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SU(4) Weak Currents and Their Experimental Implications

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A $SU_L(4)\otimes U(1)$ model of 4 quarks is presented. Three quartets of leptons are necessary in order to cancel anomalies. We propose, in the breaking of the SU(4) gauge symmetry, a set of W bosons with mass $\gtrsim$ 130 GeV. The suppression of parity violation in atomic experiments could be explained with these heavier W bosons, the neutral current cross sections are compared with the Weinberg-Salam model. The model has definite predictions for the decays of charmed mesons and the heavy lepton τ.

The Weinberg-Salam (W-S) model [1] with four quarks is quite successful in representing the neutrino induced neutral current reactions. However, the existence of the heavy lepton τ [2], the possible evidence for other heavy leptons in neutrino induced reactions [3] and the suppression of parity violation in atomic neutral current experiments [4] necessitates a modification of its conventional form. We present in this note a model which incorporates consistently all the above results.[5]

We consider the $SU_L(4)\otimes U(1)$ gauge symmetry of the 4 quarks. The charged currents being left-handed, we do not expect the "y-anomaly" in anti-neutrino reactions(beyond possible breaking of scaling). We propose that the SU(4) gauge symmetry (of the Higgs scalars) is broken in several stages, namely, $SU(4)\rightarrow O(5)$ (or $SP(4)$) $\rightarrow SU(2)$. The breaking of SU(2) with the additional U(1) gives the Weinberg-Salam model. The "super" symmetry-breaking provides us with $SU(4)-O(5) = 5$ W bosons of super heavy masses and $O(5)-SU(2) = 7$ W bosons of intermediate masses which are heavier than the SU(2) W bosons. We report below what the "medium-heavy" W bosons could do to the weak interactions.[5]

The first thing they do is to restrict severely the lepton representations. The four quarks are assigned in a 4 representation (u,d,s,c). The electron, muon and their neutrinos in this model cannot belong to the same SU(4) representation, e.g. $(\nu_e, e^-, \mu^-, \nu_\mu)$, which were proposed by Pati and Salam[6]. The reason is that the SU(4) weak currents (e.g. $\bar{\psi}\gamma_\mu\lambda_{6+i7}\psi$) with this assignment would generate strangeness-changing neutral currents and lepton number non-conservation which is untolerable in ordinary weak interactions. Another reason for rejecting this assignment is based on the fact that the small mass of the neutrinos cannot be explained, (m_{ν_μ} as compared with the charm quark mass). Instead, we identify the heavy lepton observed at SLAC and DESY as an excited electron state with an associated ν_τ. In order to cancel the anomalies with respect to

the $SU_L(4)\otimes U(1)$ gauge symmetry, the leptons belong to the $\underline{4}$ representations, and the sum of the quark and lepton charges must equal zero. This results in a proliferation of the lepton families and a quark-lepton symmetry,

$$\begin{pmatrix} u^r & u^g & u^b \\ d^r & d^g & d^b \\ s^r & s^g & s^b \\ c^r & c^g & c^b \end{pmatrix} \qquad \begin{pmatrix} e^- & \mu^- & \ell^0 \\ \nu_e & \nu_\mu & \ell^+ \\ \nu_\tau & M^0 & L^+ \\ \tau^- & M^- & L^0 \end{pmatrix}$$

where r, g, b denotes the color of the quarks. A unified picture of the quark and lepton masses could emerge from the above representation. For simplicity, we assume that the same set of Higgs scalars is responsible for generating the quark and lepton mass. We then expect the leptons to exhibit the same mass pattern as the quarks [7]. In fact, $m_e \sim m_{\nu_e} \sim m_u^{bare} \sim m_d^{bare} \sim 0$ and $m_\tau \sim m_c^{bare}$ are in accord with strong PCAC and the observed τ and the charmed meson masses. We are thus led to speculate that $m_{\nu_\tau} \sim (m_s^{bare} - m_u^{bare}) \sim m_s^{bare}$.

The second restriction is that the Cabibbo angle should be a consequence of the mixing between the u and c quarks. The SU(2) weak interaction (the W-S model) cannot distinguish this scheme from the GIM mixing [8] for the d and s quarks. But the d and s mixing with respect to SU(4) will result in strangeness-changing neutral currents in ordinary weak interactions generated by the new SU(4) currents e.g. λ_Y of eq. (2)). This is not allowed. With the Cabibbo angle generated from the u and c quark mixing, its magnitude could be estimated as follows. The strong interaction symmetry SU(4) is badly broken to SU(3). Since in gauge theories the strong symmetry breaking only appears in the quark mass terms, the c quark acquires a large mass. We assume that a second stage of symmetry breaking on a smaller scale, be it the Higgs mechanism or radiative corrections, generates the mass of the u quark and its mixing with the c quark (Δm_{uc}) . We then find, after

diagonalizing the mass matrix, that $\sin\theta_c \sim \frac{\Delta m_{uc}}{m_c - m_u} \sim \frac{m_u}{m_c}$, where m_u, m_c refer to the "physical" quark masses.

