Electroproduction and Photoproduction of
Vector Mesons and Generalized Vector Meson Dominance

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Abstract

Using generalized vector meson dominance, electro- and photoproduction
of vector mesons is studied. The unnatural parity exchange part of $\omega'(1.2)$
production is estimated to be about one fourth of that of $\omega$ production.
The off diagonal transition model suggests the suppression of diffractive
$\rho'(1.2)$ and $\omega'(1.2)$ production.

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1. Introduction

Since new data on the decay width $\Gamma(p \to \pi \gamma)$ is published $^1$, there has been much debate on SU(3) symmetry violation for radiative decays of vector mesons $^2$. In fact, the analyses of $\rho \to K\gamma$, $\omega \to \pi \gamma$ and $\phi \to \pi \gamma$ decay rates suggest substantial violation of SU(3) symmetry for the VV$^\prime$ vertex.

As a possible solution of this problem, one of the present authors proposed $^3$, on the basis of the generalized vector meson dominance (GVM$^\prime$), a simple model of $\pi^\nu$ coupling to the two virtual photons. The radiative decay widths of vector mesons predicted in this model are in good agreement with the new data. One of the characteristics of the model is that it required the existence of $\omega'$ around 1.2 GeV with the width of order of 400 MeV. There is experimental evidence $^4$ on the existence of $\rho'(1.2)$ but there is so far almost no evidence for $\omega'(1.2)$. This may be attributed to the broadness of the $\omega'$.

On the other hand, electro- and photoproduction of vector mesons give us, through the one pion exchange (OPE) amplitude, information on the P$^\nu$Y coupling constants, which are just the coupling constants that appear in radiative decays of vector mesons, and on the off-shell dependence of the virtual photon form factors. Therefore, these reactions can be used in discussing SU(3) symmetry violation of P$^\nu$Y couplings, which is indicated in the radiative decay of vector mesons. In this paper, we focus our attention on unnatural parity exchange, specifically the OPE part of these reactions and compare the GVM$^\prime$ result with the naive VMD prediction. Our calculation shows that for $\omega$-production, a sizable deviation from the SU(3) symmetry can be expected from GVM$^\prime$. Finally, using the "off-diagonal" transition model $^5$ for the natural parity exchange part, we estimate $\rho'$ and $\omega'$ production cross sections.

Based on the GVM$^\prime$ of ref. 3, the $\pi^\nu$ coupling to the two off-shell photons is expressed effectively as

$$<\pi^\nu | \gamma^\nu \gamma^\mu | q^2> = e^2 g_{\pi} \bar{q} \gamma^\nu \gamma^\mu q \epsilon_{\nu \mu \lambda}$$

$$\times \left( \frac{m_{\pi^\nu}}{m_{\pi^\nu}^2 - q^2} + \frac{m_{\omega}}{m_{\omega}^2 - q^2} \right) \left( \frac{m_{\rho}}{m_{\rho}^2 - q^2} + \frac{m_{\omega}}{m_{\omega}^2 - q^2} \right)$$

$$+ \left( p \leftrightarrow q \right) \}$$

(1)

The coupling constants are normalized such that

$$\frac{g_{\pi}}{f_\pi} + \frac{g_{\omega}}{f_\omega} + \ldots = 1 \quad \forall \rho \text{ and } \omega$$

(2)

The naive VMD is the special case of eq. (1), in which we use only the parent $\rho$ and $\omega$ and neglect daughter states. From the decay width of $\rho \to 2 \pi$ and $\omega \to 3 \pi$, we have $g_\rho^2/4\pi = 2.88 \pm 0.9$ and $g_\omega^2/4\pi = 16.5 \pm 0.8$ GeV.$^{-2}$.

The dimensionless coupling constant $f_\nu$ is determined by the leptonic decay of vector mesons, e.g., $f_{\rho^0}/4\pi = 2.1 \pm 0.4$ and $f_{\omega^0}/4\pi = 18.3 \pm 4.1$.

