Asymmetric thermal-relic dark matter

Iason Baldes
Work done in collaboration with Kalliopi Petraki

Planck 2017, Warsaw
1703.00478
(Asymmetric) Dark Matter Freezeout

Assume we have a DM asymmetry

Asymmetry $\eta_D \equiv Y^+ - Y^-$ frozen during freeze-out.
Also define $\epsilon \equiv \eta_D / \eta_B$

Fractional asymmetry

This ratio changes during freezeout.

$$r \equiv \frac{Y^-}{Y^+}$$

DM mass relation

$$M_{DM} = \frac{m_p}{\epsilon} \frac{\Omega_{DM}}{\Omega_B} \left( \frac{1 - r_\infty}{1 + r_\infty} \right)$$

- Graesser, Shoemaker, Vecchi 1103.2771; Iminniyaz, Drees, Chen 1104.5548
New here: Sommerfeld enhancement, bound state formation and unitarity
Vector mediator

\[ \mathcal{L} = \bar{X} (iD - M_{DM}) X - \frac{1}{4} F_{D\mu\nu} F_{D}^{\mu\nu} \]

- \( X \) denotes the DM particle
- Covariant derivative \( D^\mu = \partial^\mu + ig_D V_D^\mu \)
- \( F_{D}^{\mu\nu} = \partial^\mu V_D^\nu - \partial^\nu V_D^\mu \), with \( V_D^\mu \) being the dark photon field
- \( \alpha_D \equiv g_d^2 / (4\pi) \) being the dark fine-structure constant.

If \( X \) carries a particle-antiparticle asymmetry, another field is required to balance the implied \( U(1)_D \) charge asymmetry in \( X \).
Vector mediator - Sommerfeld enhancement and bound state formation

Symmetric case: - von Harling, Petraki 1407.7874

Here $\sigma_{v_{\text{rel}}} = \sigma_0(S_{\text{ann}}^{(0)} + S_{\text{BSF}})$. In the Coulomb limit, $S_{\text{ann}}^{(0)}$ and $S_{\text{BSF}}$ depend only on the ratio $\zeta \equiv \alpha_D/v_{\text{rel}}$

$$S_{\text{ann}}^{(0)}(\zeta) = \frac{2\pi \zeta}{1 - e^{-2\pi \zeta}}$$

$$S_{\text{BSF}}(\zeta) = \frac{2\pi \zeta}{1 - e^{-2\pi \zeta}} \frac{\zeta^4}{(1 + \zeta^2)^2} \frac{2^9}{3} e^{-4\zeta \arccot(\zeta)}$$

$$\sigma_0 \equiv \pi \alpha_D^2/M_{\text{DM}}^2$$
Scalar mediator

\[ \mathcal{L} = \bar{X}(i\phi - M_{DM})X + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m_\phi^2 \phi^2 - g_d \phi \bar{X}X \]

- \( \phi \) is the dark scalar force mediator with mass \( m_\phi \)
- \( \alpha_D \equiv g_d^2/(4\pi) \).

This is a p-wave process. However, as long as \( m_\phi \lesssim \alpha_D M_{DM}/2 \), the \( X - \bar{X} \) interaction manifests as long range. The velocity suppression is lifted due to the Sommerfeld enhancement!
Scalar mediator - Sommerfeld enhancement

This is a $p$-wave annihilation process

\[ \sigma_{\text{ann}} \nu_{\text{rel}} = \sigma_1 \nu_{\text{rel}}^2 S_{\text{ann}}^{(1)} \]

\[ \sigma_1 = \frac{3\pi \alpha_D^2}{8M_{\text{DM}}^2} \]

\[ S_{\text{ann}}^{(1)}(\zeta) = \frac{2\pi \zeta}{1 - e^{-2\pi \zeta}} (1 + \zeta^2) \]

- As before, $\zeta \equiv \alpha_D / \nu_{\text{rel}}$.
- At $\nu_{\text{rel}} \lesssim \alpha_D$, $\sigma_{\text{ann}} \nu_{\text{rel}} \propto 1/\nu_{\text{rel}}$.
- The $\nu_{\text{rel}}^2$ suppression of the perturbative cross-section morphs into an $\alpha_D^2$ suppression, with $\sigma_{\text{ann}} \nu_{\text{rel}} \propto \alpha_D^5$. 
Boltzmann Equations - Vector Mediator

- Three coupled equations, taking into account $Y^+ \ (Y^- = Y^+ - \eta_D)$, and the two bound states $Y_{\uparrow\downarrow}$ and $Y_{\uparrow\uparrow}$.
- At some stage $T$ drops enough so bound state decay becomes quicker than ionization.
- Annihilation through the bound state then becomes significant.
- We take into account the $T$ difference between the visible and dark sectors.

