ELECTRON-BUNCH STREAKING WITH SINGLE-CYCLE THz RADIATION USING AN NSOM-STYLE TIP *

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Abstract

Near-field scanning optical microscopy (SNOM) is a technique based on simple conical structures with relatively small apertures; the incident laser light is mostly reflected but a small evanescent field beyond the aperture can be used to make measurements beyond the diffraction limit. In this paper we discuss a scaled version of SNOM to mm-scales, suited especially for available laser-based single-cycle THz sources. We explore the potential of the device for electron-bunch diagnostics.

INTRODUCTION

A relatively difficult but invaluable measurement of an electron bunch is the longitudinal phase space (LPS). The LPS describes the longitudinal position and momentum along the bunch length \( (z, p_z) \), and is therefore a very relevant measurement for optimizing e.g. bunch-compression. This measurement is typically done by applying a transverse deflecting force along the bunch [1, 2]. The imparted transverse time-dependent force acting on the bunch ideally leads to a linearly varying transverse kick along the temporal profile of the bunch. This leads to a transverse shearing of the bunch which maps into a transverse plane along a drift. The resolution of such a measurement is given by \( R = \frac{\sigma_y \cdot \epsilon r c^2}{\gamma e k V} \), where \( \sigma_y \) is the transverse normalized emittance, \( \sigma_r \) is the transverse rms beam size, \( e \) is the electron charge, \( k \) the wavenumber of the applied field, \( V \) the integrated voltage, \( c \) the speed of light and \( m \) the electron mass.

Measurements are typically made with streaking cavities powered by MW-scale klystrons in a frequency range of 1-12 GHz. The recent emergence of semi-efficient (~1%) [3] laser-based single-cycle THz-sources, across a broad range of frequencies (~0.1-3 THz), provides an appealing route to improve the resolutions of streaking measurements and has motivated some recent research [4–12].

Near-field scanning optical microscopy (SNOM) is a technique based on simple conical structures with relatively small apertures; the incident laser light is mostly reflected but a small evanescent field beyond the aperture can be used to make measurements beyond the diffraction limit. The earliest work on SNOM dates back to E. Synge (1928) [13], who was motivated by Einstein via an exchange of letters to publish his work. Today, the technique has been largely developed to measure nm-scale phenomena [14, 15].

In this paper we explore the use of SNOM-like tips to apply a streaking field to short electron bunches with readily available single-cycle THz pulses. In our scheme, a linearly polarized (in \( \hat{y} \)) pulse propagates into the \( \hat{x} \) direction and enters a conical tip filled with dielectric permittivity \( \varepsilon_r \). An orthogonally propagating (\( \hat{y} \)) and properly delayed electron bunch samples an electric shearing force in \( \hat{y} \); see Fig. 1. We discuss the basic theory of the device and present some simulation results.

![Figure 1: A schematic of the scheme is illustrated; the cone is filled with dielectric material with permittivity \( \varepsilon_r \) and the outer surface is coated with a metal; the top and bottom have no metallic coating. Here an electron bunch propagating in the \( \hat{x} \) direction passes over the conical-tip aperture. A synchronized single-cycle THz pulse propagating in the \( \hat{y} \) direction, with \( \hat{y} \)-polarization enters the conical tip and is enhanced near the aperture. The corresponding deflecting force is entirely in the \( \hat{y} \) direction.](image)

THEORY

To gain insights into the propagation of a pulse into such a conical device, we consider a simple analytic model by dividing the domain into strips and applying a waveguide analysis. For an electromagnetic wave with wavelength \( \lambda \) propagating in a dielectric cone of half angle \( \delta \), base diameter \( D_0 \), tip aperture diameter \( D_a \), the decay of the initial transmitted power \( P_0 \) at the aperture of the cone tip \( P_a \) is given by an exponential decay mediated by the attenuation coefficient \( \alpha_{11} \) of the HE_{11} mode [16, 17],

\[
P_a = P_0 e^{-\int_{z_0}^{z_a} \gamma_{11}(z)dz},
\]

where \( \gamma_{11}(z) = \frac{\alpha_{11}}{2} \), \( \alpha_{11} \) is the attenuation coefficient of the HE_{11} mode [16, 17],

\[
\gamma_{11}(z) = \frac{\alpha_{11}}{2} \int_{z_0}^{z_a} e^{-\int_{z_0}^{z'} \gamma_{11}(z')dz'}dz,
\]

\[\int_{z_0}^{z_a} \gamma_{11}(z)dz = \frac{\alpha_{11}}{2} \int_{z_0}^{z_a} e^{-\int_{z_0}^{z'} \gamma_{11}(z')dz'}dz.
\]

This work was supported by the European Research Council (ERC) under the European Union’s Seventh Framework Programme (FP/2007-2013)/ERC Grant agreement no. 609920

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* This work was supported by the European Research Council (ERC) under the European Union’s Seventh Framework Programme (FP/2007-2013)/ERC Grant agreement no. 609920

T03 Beam Diagnostics and Instrumentation

06 Beam Instrumentation, Controls, Feedback and Operational Aspects

Proceedings of IPAC2017, Copenhagen, Denmark

MOPAB053

ISBN 978-3-95450-182-3

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where $z_a = D_0 - D a \tan(\delta)$. The attenuation of the HE$_{11}$ mode in terms of its propagation constant $\gamma_{11} = \alpha + i \beta$ has two sources. Absorption from the real part $\alpha$ is low ($\approx 0.01$ cm$^{-1}$) at 300 GHz in dielectrics like quartz and sapphire [18] and will be neglected here.

The phase constant $\beta$ is imaginary below the cutoff frequency $\omega_c = 2\pi f_c$, i.e. for an evanescent mode. The cutoff frequency depends on the refractive index $n$ of the material and waveguide aperture and can be well approximated by $\omega_c = \frac{405c}{na}$, where $a$ is the radius of the waveguide [19].