Assuming the scaling law $^6$, we evaluate $f_\nu(n)$;

$$\frac{f_\nu^2}{f_\nu^2} = \frac{m_{\nu^0}}{m_{\nu^0}^2}$$

(3)
where $\mathcal{M}_\psi^2 = \mathcal{M}_\pi^2 (1 + \lambda N)$ with $\lambda \mathcal{M}_\pi^2 = 1$ GeV$^2$. In subsequent arguments we use as the first approximation, up to the first daughter state ($n = 1$) of each vector meson series. In fact, the analyses of the nucleon form factors indicate that there is a large contribution to the isoscalar nucleon form factor from the object of mass around 1.2 GeV (7), which we identify as $\omega'(1.2)$, the first Veneziano daughter of $\omega$. The author of ref. (7) also observed that the contribution of $\omega$ and $\omega'(1.2)$ cancel at large $q^2$, indicating

$$m_\omega^2 g_{\omega}/f_\omega + m_{\omega'}^2 g_{\omega'}/f_{\omega'} \sim 0$$

(4)

In the isovector form factor, on the other hand, the contribution of $\psi(1.2)$, which corresponds to $\omega(1.2)$ turns out to be very small, which is quite consistent with our estimation

$$g_{\psi}/f_{\psi} \sim 1 - \frac{g_p}{f_p} = -0.17$$

(5)

These properties of the form factors suggest that in the VMD calculation, we should include the effect of at least the first daughter state.

From eqs. (2-5) we obtain

$$g_{\omega} = -\frac{m_\omega^2}{m_\omega^2 - m_\omega'} f_\omega$$

$$g_{\omega'} = \frac{-m_\omega^2}{m_\omega^2 - m_\omega'} f_\omega$$

(6)

The numerical values of various coupling constants are given in Table 1, where we also give values for $g_{\omega}$ in naive VMD. From this table, one obtains the resultant P$\psi'$ coupling constants as

$$g_{\psi' \pi} = 0.2 C_\pi eV^{-1}, \quad g_{\psi' \omega} = \frac{1}{3} G_\omega eV^{-1}$$

(7)

Note that the width $\Gamma(\omega \rightarrow \pi \pi) = 880$ KeV and SU(3) symmetry gives

$$g_{\psi' \pi} = 0.2 C_\pi eV^{-1}, \quad g_{\psi' \omega} = \frac{1}{3} G_\omega eV^{-1}$$

(8)

2. Photoproduction of vector mesons

Let us first discuss photoproduction of vector mesons (8). Denoting the natural and the unnatural parity exchange part of the production cross section of $\gamma P \rightarrow V P$ as $\sigma^N_\gamma$ and $\sigma^U_\gamma$ respectively, we can summarize the experimental results as follows (9). (1) $\sigma^N_\gamma$ is negligibly small compared to $\sigma^N_\gamma$. (2) $\sigma^U_\gamma$ is, on the other hand, substantial at low energy; typically $\sigma^U_\gamma \sim \sigma^N_\gamma$ around $E_\gamma \sim 3$ GeV, but at high energy $\sigma^U_\gamma$ decreases and $\sigma^N_\gamma$ is by one order of magnitude bigger than $\sigma^U_\gamma$ at $E_\gamma \sim 9.3$ GeV. (3) $\sigma^N_\gamma$ is well parametrized by diffraction

$$\sigma^N_\gamma = C \left( 1 + \frac{D}{E_\gamma} \right) e^{At}$$

with $C = 9.3 \pm 1.7$ $M^2$ GeV$^2$, $D = 1.3 \pm 1.2$ and $A = 6.7 \pm 0.6$ GeV$^{-2}$. 

(8)
while (4) $\sigma^U$ is well parametrized by OPE, where $\mathcal{G}_{\pi\pi\pi\pi}$ is deduced from $\Gamma(\omega\rightarrow\pi\pi) = 880$ KeV.

Naive VMD predicts

$$\frac{\sigma^U}{\sigma^N} \sim \frac{1}{8\pi} \frac{\sigma^U}{\sigma^N}$$  \hspace{1cm} (9)$$

if $\sigma^U$ is saturated by OPE and $\sigma^N(p\gamma\rightarrow pp) = \sigma^N(\omega\gamma\rightarrow\omega p)$. OPE is a good approximation insofar as the peripheral region is concerned and we assume, throughout this paper, the unnatural parity exchange is saturated by OPE. The latter equality is quite natural if we consider the Pomeron exchange at high energy. Equation (9) explains why $\sigma^U$ is so small.

