Perspectives on the Higgs $p_T$ as a probe of BSM physics

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Higgs production and BSM physics
Gluon Fusion

Vector Boson Fusion

Quark associated production

Higgs Strahlung

Higgs production channels at the LHC

Perspectives on the Higgs $p_T$ as a probe BSM physics
BSM physics in the Higgs sector

Characterization of the boson at 125 GeV

- Deviation of the Yukawa couplings from the SM.
- Deviation of the couplings to the EW gauge bosons.
- Gluon fusion: new states in the loop?

The differential measurements can probe NP differently from the inclusive results.

New Higgs states

- Extended Higgs sector?
- Simplest extension: Two Higgs Doublet Model (2HDM).
- MSSM has a type-II 2HDM.
- Non-SM like Yukawas can alter the hierarchy between different production processes.

Predictions required to properly recast experimental results in NP models.
$\rho^H_T$ theoretical description and its uncertainties
The $p_T^H$ distribution

- The Higgs acquires a transverse momentum due to the recoil against QCD radiation.
- At fixed order, the $p_T^H$ distribution diverges in the limit $p_T^H \to 0$.
- The physical behavior is restored by resumming the divergent terms, either analytically or numerically (i.e. through a Parton Shower).
- **Problem**: match the resummed and fixed order calculation.
- **Uncertainty estimation in this procedure is important for precision phenomenology.**

Available matched-resummed frameworks

- **Analytic resummation – b-space**: Collins, Soper, Sterman, Catani, Grazzini.
- **Analytic resummation – $p_T$-space**: Re, Torrielli, Monni et al.
- **Analytic resummation – SCET**: Becher et al.
- **NLO+PS, MC@NLO**: Frixione, Webber.
- **NLO+PS, POWHEG**: Frixione, Nason, Oleari.
- **NNLO+PS NNLOPS**: Hamilton, Nason, Re, Zanderighi

See P. Richardson’s talk for an overview of the matching and the latest developments.

[Bozzi Catani De Florian Grazzini, hep-ph/0302104]
Heavy Quark Effective Field Theory (HQEFT)

In the limit \( m_{top} \to \infty \) we can construct an effective Lagrangian for the interaction of the Higgs boson with the gluons

\[
\mathcal{L}_{\text{eff}} = \frac{\alpha_s}{12\pi} \frac{H}{v} (1 + \Delta) \text{Tr} \left[ G_{\mu\nu}^a G_{\mu\nu}^a \right]
\]

In this theory the heavy quark loop shrinks to a point vertex, simplifying the calculations.

Validity conditions

- Total cross section, \( m_H < 2m_{top} \)
- Kinematic variables, as \( p_T^H \), less than \( m_{top} \)
- No strongly coupled light particles running in the loop (e.g. bottom quark in the THDM/MSSM for large \( \tan \beta \))
The HQEFT vs the SM

- The emitted parton can resolve the internal structure of the quark loop if the $p_T^H$ is large enough (mass effects start at $p_T^H > 150$ GeV for the top loop and $p_T^H > 10$ GeV for the bottom one).

![Graph showing the comparison between HQEFT and SM for LHC 7 TeV with $m_H = 125$ GeV.](image)

$R = \frac{\frac{1}{\sigma} \frac{d\sigma}{dp_T^H}}{\frac{1}{\sigma_{\text{HQEFT}}} \frac{d\sigma_{\text{HQEFT}}}{dp_T^H}}$

[EB Degrassi Slavich Vicini, 1111.2854]
The HQEFT vs the SM

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NLO+PS

[EB Degrassi Slavich Vicini, 1111.2854]
The HQEFT vs the SM

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The inclusion of the bottom quark adds a mass scale that is much lower with respect to the others ($m_h$ and $m_t$).

