Color versus Charm

by

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Color versus Charm

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Abstract

We discuss the decay systematics (including inclusive decays) of the new mesons seen in $e^+e^-$ collisions. Our aim is to show how one can substantiate or demolish the color scheme, which we contrast to the charm model.

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The discovery of two narrow resonances which decay to $\psi^+$ has caused much agitation$^1,2,3$. At present the nature of these states is unclear. Assuming that they are new hadronic levels, the immediate task is to settle their quantum numbers and to find the other expected new states.

We consider certain classes of $\psi^+$ experiments with these points in mind. As tools we use the SU(4) charm model$^4$ and the color model (the Han-Nambu$^5$ model exorcised of its baryon number 1/3 quark states). Both models have a complex level structure and may serve as prototypes of new schemes - SU(n) or SU(3) x G. Both models may be demolished in a straightforward way. If neither is correct, one may obtain valuable hints about the proper structure in the course of the demolition work. We shall first discuss the classification of levels in the two schemes and then the decays of $\psi(3.1)$, $\psi(3.7)$ and the higher states. Perhaps the recently seen bump in $e^+e^- \rightarrow$ hadrons at $\sqrt{s} = 4.1$ GeV is one of these$^6$.

In the charm scheme, $\psi(3.1) = \psi = 1^3 S_1$ is the ground state $c\bar{c}$ system (quarks c, u, d, s). Higher states in $\psi^+$ are then radial excitations in the quark model, $\psi(3.7) = \psi' = 2^3 S_1$, etc. There may be $3^1 D_1$ states near them. The levels below $\psi'$ in mass are $3^1 P_0$, $3^1 P_1$, $1^3 P_1$, and $3^3 P_2$ about 3.4 GeV (ignoring splittings)$^7$, and a $1^3 S_0$ near the $\psi$. There are altogether $\approx 20$ states below $\psi''$ ($3^3 S_1$), expected at $4.1 - 4.2$ GeV for equal spacing in $m^2$. These states can be reached by radiative or hadronic transitions (peaks in missing mass), and ought to be narrow. Hadronic transitions violate Zweigs rule and by analogy to $\phi \rightarrow \rho \pi$ and $\phi \rightarrow \eta \gamma$ probably have widths comparable to those for radiative transitions ($10^1 - 10^2$ keV). Large mixing of $c\bar{c}$ with SU(3) singlet pseudoscalars (e.g. $\eta'$) could make the level at $4.1 - 4.2$ GeV broad even if it were below the $c\bar{c}$ threshold, $\psi'' \rightarrow (c\bar{u}) + (\bar{c}u), ..., \psi'' \rightarrow (u\bar{u}) + (d\bar{d})$. Otherwise it could only be broad if above the charm threshold.

The classification of $\psi(3.1)$ and $\psi(3.7)$ in the color scheme is less straightforward. States can be labelled by an SU(3) index and an SU(3) color index$^5$. The electromagnetic current transforms as $(8,1) + (1,8)^5,8$. Octet color states are $(1,8)$ and $(8,8)$. We shall discuss no others. An appealing assignment is to assume $(1,8) + (8,8)$ mixing and take ideally mixed $\psi \leftrightarrow \psi^\prime$ color $(u\bar{u} + d\bar{d})$ and $\psi^\prime \leftrightarrow \psi$ color $(s\bar{s})^{[2]}$. Then the color symmetry can be exact to the level of electromagnetism (of course, it need not be). It is then customary to assume dominant radiative decays of $\omega_c$ and $\phi_c$. These must be suppressed$^5$. 
The level structure is complex. As an illustration, we took full nonet symmetry for the vector states and then further assumed (approximate) degeneracy of $q\bar{q}$ orbital and spin levels. This is only meant as a guide; in particular, mixing can considerably shift the masses. For $\eta_c$ we took pure $uu + d\bar{d}$ and pure SU(3) $\eta$ as extremes. The color index $c$ runs over $c = \pi^\pm, \eta^0 ...$. The charm and color levels are shown on the figure ($\eta_c = uu + d\bar{d}$).