In our GVMN, the value $\mathcal{G}_{\pi\pi\pi\pi} = 0.83$ is within several percent the same as the $\mathcal{G}_{\pi\pi\pi\pi} = 0.78$ predicted from $\Gamma(\omega\rightarrow\pi\pi) = 880$ KeV. (cf. eqs. (7) and (8)). Therefore we cannot distinguish the two models by comparing $\sigma^U$ with experiment. Both models reproduce $\sigma^U$ successfully. On the other hand, $\mathcal{G}_{\pi\pi\pi\pi}$ is by a factor 1.3 smaller in our GVMN than $\mathcal{G}_{\pi\pi\pi\pi}$ given by SU(3) and the $\Gamma(\omega\rightarrow\pi\pi) = 880$ KeV. We can expect that $\sigma^U$ is by a factor $\sim 1.7$ smaller than in the naive VMD. But as is mentioned above, $\sigma^U$ itself is so small that it is extremely difficult to detect the effect of this factor 1.7. Nevertheless, if it turns out that $\sigma^U$ and $\sigma^N$ does not obey SU(3), it would provide more evidence for SU(3) breaking in the $\pi\pi\pi\pi$ coupling. The conclusion is, of course, subject to the assumption of OPE saturation of $\sigma^U$.

3. Electroproduction of vector mesons

In the case of electroproduction \cite{11,12}, there is an additional contribution from the longitudinal photon. General formulae for OPE are given by Praas \cite{13} and summarized in the Appendix of ref. 14. We consider here only $\omega$-electroproduction, because $\rho$-production is roughly saturated by the natural parity exchange and gives no information on $\rho\gamma\gamma$ couplings. Indeed, recent experimental data give the upper bound \cite{15}.

$$\sigma^U(\gamma\gamma\gamma\rightarrow pp) \leq (0.17\pm 0.04)\sigma_{tot}(\gamma\gamma\gamma\rightarrow pp).$$

In the OPE term of $\omega$-electroproduction we use, as in ref. 14) the Benecke-Nörr form factors \cite{16} and the diffraction term is derived from photoproduction. The results are shown in Fig. 1 together with the experimental data \cite{14}. The solid line corresponds to the naive VMD and the dashed line corresponds to our GVMN. GVMN predicts a smaller OPE cross section, e.g. at $Q^2 = 1.0$ GeV$^2$, the ratio of the two curves for the OPE part is $\sim 1.4$. However, as the diffusive cross section dominates at large $Q^2$, the difference of the two models is smeared out in $\sigma_{tot} = \sigma^U + \sigma^N$ (see Fig. 1). In this respect an experiment which separates $\sigma^U$ from $\sigma^N$ is desirable. At the present stage where we have no such experimental data, it is not possible to check the model. Again, if the OPE cross section is substantially smaller than the naive VMD, especially at not too small $Q^2$, it will be an indication of SU(3) symmetry breaking. We would like to stress the importance of separating $\sigma^U$ and $\sigma^N$ in comparing various models.

4. $\omega'$ and $\rho'$ production

Finally we estimate the cross section $\sigma(\gamma\gamma\gamma\rightarrow \omega')$ and $\sigma(\gamma\gamma\gamma\rightarrow \rho')$.
predicted in our GVMD. For this purpose one needs several additional assumptions for the natural parity exchange amplitudes in both reactions. We follow the GVMD with "off diagonal" transitions of ref. (5). This model includes contributions from a series of vector mesons \( V_N \) and introduces, in addition to the elastic amplitudes \( V_N P \rightarrow V_N P \), the "off-diagonal transitions" \( V_N P \rightarrow V_N P' \) i.e. diffraction dissociation of vector mesons. For the motivation of this procedure as well as for all details we refer to ref. (5) \(^{17}\). Here we want only to list the main assumptions. These are:

1. the same mass spectrum of vector mesons as above, \( M_{V_N}^2 = M_{V_0}^2 (1 + \lambda N) \), and the same mass dependence, eq. (2), for the photon-vector meson couplings \( f_{\gamma V} \) with alternating sign.

2. The elastic amplitudes \(^{18}\) \( T(V_N P \rightarrow V_N P) = T(V_N P \rightarrow V_N P') \) are the same for all vector mesons \( V_N \) (independent of the vector meson mass).

3. For the transition between neighbouring vector mesons \( V_N \) and \( V_{N+1} \)

   \[ \frac{T(V_N P \rightarrow V_{N+1} P)}{T(V_N P \rightarrow V_N P)} = C_N . \]

   \( C_N \) is a function of the masses \( M_N \) and \( M_{N+1} \).

4. For the mass dependence of diffraction dissociation we take a simple power law

   \[ \frac{T(V_N P \rightarrow V_N P')}{T(V_N P \rightarrow V_N P)} = \left( \frac{M_N}{M_{N+1}} \right)^{2N+1} \quad \text{for } N = 2, 3, \ldots . \]

Here we fix the parameters \( p \) and \( c_N \) to be \( p = \frac{1}{2} \) and \( c_N = \left( \frac{M_N}{M_{N+1}} \right)^2 = \frac{1}{2} \) (independent of \( N \)). For our purpose of obtaining a rough qualitative estimate for the natural parity exchange cross sections for \( \gamma P \rightarrow \omega' P \) and \( \gamma P \rightarrow P' P \) we can further simplify this model: we take only the first three states \( N = 0, 1, 2 \), corresponding to \( p, p', p'' \) and \( \omega, \omega', \omega'' \) respectively.