From the fermion representations and the Cabibbo structure described above, we see that the strangeness-changing neutral currents $(\bar{\psi}_L \gamma_\mu \lambda_{6\pm i7} \psi_L)$, in the semi-leptonic processes, always couple to at least one "new" lepton, if they are decoupled from the diagonal neutral currents. The decoupling is verified from the Higgs mechanism (see Table I below). From Table I also follows that the effective Lagrangian in lowest order of G_W has no $|\Delta s| = 2$ piece for the non-leptonic interactions. The GIM cancellation mechanism works for the higher order box diagrams, as expected.

We have explicitly constructed the Higgs sector [5] to break the SU(4) to O(5) and then to SU(2). The SU(2), in the $(u(\theta_c), d, s, c(\theta_c))$ basis $(u(\theta_c) = u\cos\theta_c - c\sin\theta_c,\ c(\theta_c) = u\sin\theta_c + c\cos\theta_c)$, consists of the following generators,

$$\lambda_{W^+} \equiv \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & h & 0 \end{pmatrix} = \frac{1}{2}[\lambda_{1+i2} + h\,\lambda_{13-i14}] = [\lambda_{W^-}]^\dagger$$

$$\lambda_{W^3} \equiv \begin{pmatrix} \frac{1}{2} & & & \\ & -\frac{1}{2} & & \\ & & -\frac{1}{2} & \\ & & & \frac{1}{2} \end{pmatrix} = \frac{1}{2}[\lambda_{W^+}, \lambda_{W^-}] = \frac{1}{2}[\lambda_3 + \frac{\lambda_8}{\sqrt{3}} - \sqrt{\tfrac{2}{3}}\,\lambda_{15}] \qquad (1)$$

where the λ's are the SU(4) λ-matrices, and where $h = \pm 1$. The sign of h is not determined from ordinary mesons or hyperons decay. We shall assume $h = 1$ for simplicity. The subgroup of SU(4) which contains the above SU(2) as a subgroup is an O(5), consisting of the following generators in addition to (1),

$$\lambda_{K^+} \equiv \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{2}\lambda_{4+i5} = [\lambda_{K^-}]^\dagger$$

$$\lambda_{D^+} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \frac{1}{2}\lambda_{11-i12} = [\lambda_{D^-}]^\dagger \qquad (2)$$

and

$$\lambda_{(K\bar{D})^0} \equiv \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{2}[\lambda_{6+i7} - \lambda_{9+i10}] = [\lambda_{(\bar{K}D)^0}]^\dagger$$

$$\lambda_Y \equiv \begin{pmatrix} \frac{1}{2} & & & \\ & \frac{1}{2} & & \\ & & -\frac{1}{2} & \\ & & & -\frac{1}{2} \end{pmatrix} = \frac{1}{\sqrt{3}}\lambda_8 + \frac{1}{\sqrt{6}}\lambda_{15}$$

Denoting the gauge couplings of $SU_L(4)\otimes U(1)$ as g and g' and their gauge bosons as W_i ($i = 1,\ldots,15$) and B, the diagonalized states after spontaneous symmetry breaking of O(5) and $SU_L(2)\otimes U(1)$ are listed in Table I. For the complete breaking of $SU_L(4)\otimes U(1)$, we refer to ref. (5). $SU_L(4)\otimes U(1)$ contains 4 neutral gauge bosons coupling to diagonal neutral currents, which we denote as A, X, Y, and Z. A and Z are the photon and the neutral gauge boson of the Weinberg-Salam model. X is super-heavy and does not interest us here. We'll study below the contribution of Y to the low energy weak interactions. The interaction Lagrangian can be obtained from

$$\mathcal{L} = \bar{\psi}^q_L \gamma_\mu (\partial^\mu + \tfrac{i}{2} g \lambda_i W^\mu_i + \tfrac{i}{2} g' y^q B^\mu)\psi^q_L + \bar{\psi}^q_R \gamma_\mu (\partial^\mu + i g' Q B^\mu)\psi^q_R$$
$$+ \bar{\psi}^\ell_L \gamma_\mu (\partial^\mu + \tfrac{i}{2} g (-\lambda^T_i) W^\mu_i - \tfrac{i}{2} g' y^\ell B^\mu)\psi^\ell_L + \bar{\psi}^\ell_R \gamma_\mu (\partial^\mu + i g' Q B^\mu)\psi^\ell_R \qquad (3)$$

where the y's are constants (matrices) determined from the charge of the quarks and leptons. Q is the charge matrix. We obtain from (3) the coupling as show in Table I. The effective Lagrangian for the neutral current induced interactions is as follows:

