Thoughts about the SUSY WIMP. ...and the state of SUSY generally...

Kai Schmidt-Hoberg

Largely based on

1603.09347
1701.03480

with G Ross and F Staub

RWTH Aachen theory seminar
Thoughts about the SUSY WIMP.
...and the state of SUSY generally...

Kai Schmidt-Hoberg

“SUSY anywhere is better than SUSY nowhere!”

b) Can you make at least the first half of the talk understandable for Master's students who know a little bit of QFT but no SUSY.

Cheers,
Jamie

RWTH Aachen theory seminar
A SUSY partner for each SM particle with $\Delta s=1/2$ with the same mass

SUSY broken in nature

Breaking mechanism unknown, parameterized by soft terms
MSSM

- Field content fixed: theory specified by superpotential and soft terms

\[ \mathcal{W} = \mu H_u H_d + \kappa_i L_i H_u \]
\[ + Y_e^i H_d L_i E_j^c + Y_d^i H_d Q_i D_j^c + Y_u^i H_u Q_i U_j^c \]
\[ + \lambda_{ijk}^0 L_i L_j E_k^c + \lambda_{ijk}^1 L_i Q_j D_k^c + \lambda_{ijk}^2 U_i^c D_j^c D_k^c \]
\[ + \kappa_{ij}^0 H_u L_i H_u L_j + \kappa_{ijk\ell}^1 Q_i Q_j Q_k L_\ell + \kappa_{ijk\ell}^2 U_i^c U_j^c D_k^c E_\ell^c \]

\[ L_{SB} = -\frac{1}{2} \sum_a M_a \bar{\lambda}_a \lambda_a - \sum_i m_{\Phi_i}^2 |\tilde{\Phi}_i|^2 + T_u H_u \tilde{Q} \tilde{u} + T_d H_d \tilde{Q} \tilde{d} + T_e H_d \tilde{L} \tilde{e} + B_\mu H_u H_d \]

- Many new parameters (>100) but likely not independent
High scale SUSY

- In many models SUSY breaking at high scale in hidden sector
- Often some universality
- Take into account running to predict SUSY spectrum at the electroweak scale.
Why we like(d) SUSY

- Hierarchy problem: stabilizes the weak against the Planck scale
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- Dark matter: If lightest SUSY particle stable $\rightarrow$ dark matter candidate
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- Gauge coupling unification:

![Graphs showing gauge coupling unification](image-url)
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- A 125 GeV Higgs boson: Additional hint for SUSY?

...a TC guy from Odense still owes me...
Why we like(d) SUSY

- Hierarchy problem: stabilizes the weak against the Planck scale
- Dark matter: If lightest SUSY particle stable → dark matter candidate
- Gauge coupling unification:
- A 125 GeV Higgs boson: Additional hint for SUSY?
- Also hard to get 750 GeV diphoton excess ;-)
Why we like(d) SUSY

> Hierarchy problem: stabilizes the weak against the Planck scale
> Dark matter: If lightest SUSY particle stable → dark matter candidate
> Gauge coupling unification:
> A 125 GeV Higgs boson: Additional hint for SUSY?

> So why do people get worried?

~ 2 TeV
Why we like(d) SUSY

- Hierarchy problem: stabilizes the weak against the Planck scale
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- So why do people get worried?

NATURALNESS!

But maybe what looks unnatural can still be natural? Nature decides...
The LSP – a DM candidate.

- Neutralinos are mixtures of bino, Wino and Higgsinos
- EM and colour neutral → potentially interesting dark matter candidates
The LSP – a DM candidate.

- Neutralinos are mixtures of bino, Wino and Higgsinos
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\[ W = \mu H_u H_d + \kappa_i L_i H_u + Y_e^i H_d L_i E_j^c + Y_d^i H_d Q_i D_j^c + Y_u^i H_u Q_i U_j^c \]
\[ + \lambda_{ijk}^{(0)} L_i L_j E_k^c + \lambda_{ijk}^{(1)} L_i Q_j D_k^c + \lambda_{ijk}^{(2)} U_i^c D_j^c D_k^c \]
\[ + \kappa_{ij}^{(0)} H_u L_i H_u L_j + \kappa_{ijk\ell}^{(1)} Q_i Q_j Q_k L_\ell + \kappa_{ijk\ell}^{(2)} U_i^c U_j^c D_k^c E_\ell^c \]

- With only SUSY and gauge invariance: extra terms leading to proton and dark matter decay → need additional symmetry.
- Standard assumption: R-parity conservation (good enough for dark matter – not good enough for the proton, need a better symmetry such as $Z_4^R$)
Neutralinos are produced in the early universe via thermal freeze out

1. Neutralino $X$ is initially in thermal equilibrium:
   \[ XX \leftrightarrow qq \]

2. Universe cools:
   \[ XX \rightarrow qq \]

3. Universe expands:
   \[ XX \rightarrow qq \]

Relic abundance depends on annihilation cross section
Neutralinos are produced in the early universe via thermal freeze out

Jungman, Kamionkowski, Griest (1995)
How naturally can the dark matter relic abundance be achieved?