Then after a simple calculation we find that the natural parity exchange cross section for photoproduction of \( p' \) or \( \omega' \) is strongly suppressed:

\[ \sigma_N(\gamma P \rightarrow \pm |p'\rangle |p\rangle) \sim \left( \frac{1}{2c} \sqrt{\frac{1}{200}} \right) \sigma_N(\gamma P \rightarrow |p\rangle |p\rangle) \]

This is clearly a consequence of including diffraction dissociation of vector mesons and of the alternative sign assumption for the \( \gamma - V_N \) couplings. Of course, the numerical value of the suppression factor in the above equation cannot be taken too seriously. The qualitative result of strong suppression of diffractive \( \omega' \) - and \( P' \)-photoproduction does, however, not depend crucially on the detailed assumptions (i.e. the values of \( c_N \) and \( p \)) of the model discussed. This would explain why it is so hard at high energy to produce \( p' \) and \( \omega' \). In contrast, \( p'' \) and \( \omega'' \) production is estimated in this model as

\[ \sigma_N(\gamma P \rightarrow |p''\rangle |p\rangle) \sim \frac{1}{4} \sigma_N(\gamma P \rightarrow |p\rangle |p\rangle) \]

that is, we can expect a sizable diffractive production of \( p'' \) and \( \omega'' \).
Note that in photoproduction experiments, $J^P = 1^+ \eta \omega'$ and $J^P = 2^+ \omega \omega'$, a resonance state of mass 1250 MeV has been seen \(^{19}\). The production cross section is as big as $1 \sim 2 \mu_b$ and the dominant decay mode is into $\omega \pi^0$. The spin and parity of this state is either $J^P = 1^+$ or $1^-$, corresponding to $B$ or $\omega'$, and both assignments are compatible with the data. In view of the smallness of $\sigma^N(p \to p' \omega)$ predicted in our model, we regard this object as $J^P = 1^+$ a meson and we don't identify it with the object of mass 1250 MeV, which is found as an enhancement in the $e^+ e^- \to \omega \pi^0$ reaction. The latter is regarded as $p'(1,2)$. (See Leith in ref. 4). It is therefore crucial in the "off diagonal transition" model that the former object is not $p'(1,2)$.

As to the OPE part of $\omega'$ and $\omega'$ one notices from Table 1 that $(\sigma_{\omega'}/\sigma_{\omega'})_0 \approx 0.0$. Therefore $\sigma_{\omega'}^N(\pi \to \omega')$ is expected to be quite small. For OPE'-production, however, one has considerable contributions from OPE so that at not too high energies it should be possible to see $\omega'$. Expected $\omega'$ production cross sections are shown in Fig. 2. The dominant decay mode of $\omega'$ is $\omega' \to \pi^+ \pi^- (e \to \eta \pi^0\pi^0)$.

The results do not change much for $Q^2 \neq 0$ so that in electroproduction, the diffractive contributions is small and the OPE part of the $\omega'$ production cross section is about one third of the OPE part of the $\omega$ production cross section.

5. Comments

Finally several comments are in order. (1) one obtains the ratio $\sigma_{\omega'}/\sigma_{\omega}$ at $Q^2 = 0$ from the photoproduction experiment

$$\frac{\sigma_{\omega'}}{\sigma_{\omega}} \approx \frac{\sigma_{\omega'}}{\sigma_{\omega}} \cdot \text{e}, \frac{1}{2} \text{f}.$$ 

This ratio decreases as $Q^2$ increases in both GMD and naive VMD, since, as is seen from Fig. 1, $\sigma_{\omega'}/\sigma_{\omega}$ is a decreasing function of $Q^2$. Experimental data show a flat or rather a slightly increasing behaviour of $\sigma_{\omega'}/\sigma_{\omega}$ with increasing energy. Possibly this discrepancy could for the peripheral region be due to the Benecke-Dörr form factors we used. By these form factors a rather strong additional $Q^2$-dependence is introduced which with increasing $Q^2$ damps the OPE cross section.