We can always rewrite the full amplitude as

$$|\mathcal{M}(t + b)|^2 = |\mathcal{M}(t)|^2 + |\mathcal{M}(b)|^2 + \left[|\mathcal{M}(t + b)|^2 - |\mathcal{M}(t)|^2 - |\mathcal{M}(b)|^2 \right].$$

One should introduce separate resummation scales for the top ($Q_t$), the bottom ($Q_b$) and the interference ($Q_{\text{int}}$) contributions and rewrite the formula for the total cross section as

$$\sigma(t + b) = \sigma(t, Q_t) + \sigma(b, Q_b) + [\sigma(t + b, Q_{\text{int}}) - \sigma(t, Q_{\text{int}}) - \sigma(b, Q_{\text{int}})].$$

Extend the same reasoning to differential distributions.
Theoretical uncertainties

- We discuss the NLO-matched case for simplicity.

Analytic resummation

- In parameter space the cross section for multiple emissions factorizes and can be resummed. The factorization is defined at the unphysical resummation scale $Q$.
- $Q$ is unphysical and the complete result does not depend on it. However, at fixed resummation-accuracy one has a residual dependency on it.
- This dependency can be used to probe the theoretically uncertainty on the resummed spectrum.

Matched NLO+PS

- In MC@NLO, the SCALUP parameters determines the effective range of application of the resummation. It is chosen in such a way to recover the NLO behavior in the high-$p_T$ region.
- In POWHEG we can use effectively the $h$ parameter (that controls the higher order terms) to recover the NLO behavior in the high-$p_T$ region and then, with some caveats, again the SCALUP parameter to define the shower phase space.
- Scale determination oriented towards BSM physics.
- Two different approaches, based on different theoretical assumptions: collinear factorization ([Bagnaschi, Vicini '15]); high-$p_T$ matching ([Harlander et al, '15])
- Both approaches yield results that diverge from the customary choices at large Higgs masses.
BSM Physics perspectives
The 2HDM and the MSSM

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<th>Coupling</th>
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- Two Higgs doublets. Enlarged physical spectrum: $h/H/A$ and $H^\pm$.
- MSSM is a type II 2HDM; squark (and gluino at NLO) enters in the gluon fusion loops (many mass scales).
- Rescaled couplings to quarks. Change in the relative weight of the quarks in the gluon fusion process (e.g. bottom contribution larger than the top).
- If the bottom quark coupling to the Higgs is enhanced, the bottom annihilation process can be the dominant one.
In the MSSM the matrix element squared for the $gg \to gH$ channel is given by

$$|\mathcal{M}(gg \to gH)|^2 = |\mathcal{M}_t + \mathcal{M}_b + \mathcal{M}_{\tilde{q}}|^2 =$$

$$= |\mathcal{M}_t|^2 + 2\text{Re}(\mathcal{M}_t\mathcal{M}_b^\dagger) + |\mathcal{M}_b|^2 + 2\text{Re}(\mathcal{M}_t\mathcal{M}_{\tilde{q}}^\dagger) + 2\text{Re}(\mathcal{M}_b\mathcal{M}_{\tilde{q}}^\dagger) + |\mathcal{M}_{\tilde{q}}|^2$$

- The bottom contribution can be sizable if not dominant (i.e. $\tan \beta$ enhancement).
- This in turn could make the contribution of the interference between the bottom and the squarks relevant.
- Exact treatment of mass effects could not be neglected.
Higgs $p_T^H$ distribution in the MSSM

- $R = \frac{d\sigma_{\text{MSSM}}/dp_T^\phi}{d\sigma_{\text{SM}}/dp_T^H}$, where the SM spectrum is computed for an Higgs of equal mass to the SUSY one
- In red SusHi and POWHEG fixed order. In blue, POWHEG+PS.

- Light Higgs ($m_h \sim 123$ GeV)
  - light-stop scenario, $M_A = 130$ GeV, $\tan \beta = 40$ – illustrative purpose of the effect of non-standard couplings.

- Heavy Higgs ($m_H \sim 130$ GeV)

[HXSWG YR3, 1307.1347]
Uncertainty on the heavy Higgs $p_T^H$ distribution in the MSSM

$m_{h^{mod+}}$ scenario, $\tan \beta = 17$, $m_A = 500$ GeV, $m_H = 499.9$ GeV

- ±40% variation between the “normal” scale choices and the specific ones.