Often universal gaugino masses assumed at high scale, at low scale $M3: M2: M1 \sim 6:2:1 \rightarrow \text{bino LSP}$

Bino: Typically need to finely tune relic density via coannihilations or resonances :-(

Crucially depends on assumption of SUSY breaking terms! Other patterns possible...

2-3 TeV Wino challenged by ID
Mariengela Lisanti et al 1307.4082

1 TeV Higgsino looking good :-)
EW naturalness in the MSSM

> How naturally can we achieve the correct Higgs vev?

> Electroweak vev (or $M_Z$) determined by SUSY parameters (from minimization condition for scalar potential)

\[
\frac{m_Z^2}{2} = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2 \approx -m_{H_u}^2 - \mu^2
\]

> Cancellation (tuning) needed for large SUSY masses

> How to quantify this?

\[
\Delta_p \equiv \frac{\partial \ln v^2}{\partial \ln p} = \frac{p}{v^2} \frac{\partial v^2}{\partial p}
\]

'sensitivity measure'

> Large $\Delta$ implies large tuning
Caveats of the sensitivity measure

- What fundamental parameters should be included (and what are the fundamental parameters)?
- Also depends on parameterization of fundamental parameters
- At which scale?
- It measures sensitivity rather than 'tuning' – can be different
- The acceptable values $\Delta$ depends on taste – no absolute measure

$\rightarrow$ While for a given definition it can be calculated precisely, its physical interpretation is somewhat blurred.
The usual story

What does this tell us about a natural SUSY spectrum?

- $\mu$ is a superpotential parameter and hardly runs: $\mu_{\text{EW}} \sim \mu_{\text{GUT}}$

$$\text{Higgsino mass} \sim \mu \sim 1 \text{ TeV} \rightarrow \Delta_\mu \sim \frac{2\mu^2}{M_Z^2} \sim 250$$

- “Natural SUSY requires light Higgsino”

What about the $m_{H_u}$ part?

- Loop effects introduce a large sensitivity to stop and gluino masses

\[
\delta m_{H_u}^2 = -\frac{3y_t^2}{4\pi^2} m_t^2 \ln \left( \Lambda / m_t \right)
\]

\[
\delta m_{\tilde{t}}^2 = \frac{2g_s^2}{3\pi^2} m_{\tilde{g}}^2 \ln \left( \Lambda / m_{\tilde{g}} \right)
\]
The 'natural SUSY' spectrum

Naturalness

TeV

\( m_2 \)

\( h \)

\( t \)

\( \tilde{W} \)

\( \tilde{g} \)

\( \tilde{q} \)

\( \tilde{\ell} \)

N Craig
EW naturalness in the MSSM

- Starting from the high scale, all soft terms contribute to $m_{Hu}$ and $m_Z$

\[
m_Z^2 \approx -2.18\mu^2 + 3.84M_3^2 + 0.32M_3M_2 + 0.047M_1M_3 - 0.42M_2^2 + 0.011M_2M_1 - 0.012M_1^2 - 0.65M_3A_t - 0.15M_2A_t - 0.025M_1A_t + 0.22A_t^2 + 0.004M_3A_b - 1.27m_{Hu}^2 - 0.053m_{Hd}^2 + 0.73m_{Q_3}^2 + 0.57m_{U_3}^2 + 0.049m_{D_3}^2 - 0.052m_{L_3}^2 + 0.053m_{E_3}^2 + 0.051m_{Q_2}^2 - 0.11m_{U_2}^2 + 0.051m_{D_2}^2 - 0.052m_{L_2}^2 + 0.053m_{E_2}^2 + 0.051m_{Q_1}^2 - 0.11m_{U_1}^2 + 0.051m_{D_1}^2 - 0.052m_{L_1}^2 + 0.053m_{E_1}^2,
\]

- We don't just want $m_{Hu}$ to be small, but every contribution to it. Assuming no correlations among the terms, need rather light stops and gluinos
Starting from the high scale, all soft terms contribute to $m_{Hu}$ and $m_Z$

$$m_Z^2 \simeq -2.18\mu^2 + 3.84M_3^2 + 0.32M_3M_2 + 0.047M_1M_3 - 0.42M_2^2$$
$$+ 0.011M_2M_1 - 0.012M_1^2 - 0.65M_3A_t - 0.15M_2A_t$$
$$- 0.025M_1A_t + 0.22A_t^2 + 0.004M_3A_b$$
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$$+ 0.051m_{Q_2}^2 - 0.11m_{U_2}^2 + 0.051m_{D_2}^2 - 0.052m_{L_2}^2 + 0.053m_{E_2}^2$$
$$+ 0.051m_{Q_1}^2 - 0.11m_{U_1}^2 + 0.051m_{D_1}^2 - 0.052m_{L_1}^2 + 0.053m_{E_1}^2$$