(2) If the narrow vector resonance recently discovered \(^{20}\) around 1100 MeV is isoscalar, we can alternatively use it in place of $\omega' (1,2)$, because what is essential in our GMD is the existence of an isoscalar object around 1.2 GeV, which contributes strongly to the isoscalar form factor. Without further information we cannot go into detail at this point.

(3) An analytic model of pion coupling to the two virtual photons is proposed in terms of the hypergeometric function \(^{21}\). This model is completely within the framework of SU(3) symmetry and cannot explain the small width $\Gamma (\eta \to \pi 
p)$ Nevertheless it is interesting to compare the model with ours. Pulling out $\frac{q_{\omega' \pi}}{q_{\eta \pi}}$ and $\frac{q_{\omega' \pi}}{q_{\eta \pi}}$ in both models and comparing $q_{\omega' \pi}/q_{\eta \pi}$ and $q_{\omega' \pi}/q_{\eta \pi}$, we find that our coupling is reproduced by setting $\gamma = 2$ for the $\omega$ series and $\gamma = 1$ for the $\omega'$ series, while $\gamma = \frac{3}{2}$ in the original
model. Here, $\gamma$ is a fixed parameter corresponding to quark-pseudoscalar meson trajectories appearing in ref. (21). The fact that we need two different values of $\gamma$ for the $\omega$ series and the $\rho$ series is the mere reflection of the SU(3) breaking nature of our GVMD. For these values of $\gamma$, 
\[ g_{\omega\pi} = \frac{1}{3m_{\omega}c_f} \quad \text{and} \quad g_{\omega\pi} = \frac{1}{2m_{\omega}c_f} \quad \text{which indicates}\]
that SU(3) is broken by a factor $g_{\omega\pi}/3g_{\omega\pi} = 2/3$ but in an opposite direction to what the experimental data show.

(4) We have shown in this paper that our GVMD is consistent with the present experimental data on $\omega$ and $\rho$ photo- and electroproduction and that to distinguish various models a very accurate experiment which separates natural and unnatural parity exchange is required. We have also shown that the $\gamma'$ and $\omega'$ diffractive production would be quite small while $\gamma$ and $\omega$ diffractive production can be as big as one forth of $\rho$ and $\omega$ production.

(5) The experiment which allows us to separate $\gamma'$ from $\gamma$ is highly desirable in comparing the various models for OPE amplitude and $V^H$ couplings. Experiments which definitely determine the spin and parity of the object at 1250 MeV observed in photoproduction experiments would also be desirable.

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2. For a review see for example R.G. Moorhouse, CERN report CERN TH 2103 (1976).
G. Cosme et al., Orsay preprint LAI 1287 (1976)
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J.J. Sakurai and D. Schildknecht, Phys. Lett. 40B, 121 (1972)
A. Bramon et al., Phys. Lett. 41B, 309 (1972)

10. It is quoted in ref. 8 that \[ \frac{\Gamma_{s}}{\Gamma_{a}} = 0.03 - 0.07 \text{ at } p_{T} \sim 3 \text{ GeV}, \]
where the suffix denotes the isospin exchanged in the t-channel.


14. F. Joos et al., DESY report 77/09


17. The first part of this section is based on the discussion with B.J. Read and D. Schildknecht.

18. Since we are discussing photoproduction in the forward direction, we can restrict ourselves to the s-channel helicity amplitudes for transversely polarized photons and vector mesons.

F. Rabe et al., DESY report DESY-F1-F1/2

20. S. Bartalucci et al., DESY report 76/63 and Frascari report LNF-77/1(F)


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Table 1

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Table 1. Various coupling constants in our model. In the last column we give the values derived from \( \Gamma_{\nu} \rightarrow \gamma \gamma \) and \( \text{SU}(3) \).

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Figure captions

Fig. 1: \( Q^{2} \) dependence of \( \Gamma(\nu p \rightarrow \omega p) \) as predicted by the naive VMD (full curves) and by GVMD (dashed curves) for \( 2.0 \text{ GeV} \times \text{GeV} < 2.8 \text{ GeV}^{2} \text{ and } |t| < 0.5 \text{ GeV}^{2} \). The experimental data are from Ref. 14. The dashed-dotted line is the diffractive cross section \( \chi_{N}(\nu p \rightarrow \omega p) \), common to both models.

Fig. 2: \( Q^{2} \) dependence of \( \Gamma(\nu p \rightarrow \omega p) \) as predicted by the GVMD of Ref. 3 for \( 2.5 \text{ GeV} < 3.0 \text{ GeV} \text{ and } |t| < 0.5 \text{ GeV}^{2} \).