[EB, Harlander, Mantler, Wiesemann, Vicini 1510.08850]
Modifications of light-quark Yukawa couplings

- [Bishara et al. ’16, 1606.09253]
- Follows the same reasoning as in the MSSM/2HDM studies.
- Assumes only quarks running in the gluon-fusion loop.
- Includes HQEFT NLO corrections to the Higgs $p_T^H$ and NNLL resummation.
- Provides limits (8 TeV ATLAS data) and projections for the rescaling factor of the bottom and charm Yukawa couplings, $k_b$ and $k_c$.

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Perspectives on the Higgs $p_T$ as a probe BSM physics
Overview of other analyses

- Many other studies have been published on this subject that could not be covered here. For example:
  - EFT approach – Use the Higgs $p_T$ to break the degeneracy between different operators ([Grazzini et al, 1612.00283])
  - And many others...

- Probing non-SM top-quark coupling in H+jet (high $p_T$ region ([Azatov et al, 1309.5273] [Azatov et al, 1608.00977])).
- Other studies on light-quark Yukawa couplings modifications ([Cohen at al, 1705.09295], [Soreq et al., 1606.09621]).
Conclusions
Conclusions

- The Higgs transverse momentum distribution is sensitive to new physics in non-trivial ways.
- The measurement of this observable is going to be one of the focus of Higgs characterization in the current and future runs of LHC.

Properties

- Modifications of the Yukawa couplings (e.g. what happens in the 2HDM, MSSM etc.), impact the shape of the distribution.
- A proper treatment of the theoretical uncertainties is necessary for any meaningful discussion.

New colored states in gluon fusion

- Example: squarks in the MSSM.
- Non trivial shape modification when the emitted gluon starts probing the relevant mass scale.
Backup slides
References
Results overview

- To properly understand the sensitivity to BSM in the comparison with the data, precise SM predictions are required. It has a long history but new developments are still being made nowadays (see S. Forte talk for an overview).


- mixed NLO EWxQCD: [Anastasiou Boughezal Petriello] (2009).


- More work on the $p_T$ at higher-orders: [Caola et al], [Monni et al] (2016).
To properly understand the sensitivity to BSM in the comparison with the data, precise SM predictions are required. It has a long history but new developments are still being made nowadays (see S. Forte talk for an overview).

- H+J NLO-QCD with finite $m_t$ effects: [Harlander, Neumann, Ozeren, Wiesemann 2012].
- Pheno-analysis in GoSam, HQEFT, H+1,2,3J @ NLO-QCD, [Greiner et al 2015].

**Codes**

- HIGLU, Fehip - NLO full theory
- ggh@nnlo, HNNLO - NNLO-QCD HQEFT
- iHixs - NNLO-QCD HQEFT, NLO-EW,NLO-EW-QCD
- Pythia/Herwig
- HqT - (NNLO+NNLL) - QCD HQEFT
- HRES - MC NNLO+NNLL QCD (full theory@NLO)
- SusHi/ MoreSuShi / SusHi Bento - N3LO QCD HQEFT+SM/2HDM/MSSM
- CuTe - HQEFT, NNLO+NNLL
- aMC@NLO/aMCSushi/POWHEG - MC NLO + PS full theory/2HDM/MSSM;
- HNNLOPS, MiNLO merging.
- aMC@NLO with FxFx merging
Formalism
Matching in an analytic resummation framework