(1) Atomic neutral currents. Here the dominant contribution comes from the quark vector currents and electron axial-vector current which are

$$\frac{G_W}{2\sqrt{2}}\left\{\left[\bar{u}\gamma_\mu u-\bar{d}\gamma_\mu d-4X_W\left(\frac{2}{3}\bar{u}\gamma_\mu u-\frac{1}{3}\bar{d}\gamma_\mu d\right)\right]\bar{e}\gamma^\mu\gamma_5 e + \left(\frac{m_W^2}{m_Y^2}\right)\left[\bar{u}\gamma_\mu u+\bar{d}\gamma_\mu d\right]\bar{e}\gamma^\mu\gamma_5 e\right\} \tag{4}$$

The first term is the W-S model prediction $(X_W = \sin^2\theta_W)$ and the second term is the added contribution from the Y exchange diagram. The relevant quantity for the Atomic Bi^{209}_{83} experiments is $Q^W = Z(1-4X_W)-N + (m_W^2/m_Y^2)\cdot 3(Z+N) = -167+\left(\frac{m_W^2}{m_Y^2}\right)627$ for $X_W \sim \frac{3}{8}$. If Q^W is near zero, we find $m_Y \sim 2m_W \sim$ 130 GeV. But m_Y could of course be heavier, depending on the final experimental results.

(2) Neutral currents in neutrino reactions. The W-S model is modified as follows

$$\begin{aligned}\frac{G_W}{2\sqrt{2}}\Big\{ & -\bar{e}\gamma_\mu(1-4X_W-\gamma_5)e + [\bar{u}\gamma_\mu(1-\gamma_5)u-\bar{d}\gamma_\mu(1-\gamma_5)d \\ & -4X_W(\tfrac{2}{3}\bar{u}\gamma_\mu u-\tfrac{1}{3}\bar{d}\gamma_\mu d)] \\ & +\left(\frac{m_W^2}{m_Y^2}\right)[\bar{e}\gamma_\mu(1-\gamma_5)e-\bar{u}\gamma_\mu(1-\gamma_5)u-\bar{d}\gamma_\mu(1-\gamma_5)d]\Big\} \\ & \times\ \bar{\nu}_\mu\gamma^\mu(1-\gamma_5)\nu_\mu\end{aligned} \tag{5}$$

where the last term is the new contribution from the Y exchange. For $X_W \sim \frac{3}{8}$ and $m_Y \sim 2m_W$, the new contribution suppresses the left-handed component of the electron neutral current. The cross sections $\sigma(\bar{\nu}_\mu e)$ and $\sigma(\nu_\mu e)$ are modified by 4 % and 30 % respectively as compared with the W-S model predictions. We find $\sigma(\nu_\mu e) \sim 0.8\times10^{-42}E_\nu\,cm^2GeV^{-1}$ and $\sigma(\bar{\nu}_\mu e)\sim 2.4\times10^{-42}E_\nu\,cm^2GeV^{-1}$. For comparisons with the recent data, see ref. (5). The effect of the new contribution for the inclusive neutral current cross sections turns out to be negligibly small. For the elastic νp and $\bar{\nu}p$ cross sections, the effect is at the 10 ~ 20 % level; the modification appears only in the isoscalar currents (5).

(3) Strangeness-changing and charm-changing neutral currents. From the W^0_{KD} exchange, we arrive at the effective Lagrangian $(m_{W^0_{KD}} = m_{W'})$

$$\frac{G_W}{\sqrt{2}}\left(m_W^2/m_{W'}^2\right)[\bar{d}\gamma_\mu(1-\gamma_5)s+\bar{u}\gamma_\mu(1-\gamma_5)c][\bar{\nu}_e\gamma^\mu(1-\gamma_5)\nu_\tau]+h.c.$$

Thus, unless W^0_{KD} is very heavy, the lower limit of m_{ν_τ} is given by the absence of $K^+\to\pi^+\nu\nu$, namely, $m_{\nu_\tau} \gtrsim (m_{K^+}-m_{\pi^+}) \sim 350$ MeV. This value is in accord with the theoretical expectation ($m_{\nu_\tau}\sim m_s^{bare}$, see above) - the present experimental upper limit from τ decay is $m_{\nu_\tau} <$ 500 - 600 MeV.(9) If $m_{W'}\sim m_Y \sim 2m_W$, we predict that $\Gamma(D^+\to\pi^+\bar{\nu}_\tau\nu_e)/\Gamma(D^+\to\bar{K}^0e^+\bar{\nu}_e)\sim 7$ % for m_{ν_τ} ~ 0.35-0.45 GeV and having assumed the same constant form factors for both processes.