Lastly, we remark on the degeneracy of $\omega_c = \omega_\eta$ and $\phi_c = \phi_\eta$. Because of the arbitrary orientation of the color isospin axis, one of these can be chosen to decouple from the photon in the symmetry limit. Then only $\omega_\eta$ and $\phi_\eta$ are produced and hadronic decays of higher $\omega_\eta$, $\phi_\eta$ feed only the $\eta$-level in color space.

We now go on to discuss inclusive spectra, chain and radiative decays of the lowest levels in $e^+e^-$. Because of the high mass of $\psi(3.1)$ and $\psi(3.7)$, it is natural to consider inclusive decay spectra. Pion spectra in the charm model are familiar: We expect $\pi^0 = \pi^+$ for final states without baryons or $K\bar{K}$. States with an even number of pions arise only via $\psi\psi' \to l\gamma \to$ hadrons. For the color model the two lowest states should have dominant radiative decays. There will be (O($a$)) effects arising from $\omega_c, \phi_c \to l\gamma \to$ hadrons, and also non radiative decays due to virtual emission and absorption of a photon. From the example of $p - \omega$ mixing, we expect that the latter may well be bigger than the former. The only restriction on the final state is then $I \leq 1$. We discuss chain decays later. For radiative decays, the hadron system recoiling against the "colored" photon has $I = 0$, $G = +$ and the number of pions is even. Amusingly, the ratio $\Gamma(\phi_c \to \pi + X)/\Gamma(\omega_c \to \pi + X) \to 0$ as $2p_\pi/m \to 1$ (the fastest particle in $\phi_c$ decay is always a $K$ or $\eta$).

For $K$ and $\eta$ spectra in the charm model, we expect $K^+ = K^0_S = \eta$ from SU(3) (pure octet $\eta$). The color case with $\psi(3.1) = \omega_c$ and $\psi(3.7) = \phi_c$ is interesting. If we assume dominance of radiative decays (or subtract the others), the hadron system has the same quantum numbers as $\omega_c$ or $\phi_c$. From isospin alone $K^+ = K^0_S$. Besides this, we can get bounds from the parton model for decay of a heavy state $(n(\phi_c) \equiv \Gamma^{-1}d\Gamma/\eta dp$, etc.)
\[ \omega_c: \quad 2/3 \leq \eta/K^+ \leq 4/3 \]
\[ \phi_c: \quad 0 \leq \eta/K^+ \leq 4/3 \]

and from the \( \omega_c \) distributions we get (approximately, ignoring mass differences or using \( x_p = 2 \ p/m \)) \( \eta(\phi_c) = 6 \ K^+(\omega_c) - 5 \eta(\omega_c) \), \( K^+(\phi_c) = 4 \ K^+(\omega_c) - 3 \eta(\omega_c) \).

Of course, this all depends on \( \text{SU}(3) \). With luck, the above may be good to \( \sim 20 \% \). Near the kinematical limit \( 2 \ p/m + 1 \) familiar considerations give for \( \omega_c, \eta/K^+ \to 2/3 \) and for \( \phi_c, \eta/K^+ \to 4/3 \).

There are a number of chain decays common to the charm and color models, and some characteristic of the latter. In turn: \( \psi(3.7) \to \psi(3.1) \eta \). In the charm case this is a Zweig rule violating decay and is consistent with the order of magnitude \( G_{\psi_c}^2/G_{\psi}^2 p^2 \rho_p \sim 10^{-2} \) expected for such decays. We used an \( \eta \)-meson pole model for this estimate (see also J.D. Jackson \( ^{10} \)). The couplings are dimensional, so the discrepancy between this and the factor \( \sim 10^3 \) suppression of \( \Gamma(\psi) \) compared to a typical hadron decay is not alarming. In the color model it is perhaps natural that such decays are suppressed to the level of radiative decays, as for the \( \phi \).