The master formula for the analytic matching is given by

\[
\frac{d\sigma}{dp_{\perp}^2} = \int \frac{d\Phi_B}{dp_{\perp}^2} (B + \hat{V}_{\text{fin}}) F_{\text{NLL}}(Q_{\text{res}}) + \int \frac{d\Phi}{dp_{\perp}^2} R \otimes \Gamma - \int \frac{d\Phi_B}{dp_{\perp}^2} B F_{\text{NLO}}(Q_{\text{res}}),
\]

with

\[
F_{\text{NLL}}(Q_{\text{res}}, p_{\perp}) = \frac{m_\phi^2}{S} \int_0^\infty db \frac{b}{2} J_0(b p_{\perp}) S(\alpha_s, \tilde{L})
\times \sum_{i,j} \int dz_1 dz_2 \left[ \delta z_1 \delta z_2 + \frac{\alpha_s(b_0/b)}{\pi} C_{gi}^{(1)}(z_1) \delta z_2 + \frac{\alpha_s(b_0/b)}{\pi} \delta z_1 C_{gj}^{(1)}(z_2) \right] \Gamma_{ij}(b_0/b, z_1, z_2),
\]

with

\[
S(\alpha_s, \tilde{L}) = \exp \left\{ \tilde{L} g^{(1)}(\alpha_s, \tilde{L}) + g^{(2)}(\alpha_s, \tilde{L}) \right\}
\]

- Additive matching. Remove explicitly the terms that are double counted.
- \( \tilde{L} = \ln(b^2 Q_{\text{res}}^2/b_0^2 + 1) \)
- The scale \( Q_{\text{res}} \) determines the \( p_{\perp} \)-range where the resummation is applied.
Matching in a NLO+PS framework

\[ d\sigma = d\Phi_B \overline{B}^s(\Phi_b) \left[ \Delta^s(p_{\perp}^{\text{min}}) + d\Phi_R|_B \frac{R^s(\Phi_R)}{B(\Phi_B)} \Delta^s(p_T(\Phi)) \right] + d\Phi_R R^f(\Phi_R) \]

\[ \overline{B}^s = B(\Phi_b) + \left[ V(\Phi_b) + \int d\Phi_R|_B \hat{R}^s(\Phi_R|_B) \right] \]

\[ \Delta(\Phi_B, p_T) = \exp \left\{ - \int d\Phi_{\text{rad}} \frac{R^s(\Phi_B, \Phi_{\text{rad}})}{B(\Phi_1)} \theta(k_T - p_T) \right\} \]

**MC@NLO**

\[ R^s \propto \frac{\alpha_s}{t} P_{ij}(z) B(\Phi_B) \quad , \quad R^f = R - R^s \]

**POWHEG**

\[ R^s = \frac{h^2}{h^2 + p_T^2} R \quad , \quad R^f = \frac{p_T^2}{h^2 + p_T^2} R \]

- The Sudakov form factor is the one from the P.S., i.e. it uses the collinear splitting function in the exponent.
- The full matrix element appears only in the regular contribution.
- \( h\) fact controls high order effects
- At low \( p_T \) \( R \) goes into collinear factorization and the Sudakov regains the splitting function in the exponent.

The two approaches differs by higher order terms.

See P. Richardson talk for a detailed talk on the topic and the latest developments.
Matching scales determination in gluon fusion
Collinear behavior of the $gg \rightarrow Hg$ amplitudes

\[ C \equiv \frac{|M_{gg \rightarrow Hg}(s,p_T^H,m_Q)|^2}{|M_{gg \rightarrow Hg}^{div}(s,p_T^H,m_Q) + p_T^H|^2} \]

Relative deviation from the collinear limit.

- The $p_T^H$ at which the deviation reach $\bar{C} = 0.9/1.1$ gives us our preferred value for the factor $h$.
- We choose a value of $s = s_{min} + s_{soft}$ close to the production threshold. Larger values should be PDF suppressed.
- $s_{min} = m_H^2 + 2(p_T^H)^2 + 2p_T^H\sqrt{(p_T^H)^2 + m_H^2}$.
- $s_{soft}$ is used to move away from the soft divergence.
- Analogous study for the $qg$ channel yields much lower scales.
- Combination of the two channels using a differential weights.
- Manifest effect of the top threshold.
- Monotonous line for HQEFT and the bottom since no relevant scales are crossed.
- For heavy Higgs masses, our scales lower than the extrapolation of the “canonical” ones ($m_H/2, m_H/1.2$), currently used for a light Higgs.
High-$p_T$ matching, example for $m_h = 125$ GeV