$m_i^2 = m_0^2$

$\sim 0.01 m_0^2$

We don't just want $m_{Hu}$ to be small, but every contribution to it. Assuming no correlations among the terms, need rather light stops and gluinos

But we know correlations should be present…

Example: the scalar focus point.
The gaugino focus point

> Assume fixed ratios of gaugino masses

\[ M_1 = a \cdot m_{1/2} \]
\[ M_2 = b \cdot m_{1/2} \]
\[ M_3 = m_{1/2} \]

> Possible also in GUTs
Comment on loop corrections

- So far assumed tree-level relation for EWSB condition

\[
\left. \frac{\partial V^{(L)}}{\partial v_u} \right|_{\tan \beta \to \infty} \equiv 0 = \left( m_{h_u}^2 + \mu^2 + \frac{1}{8} (g_1^2 + g_2^2) v^2 \right) v + \Sigma_u
\]

**How to parametrise \( \Sigma_u \)?**

1. \( \Sigma_u \equiv v \Sigma_{uu} \)

2. \( \Sigma_u \equiv \Sigma_1 v + \Sigma_2 v^2 + \Sigma_3 v^3 \)

\[\rightarrow \quad \frac{1}{2} M_Z^2 = -|\mu|^2 - m_{H_u}^2 + \Sigma_{uu} \]

no change in FT; only valid if \( \Sigma_{uu} \) is independent of \( v \! \)!

\[\rightarrow \quad \Delta \mu = \frac{8 \mu^2}{(g_1^2 + g_2^2 + 8 \Sigma_3) v^2 + 4 \Sigma_2 v} \]
Comment on loop corrections

Result for leading stop correction

\[
\Sigma_1 = \frac{3m^2_t Y^2_t \left( -2 \log \left( \frac{2m^2_t + \nu^2 Y^2_t}{Q^2} \right) \right) + 2 + \log(4)}{32\pi^2}
\]

\[
\Sigma_2 = 0
\]

\[
\Sigma_3 = \frac{3 Y^4_t \left( \log \left( \frac{2m^2_t + \nu^2 Y^2_t}{Q^2} \right) \right) - \log \left( \frac{\nu^2 Y^2_t}{Q^2} \right)}{32\pi^2}
\]

\[
r^{FT} = \frac{\Delta_\mu^{\text{Loop}}}{\Delta_\mu^{\text{Tree}}} \approx \left( 1 + \frac{3}{4\pi^2} \frac{Y^4_t}{g_1^2 + g_2^2} \log \left( \frac{m^2_t}{m^2_t} \right) \right)^{-1} \approx \frac{1}{2}
\]

Reduction of about \( \frac{1}{2} \) when including loop corrections
A new contribution to the Higgsino mass

> Non-standard SUSY breaking terms (in the classification of S Martin: 'maybe-soft')

\[ \mathcal{L}_{NH} = \mu' \tilde{h}_d \tilde{h}_u + T'_{u,ij} h^*_d \tilde{u}^*_R, i \tilde{q}_j + T'_{d,ij} h^*_u \tilde{d}^*_R, i \tilde{q}_j + T'_{e,ij} h^*_u \tilde{e}^*_R, i \tilde{l}_j + \text{h.c.} \]

> \( \mu' \) contributes to the Higgsino mass (\( m_h \sim \mu + \mu' \)) but does not enter the tadpole equations

> Can significantly reduce FT

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Kai Schmidt-Hoberg | Thoughts about the SUSY WIMP | 6 July 2017 | Page 27
Studied different MSSM variants with GUT boundary conditions

<table>
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<th>$m^2_{h_u}$</th>
<th>$m^2_{h_d}$</th>
<th>$M_1$</th>
<th>$M_2$</th>
<th>$M_3$</th>
<th>$\mu'$</th>
<th>$A'_0$</th>
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1701.03480
Results non-universal gaugino masses

- Region of small FT can be well beyond LHC reach

\[ \mu' = 0 \]

- Allowing for DM underabundance FT can be as small as 10.
A 1 TeV Higgsino can be quite natural

\[ \mu' = 0 \]

\[ \Delta \mu \sim \frac{2\mu^2}{M_Z^2} \]
What does the spectrum in the regions with low FT look like?
Prospects for direct detection

- No lower bound on relic abundance (and rescaled) – other DM component
Prospects for direct detection

- No lower bound on relic abundance (not rescaled) – non-thermal production (gravitino decay)
Prospects for direct detection

Correct (thermal) relic abundance
What looks unnatural from an IR perspective **might** still look natural from the UV

Extra Higgsino mass contribution $\mu'$ could help

To do: build a UV model

SUSY could well be beyond the LHC reach

Good chances at direct detection experiments to find it
What looks unnatural from an IR perspective might still look natural from the UV

Extra Higgsino mass contribution $\mu'$ could help

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Thank you!

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