Similarly for the scalar mediator but without the bound states.
Relic abundance - Example

\[ \text{Y}_X, \text{Y}_{\text{para}}, \text{Y}_{\text{ortho}} \]
Required couplings/cross-section - vector mediator
Required couplings - scalar mediator
The effective cross-section for indirect detection signals,

\[ \sigma_{ID} v_{rel} = \left[ \frac{4r_\infty}{(1 + r_\infty)^2} \right] \sigma_{inel} v_{rel}. \]

We have used \( v_{rel} = 10^{-3} \), which is relevant for indirect searches in the Milky Way.
The effective cross-section for indirect detection signals,

\[ \sigma_{\text{ID}} v_{\text{rel}} = \left[ \frac{4 r_\infty}{(1 + r_\infty)^2} \right] \sigma_{\text{inel}} v_{\text{rel}}. \]

We have used \( v_{\text{rel}} = 10^{-3} \), which is relevant for indirect searches in the Milky Way.
Unitarity constraint

In the non-relativistic regime

\[ \sigma_{\text{inel}} v_{\text{rel}} \leq \sigma_{\text{uni}} v_{\text{rel}} = \frac{4\pi(2J + 1)}{M_{DM}^2 v_{\text{rel}}} \]

- Note that with SE \( \sigma v_{\text{rel}} \propto 1/v_{\text{rel}} \), meaning there is no need to insert an arbitrary \( v_{\text{rel}} \) on the RHS of the inequality, as would be the case if naively using \( \sigma v_{\text{rel}} \sim \alpha_D^2/M_{DM}^2 \) or \( \sigma v_{\text{rel}} \sim \alpha_D^2 M_{DM}^2/m_{\text{med}}^4 \).
- We obtain some \( \alpha_{\text{uni}} \) above which the unitarity constraint is violated. However, \( \sigma v_{\text{rel}} \) is based on a perturbative calculation - the relevant approximations will break down before this.
- The \( \sigma^{(J)}_{\text{uni}} v_{\text{rel}} \propto 1/v_{\text{rel}} \) behaviour indicates that to approach the unitarity limit, the cross section will necessarily display some long range \( 1/v_{\text{rel}} \) behaviour, at least in the types of scenarios explored here.
Unitarity constraint - Results

![Graphs showing unitarity constraints for different wave types and parameters.](image-url)
Approaching Unitarity constraint implies a long range interaction

In the non-relativistic regime

\[ \sigma^{(J)}_{\text{inel}} v_{\text{rel}} \leq \sigma^{(J)}_{\text{uni}} v_{\text{rel}} = \frac{4\pi (2J + 1)}{M_{\text{DM}}^2 v_{\text{rel}}} \]

- Interaction mediated by a heavy force carrier of mass \( m_{\text{med}} \gtrsim M_{\text{DM}} \).
- \( \sigma v_{\text{rel}} \sim \alpha_D^2 M_{\text{DM}}^2 / m_{\text{med}}^4 \).
- Realising unitarity limit
  \[ \alpha_D^{\text{uni}} \sim (m_{\text{med}} / M_{\text{DM}})^2 / \sqrt{v_{\text{rel}}} \gtrsim m_{\text{med}} / M_{\text{DM}} \gtrsim 1. \]
- This implies \( m_{\text{med}} \lesssim \alpha_D^{\text{uni}} M_{\text{DM}} \).
- That is range of the interaction between two DM particles, \( m_{\text{med}}^{-1} \), is comparable or larger than their Bohr radius, \( (\alpha_D^{\text{uni}} M_{\text{DM}} / 2)^{-1} \).
- Interaction manifests as long-range, thereby contradicting the original premise of a contact-type interaction.
Asymmetric DM scenarios require a slightly larger annihilation cross section.

We have calculated the required $\alpha_D$ in some simple example scenarios including Sommerfeld enhancement and bound state formation.

We have explored the unitarity constraint.

This is a first step needed in order to constrain these models experimentally.

Thanks.