Finally, we must account for the Fresnel reflection at the upper and bottom boundary of the structure; we account for this via $R = |\frac{n-1}{n+1}|^2$; the transmission is given by $T = 1 - R$, and finally due to two interfaces the full transmission is given by $T^2$. Collecting all the effects, we can obtain the transmitted field strength as a function of input power by the appropriate scaling with the aperture area ($E \propto P/A$), for a structure with a $45^\circ$ angle and input aperture of diameter 1 cm; we show the resulting transmitted field strengths for various $n$ as a function of aperture in Fig. 2. Larger $n$ allow the higher transmitted powers to reach the device tip but also suffer from larger Fresnel losses; this could be improved with e.g. an anti-reflection coating.

With an overview on the electromagnetic propagation in the structure, we are now ready to discuss the resulting forces and transverse dynamics a properly delayed electron bunch will experience. Let us first define some parameters: $\lambda$ as the input wavelength, $c$ the speed of light, $D_a$ the aperture of the device-tip, $\sigma_x$ the longitudinal rms bunch length and $\beta$ the normalized velocity of the electron bunch.

The Lorentz force describes the force acting on a charged particle in a magnetic field, $F = e(E + v \times B)$. Here however, we have chosen the $B$-polarization to be parallel to the electron bunch velocity, therefore the there is no magnetic contribution and the transverse force is entirely due to the electric field $F_y = eE_0 \cos(\omega t - k_z z)$, here $E_0$ is the electric field strength at the tip, $k_z$ is the wavenumber, and $\omega$ is the angular frequency.

To ensure a linear streaking action on the electron bunch, we should require that the centroid of the bunch sample a linear portion of the field i.e. near the zero crossing. This additionally implies that the bunch length should be small relative to the wavelength, $\sigma_x \ll \lambda$. With these considerations, we make the following loose criterion that the bunch centroid should pass through a quarter of a wavelength about the zero crossing,

$$t_d \approx \frac{\lambda}{4c}.$$  

The time required for the electron bunch to traverse the aperture is given by

$$t_d = D_a / v_e$$

where $v_e$ is the electron velocity. These equations can be rearranged to set a restraint on $D_a$,  

$$D_a < \frac{\lambda \beta}{4}$$  

where $\beta$ is the normalized velocity $v_e/c$.

If we assume the bunch samples only a linear portion of the electric field and also travels through a symmetric portion of the field (e.g. from $\sin -\phi \rightarrow \sin \phi$), then the centroid of the bunch will not have any deflection. Moreover, the net momentum acquired onto the centroid of the bunch will be zero. In this scheme, the streaking is entirely due to the longitudinal extent of the bunch; consider for example a displacement offset $\delta$ from the bunch centroid starting location $x_0$, after passing through a linear portion of the electric field with slope $m$, the final transverse energy of such a slice is given by

$$\int_{-x_0+\delta}^{x_0+\delta} F \cdot dx \approx 2eE_0 k_x dx \approx 2eE_0 k x_0 \delta.$$  

Here we note that the zero offset of the centroid should provide a calibration in an experimental realization.

**SIMULATION**

Finally, to validate the method, we simulate such an interaction using CYRUS, a discrete Gallerkin finite time-domain code and use a finite-element mesh generator GMSH [20] to create the structure. The parameters for the simulation can be found in Table 1; as described above, a linearly polarized single-cycle pulse polarized in $\hat{y}$ is launched into the $\hat{z}$ direction into the deflection probe; a properly delayed electron bunch travelling in the $\hat{x}$ direction is deflected in the $\hat{x} - \hat{y}$ plane.

We present snapshots of the simulation in the $y-z$ plane where the pulse is approaching the interaction region and at the time of maximum field enhancement in Fig. 3(left) and Fig. 3(right) respectively.
Figure 3: A contour plot of a snapshot in time of the electric field \(E_y\) traveling toward the interaction region is shown in (left). (Right) we show a snapshot in time of \(E_y\) at the maximum intensity. Here the scale shown in the bottom right has a maximum value of 30 MV/m.

<table>
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<th>parameter</th>
<th>symbol</th>
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<tr>
<td>relative permittivity</td>
<td>(\epsilon)</td>
<td>4.41</td>
<td>–</td>
</tr>
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Table 1: Parameters Associated in the Numerical Simulation.

In Fig. 4(top) we show simulation results for the net force acting on the bunch as a function of position over the deflection probe; the asymmetry of the force acting on the bunch is due to a non-optimized configuration. Finally in Fig. 4(bottom), we show the transverse momentum chirp over the bunch. Finally, the chirp over the bunch over a drift will generate a shearing of the bunch and lead to a transverse mapping onto e.g. a screen located downstream. The resolution is governed by the equations given above.

**CONCLUSION**

We have described a relatively simple method to streak electron bunches using single-cycle THz radiation. We launch a linearly polarized single-cycle into a metallic-coated dielectric cone which couples to a pseudo-\(HE_{11}\) mode; the dielectric reduced the cutoff frequency in the cone to effectively allow a THz enhancement at a relatively small aperture at the tip of the structure. The scheme uses a \(~\text{quater-cycle of the THz pulse to generate a linear transverse momentum chirp over the length of the electron bunch. Our future work looks to develop the theoretical model further and investigate alternative polarizations for other manipulation possibilities. Finally, the original work and idea of SNOM – to image objects beyond the diffraction limit – motivates us to consider this possibility with an electron bunch; i.e. could we image an electron bunch, or its shadow using electromagnetic radiation? We hope to extend this possibility with a theoretical investigation between non-relativistic interactions between light and electrons.**

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