(4) Implications for τ decays. With respect to the SU(2) gauge interactions, τ behaves just like a true sequential heavy lepton. Modification arises from the W^0_{KD} exchange diagrams (other contributions are much suppressed by phase space). Specifically, the electron quantum number carried by τ will manifest itself in the following decay modes (but $\tau^-\not\to e^-e^+e^-, \tau^-\not\to e^-\mu^+\mu^-$), neglecting m_{ν_τ}/m_τ,

$$\frac{\Gamma(\tau^-\to e^-K^0)}{\Gamma(\tau^-\to\nu_\tau\mu^-\bar{\nu}_\mu)} = \frac{12\pi^2 f_K^2}{m_\tau^2}\left(1-\frac{m_K^2}{m_\tau^2}\right)^2\left(\frac{m_W^2}{m_{W'}^2}\right)^2, \tag{6}$$

$$\frac{\Gamma(\tau^-\to\nu_\tau e^-\bar{\nu}_e)}{\Gamma(\tau^-\to\nu_\tau\mu^-\bar{\nu}_\mu)} = \left[1-h\frac{m_W^2}{m_{W'}^2}\right]^2, \quad h=\pm1. \tag{7}$$

Thus we'll expect that the ratio of e^+e^- versus μ^+e^- from the heavy lepton pair decay in e^+e^- annihilation will not be 1. It is interesting to remark that the sign of h could be measured by this ratio. For $m_{W'}\sim 2m_W$, one finds $\sigma(e^+e^-)/\sigma(\mu e)$ = 0.28 or 0.78 as compared with the preliminary data(9) $\sigma(ee)/\sigma(\mu e)$ = 0.57 ± 0.3. More precise measurement should be interesting. The corresponding prediction for $\frac{\sigma(\mu e K_s)}{\sigma(\mu e)}$ from (6) is at the level of 1%. The search for μeK_s+nothing is a difficult experiment, but a resonance in eK_s, if seen, could be a direct proof of the existence of the heavy lepton. ν_τ in this model is a long lived particle, since it is forbidden

to decay by phase space. We predict $m_{\nu_\tau} \gtrsim 350$ MeV. (see above). Thus the measurement of m_{ν_τ} will be the first crucial test of this model. The two-body decay modes of τ (eg. $\nu\pi, \nu\rho$) could be the best place to measure m_{ν_τ}.

(5) Implication for high energy neutrino and e^+e^- experiments.

From Table I, one immediately concludes that the semi-leptonic decay of M^0 always involve strangeness non-zero final states. (M^- is heavier than M^0 by the quark-lepton mass analogy). M^0 could be found in high energy e^+e^- annihilation through pair production of M^+M^- and their subsequent decays. In particular, M^0 could be looked for in the invariant mass plot of $\mu^\pm K^\mp$. In neutrino reactions, M^- can be produced via the charm changing current only. This and the decay[10] of M^- and M^0 leads us to speculate that trimuon events are associated with two strange particles.

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Table I

W bosons	coupling to	masses
$A = \sin\theta_W A_3 + \cos\theta_W B$	Q	0
$Z = \cos\theta_W A_3 - \sin\theta_W B$,	$\sqrt{\frac{g^2}{2}+g'^2}\,[\lambda_{W^3}\frac{1-\gamma_5}{2} - \sin^2\theta_W Q]$,	$m_Z^2 = \frac{m_W^2}{\cos^2\theta_W}$
$W^\mp$	$\frac{g}{2}\lambda_{W^\pm}\frac{1-\gamma_5}{2}$	m_W^2
$W_{K^\mp}$	$\frac{g}{\sqrt{2}}\lambda_{K^\pm}\frac{1-\gamma_5}{2}$	$m_{W'}^2 \sim m_Y^2$
$W_{D^\mp}$	$\frac{g}{\sqrt{2}}\lambda_{D^\pm}\frac{1-\gamma_5}{2}$	
$W_{(K\bar{D})^0}, [W_{(\bar{K}D)^0}]$,	$\lambda_{(K\bar{D})^0}[\lambda_{(\bar{K}D)^0}]\frac{1-\gamma_5}{2}$	
$Y = \sqrt{\frac{2}{3}}W_8 + \frac{1}{\sqrt{3}}W_{15}$	$\frac{g}{\sqrt{2}}\lambda_Y\frac{1-\gamma_5}{2}$	m_Y^2

Table I Caption

Eigenstates and masses of the W bosons with their coupling to the fermion currents. $A_3 \equiv \frac{1}{\sqrt{2}}(W_3 + \frac{1}{\sqrt{3}}W_8 - \sqrt{\frac{2}{3}}W_{15})$, $\tan^2\theta_W = g'^2/(\frac{g^2}{2})$.
For the notation of the λ's see text.

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(10) Note that $M^0 \not\rightarrow \mu^+\mu^-\nu_\mu$, but $M^0 \rightarrow \mu^- e^+ \nu_\tau$