- Decomposition of the cross section in three contributions:

$$\sigma(t + b) = \sigma(t, Q_t) + \sigma(b, Q_b) + \sigma(\text{interference}, Q_{\text{int}})$$
Comparison of the scale sets

- Similar behavior for the bottom scale.
- Different behavior (especially around the two-top threshold), though compatible, for the top quark contribution.
- Opposite behavior for the interference scale when the interference terms goes to zero.
SM results
- **HRes** recovers the fixed order distribution at $\mathcal{O}(m_h)$ with a forced matching.
- Difference in the intermediate region due to different matching and possibly due to the structure of the **POWHEG** Sudakov form factor.
- Change the default shower scale choice in POWHEG, by capping it at the same value used for $h$.
- Tail of the distribution goes over the fixed order results.
Analytic resummation vs POWHEG vs MC@NLO in the SM and in the 2HDM
Comparison of the hadronic predictions

We show the comparison of the results obtained with

Analytic resummation (NLO+NLL),

NLO+PS POWHEG (NLO+LL),

NLO+PS MC@NLO (NLO+LL).

  \( \tan \beta = 50, \sin(\beta - \alpha) = 0.999, m_h = 125 \text{ GeV}, \ m_H = 300 \text{ GeV}, \ m_A = 270 \text{ GeV} \).

- 2HDM large-top scenario
  \( \tan \beta = 1, \sin(\beta - \alpha) = 0.999, m_h = 125 \text{ GeV}, \ m_H = 300 \text{ GeV}, \ m_A = 270 \text{ GeV} \).

- We have considered the shape of the distribution (i.e. \( 1/\sigma d\sigma/dp_T \)) for \( h, H \) and \( A \) production.

- Uncertainty band computed by varying only the matching scale using the rescaling-factor combination
  \( \{ Q_t/2, \ Q_t, \ 2 \cdot Q_t \} \times \{ Q_b/2, \ Q_b, \ 2 \cdot Q_b \} \times \{ Q_i/2, \ Q_b, \ 2 \cdot Q_i \} \) and then taking the envelope of the results.

- A more complete study, considering also different scenarios, is available in hep-ph/1510.08850
Scenario B – Matching uncertainty

- Bottom dominated scenario.
- Comparison at fixed scales (BV) of the different tools.
- Same behavior of the MCs up to 25 GeV. In the intermediate region POWHEG is flatter, then the two curves cross at $p_T \approx 150$ GeV.
- Overlap of the uncertainty bands.
- Different shape of the POWHEG vs MC@NLO band understood to be due to the very different distribution of the shower scale.

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- High-pt tail behavior enhanced in the case of bottom dominated models.
- Changing the default prescription for the shower scale (mPOWHEG) allow for the recovery of the fixed order at high-pt.
Sensitivity to the shower scale choice in \(\text{aMC@NLO}\)

In the default \(\text{aMC@NLO}\) implementation, the shower scale is chosen as

- **S–events**: it uses a probability density distribution, which depends on the kinematic of the event, and that results in relatively low scales.
- **H–events**: the scale is taken equal to the maximum of the distribution for the S–events.
- Probe the sensitivity to these choices by using instead a \(\delta\)-function distribution.

![Graph showing differential cross-sections](image)
H, large t scenario – Matching uncertainty

- Top dominated scenario.
- Comparison at fixed scales (BV) of the different tools.
- Very compatible behavior of the central predictions between the two MC.
- Overlap of the uncertainty bands.
- Very similar shape of the uncertainty band for the MCs.
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Top dominated scenario.
Fixed tool (AR), all scales compared.
Different scales for the top quark.
Deviation of the central value predictions.
H, large t – Scale sensitivity

- Top dominated scenario.
- Fixed tool (MC@NLO), all scales compared.
- Different scales for the top quark.
- Deviation of the central value predictions.
H, large t – Scale sensitivity

- Top dominated scenario.
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