress tensor multiplet $\Rightarrow T(z)$
Space of $\mathcal{N} = 2$ SCFTs

$4d \mathcal{N} = 2$ SCFT with flavor symmetry

$\rightarrow$ Stress tensor multiplet $\Rightarrow T(z)$
$\rightarrow$ Flavor symmetry current multiplet
Space of $\mathcal{N} = 2$ SCFTs

4d $\mathcal{N} = 2$ SCFT with flavor symmetry

→ Stress tensor multiplet $\Rightarrow T(z)$
→ Flavor symmetry current multiplet $\Rightarrow J^a(z)$
Space of $\mathcal{N} = 2$ SCFTs

4d $\mathcal{N} = 2$ SCFT with flavor symmetry

$\rightarrow$ Stress tensor multiplet $\Rightarrow T(z)$

$\rightarrow$ Flavor symmetry current multiplet $\Rightarrow J^a(z)$

\[
T(z)T(0) \sim -12\frac{c_{4d}/2}{z^4} + 2\frac{T(0)}{z^2} + \frac{\partial T(0)}{z} + \ldots ,
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Space of $\mathcal{N} = 2$ SCFTs

4d $\mathcal{N} = 2$ SCFT with flavor symmetry

$\rightarrow$ Stress tensor multiplet $\Rightarrow T(z)$

$\rightarrow$ Flavor symmetry current multiplet $\Rightarrow J^a(z)$

$$T(z)T(0) \sim -12 \frac{c_{4d}/2}{z^4} + 2 \frac{T(0)}{z^2} + \frac{\partial T(0)}{z} + \ldots,$$

$$J^a(z)J^b(0) \sim -\frac{k_{4d}/2\delta^{ab}}{z^2} + i f^{abc} \frac{J_c(0)}{z} + \ldots,$$
Space of $\mathcal{N} = 2$ SCFTs

4d $\mathcal{N} = 2$ SCFT with flavor symmetry

$\rightarrow$ Stress tensor multiplet $\Rightarrow T(z)$
$\rightarrow$ Flavor symmetry current multiplet $\Rightarrow J^a(z)$

$$T(z)T(0) \sim -12 \frac{c_{4d}/2}{z^4} + 2 \frac{T(0)}{z^2} + \frac{\partial T(0)}{z} + \ldots ,$$

$$J^a(z)J^b(0) \sim - \frac{k_{4d}/2\delta^{ab}}{z^2} + if^{abc} \frac{J^c(0)}{z} + \ldots ,$$

$n$ Fix in terms of $c_{4d}$ and $k_{4d}$:

$$\langle TTTT \rangle, \quad \langle J^a J^b J^c J^d \rangle, \quad \langle TTJ^a J^b \rangle$$
Constraining the space of $4d \mathcal{N} = 2$ SCFTs

$E_6$ flavor symmetry

![Graph showing the ruled out and numerically ruled out regions for different $e$ values.

$[\text{Beem, ML, Liendo, Peelaers, Rastelli, van Rees}]$ $[\text{ML, Liendo}]$

$[\text{Beem, ML, Liendo, Rastelli, van Rees}]$