\[ \psi(3.7) \to \psi(3.1) \eta \] It is common knowledge that this is suppressed in the charm model because the \( \eta \) is mostly octet. Besides this, \( \psi(3.7) \) is \( 2^2S_1 \) (a radial excitation), probably giving a further suppression. If we estimate this to be at least a factor \( \sim 10 \) from nonobservance of \( e^+e^- + \rho' + \pi^- \) relative to \( e^+e^- + \rho' + \rho \) we have \( \Gamma(\psi' + \psi \eta) - \sin^2 \theta_{\eta_e - \eta_{\eta}} \times (0.1) \times \Gamma(\phi + \rho \pi) \sim 2 \text{ KeV} \). For the color case we include the (small) effect of relative phase space and find \( \Gamma(\phi_c + \omega_c \eta) = \Gamma(\phi + \rho \pi) \ p_{\eta}^3/p_{\pi}^2 \sim .9 \text{ MeV} \). This is way too big, but it indicates that a large \( \phi_c + \omega_c \eta \) branching ratio is to be expected.

\[ \phi_c + \rho_c \pi \] This decay is characteristic of color: The ground state with \( I = 0 \) \( \omega_c \) is degenerate with an \( I = 1 \) state \( \rho_c \) which does not couple to a single photon. The decay \( \phi_c + \rho_c \pi \) violates Zweig's rule. Comparing to \( \phi + \rho \pi \) we would expect \( \Gamma(\phi_c + \rho_c \pi) - (p_{\rho_c}/p_{\rho})^2 \Gamma(\phi + \rho \pi) \sim 16 \text{ MeV} \); but a remark is in order. The large suppression of radiative decays in the color model may even require that such hadronic vertices be smaller than our estimate uses. In any event, the existence of such a decay would eliminate charm.

Chain decays of the higher states are even more interesting. The \( \psi''(4.2) \) in the charm model is \( 3^3S_1 \). For a simple harmonic potential there are 20 cc
states below it in mass, and of these \( c = - \) states can be reached by \( s \)-wave \( \pi \pi \)
emission and \( c = + \) states by \( 3 \pi \) or photon emission (and maybe \( J = 1 \) KK). A
detailed model is needed to estimate widths, and this is not our aim. We just
note that there seems to be no reason to expect individual decays of this class
to be much bigger than a few hundred KeV, with perhaps one sort of exception;
\( \psi'' \to \psi' \) can be a strong decay if \( \eta' \) has a large mixing with \( \eta \). We ought
to note, however, that \( \psi'' \to \psi' \) involves dropping two radial modes and could
be suppressed. It might be that \( \psi'' \to \psi'S' \) is also a strong decay. Then one has
to keep \( \psi' \to \psi' + \pi \) small, perhaps by making the \( \varepsilon \) nearly pure octet (mixing
angle with the singlet \( \theta < 1 \)). A broad resonance near 4.2 GeV may be a
difficulty for this scheme, perhaps even if the \( \psi'' \) is above the \( \eta \) threshold.

In the color scheme a state near 4.2 GeV would be broad \( (\omega_C') \) (with \( \phi_C' \)
and 4.6 GeV). To get crude estimates for the decays we did the following.
All levels kinematically accessible from \( \omega_C' \) are counted (see the classification
schemes). Decays like \( \rho' \to \rho \varepsilon \) are normalized to this process and symmetry
relations like \( G_{\omega_C \omega_C} = G_{\rho' \rho} \) (we choose \( \varepsilon, \delta^0 \) and \( \eta_C \) pure \( uu + dd \)). Then \( \Gamma(\omega_C' \to \omega_C) \sim 90 \text{ MeV}, \Gamma(\omega_C' \to \rho_C \delta^0) \sim 27 \text{ MeV}, \Gamma(\omega_C' \to K^{+} K^{-}) \sim 80 \text{ MeV}. [4] Another set is \( \omega_C' \to \rho_C \pi, \rho \pi_C, \omega_C \eta, \omega \eta_C, K^{+} K^{-}, K^{+} K^{-} \) (and charge conjugates), which would have to be
normalized to the unknown (small?) \( \rho' \to \omega \eta^0 \). Finally, \( \omega_C' \to \pi \pi, K^{+} K^{-} \).

Such decays provide a copious source of new mesons; some dramatic effects
such as narrow spikes in inclusive spectra \( \omega_C' \to \pi^+ + X, K^+ + X \) are expected.

Finally, a remark on radiative decays. In the charm model these are only
significant when the decay is to \( \gamma (\eta \eta) \) state[5] In the color quark model scheme,
decays to colored and uncolored mesons are related. Further, there are two co-
lored pseudoscalars, etc., compared to the \( \eta \eta \) case where there is only one. De-
fining \( \Gamma = \Gamma(V + P) \rho_{\gamma}^3 \), then \( \Gamma(\omega_C \to \eta \gamma): \Gamma(\omega_C' \to \eta \gamma): \Gamma(\phi_C \to \eta \gamma) = 1 : 2 : 2 : 1 \) ignoring the small mixing angles. For a \( uu + ss \) \( \eta_C \) and \( ss \)
\( \eta_C' \), \( \phi_C \to \eta_C \eta \) and \( \phi_C \to \eta_C' \eta \) if both decays are allowed by kinematics. With
the same proviso, an octet (singlet) \( \eta_C \) (\( \eta_C' \)) satisfy \( \Gamma(\omega_C \to \eta \gamma): \Gamma(\omega_C' \to \eta \gamma): \Gamma(\phi_C \to \eta \gamma) = 1/8 : 1/4 : 4 \) : 2 \( \Gamma(\omega \to \eta \gamma) \) normalized to unity). Simi-
lar arguments can be applied to colored \( J^{PC} = 0^{+}, 2^{+} \) states. If a vector
meson radiative decay does not occur because of kinematics, it may be possible
to observe the radiative pseudoscalar (scalar, tensor) decay to vector plus
photon.
It is useful to remark that a predominance of directly produced inclusive single photons at large $p$ (compared to photons from $\pi^0, \eta, \omega$ decay) would be evidence that the color scheme is correct. The ratio of directly produced photons to photons from decay of light states in the charm case is expected to be $O(\alpha)$. Of course, one needs to subtract photons from pseudoscalar, scalar and tensor $c\bar{c}$ decays to $\gamma\gamma$.

In order to substantiate any new scheme it is obviously necessary to find the hadronic states characteristic of it and of no other model. In the case of color or charm this process is straightforward as we have seen. Of course, both models contain states we have not mentioned - charmed or colored baryons and (in the color model) doubly charged meson states and perhaps even triply charged baryon states.

It may even happen that the charm model turns out to be correct at low ($\sqrt{s} \sim 3 - 10$ GeV) energies and a color degree of freedom (three quartets) appears at very high energy.

Acknowledgement

We want to thank S. Orito for comments. After this was worked out we were informed by V. Rittenberg that some of our points have also occurred to others.

Added Note

Recently a number of papers on color have come to our attention.
Footnotes

[1] We limit the treatment of this option since we have already discussed it. 

[2] This assignment has the nice feature that \( \Gamma(\phi_c \to e^+ e^-) = (1/2) \Gamma(\omega_c \to e^+ e^-) \). The identification \( \psi(3.1) \leftrightarrow \omega_c; \psi(3.7) \leftrightarrow \phi_c \) must have occurred to many people.

[3] This has been discussed by M. Krammer, D. Schildknecht and F. Steiner (private communication) who also take \( \omega_c, \phi_c \).

[4] Here and elsewhere \( \Gamma(V^+ \to V \bar{e}) = \frac{1}{2M_V} \frac{G^2}{4\pi} \frac{P_{CM}^2}{M_V} \left(1 + \frac{1}{3} \frac{P_{CM}^2}{M_V^2}\right) \) also, \( \Gamma(\rho^+ \to \pi \bar{e}) = 350 \text{ MeV} \).

[5] For an attempt to estimate radiative widths, see Ref. 11.
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Figure Caption

Levels in the charm and color schemes. For charm we took a quadratic mass formula for vector states plus degeneracy of L-levels in the quark model. Some states have been omitted at the $\psi'$ level to avoid confusion. For color we took $\psi(3.1) \leftrightarrow \omega_c$, $\psi(3.7) \leftrightarrow \phi_c$ and nonet symmetry for the other states with $\epsilon_c$, $\delta_c$, $\eta_c$ pure $c \bar{u} + d \bar{d}$. Both level schemes should be taken only as a guide; states may easily be drifted by several hundred MeV with respect to our estimates.