sured with the Brookhaven flying spot digitizer and analyzed with the Brookhaven QLOD system of programs. The selection on competing hypotheses was made from a fit to the measured ionization and the usual chi-square value from the kinematic fit. Events for which there were insufficient or no measurements of ionization available were examined visually. Spectator protons with insufficient energy to leave a visible track were assumed to have momentum components 0 ± 50 MeV/c for the purposes of fitting and for calculations of missing mass.

⁴The charged-to-neutral decay ratio for the η is 0.29:0.71. This rate is corrected for the 15% loss of neutrally decaying η 's which are outside the " η_N " region, giving a ratio of 0.33:0.67 for the rates of η_C to η_N which we expect to see in our data. Since the η_C events have more tracks measured, their failure rate in the analysis programs is slightly greater (by $\sim 1\%$). This makes an insignificant change in the above ratio.

⁵We have considered and eliminated the following possible causes: (1) It is not a reflection of the $\Sigma(1385)^-$ in the $\Sigma(1385)^-\pi^-\pi^+$ final state since the effect remains with equal strength when events showing $\Sigma(1385)^-$ are removed. (2) It is not due to the tail of the π^0 peak in Fig. 1(b) contaminating the " η_N " sample since we estimate this at less than 15 events in the whole $\pi^-\pi^+\pi^0$ effective mass spectrum. (3) The final state $\Sigma^0\pi^0\pi^-$ will appear in Reaction (2). From the similar final state $\Sigma^-\pi^+\pi^-$ we find predominant $Y_0^*(1520)\pi^-$ and some $\Sigma^-\rho^0$ production, and predict that from the decay $Y_0^*(1520) \rightarrow \Sigma^0\pi^0$ there will be essentially no events with the missing mass ($\pi^0\gamma$) in the η region. From Monte Carlo calculations on the $\Sigma^0\rho$ final state, by selecting events with $\pi^0\gamma$ mass lying in the η region, we

find a 300-MeV-broad distribution centered at 1 GeV consisting of about 40 events. This is much too wide to account for the effect in question. (4) From the $\Sigma^0\pi^+\pi^-\pi^-$ final state we estimate that there will be about 70 events from the reaction $\Sigma^0\pi^-\pi^0\pi^0$ which appear in Reaction (2) with missing mass in the " η_N ." These events populate the $\pi^-\pi^+\pi^0$ spectrum uniformly with ~ 6 events in the " η_N " region. (5) The $\pi^0\pi^0\pi^0$ spectrum from the $\Lambda\pi^-\pi^0\pi^0\pi^0$ final state is limited by phase space in the η region so that contributions from it are negligible.

⁶The region from 980 to 1060 MeV is taken as the " π_N " region. Although the enhancement in Fig. 1(d) peaks below 1000 MeV, the center was at 1020 MeV before the events were kinematically constrained to the η mass.

⁷See D. J. Crennell *et al.*, Phys. Rev. Letters **22**, 1327 (1969).

⁸Inclusion of events showing $\Sigma(1385)$ production does not materially change the shape of the distribution in the 1 GeV region.

⁹Although the total $\rho^0\pi^-$ mass spectrum shows a broad enhancement at 1.06 GeV, the " $\eta^-\pi^+$ " effective-mass selection creates a narrower peak shifted towards 1.0 GeV.

¹⁰From Figs. 2(a)-2(c) we find that in the Dalitz plot of the " A_2 " $\rightarrow \pi^+\pi^-\pi^-$ region the " $\eta^-\pi^+$ " area is populated more densely than the adjacent $\pi^-\pi^+$ regions.

¹¹All the other figures of Ref. 1 compare well with our data from Reaction (4).

¹²For a review see B. French, in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, September, 1968 (CERN Scientific Information Service, Geneva, Switzerland, 1968), p. 104.

COMPARISON OF PHASE-SHIFT ANALYSIS SOLUTIONS TO π^-p SCATTERING DATA*

A. D. Brody, D. W. G. S. Leith, B. G. Levi, and B. C. Shen

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

and

D. Herndon, R. Longacre, L. Price, A. H. Rosenfeld, and P. Söding†

Lawrence Radiation Laboratory, University of California, Berkeley, California 94720

(Received 28 April 1969)

A comparison between the π^-p scattering data and the predictions of the various phase-shift analyses is presented. In particular, it is pointed out that the dispersion relation fit of the CERN group fails to represent the data.

In the past few years, several experiments making precise determinations of the $\pi^\pm p$ elastic differential cross section¹⁻⁵ and proton polarization,⁶⁻¹¹ over wide energy regions, have made possible extensive phase-shift analyses.¹²⁻¹⁸ The results of these analyses have led to substantial progress in the understanding of pion-nucleon scattering. One of the features of these studies

was the discovery, in different partial waves, of several resonances which could not be readily identified as peaks in total cross sections.¹³⁻¹⁵ A notable example of this is the so-called "third pion-nucleon resonance" at 1688 MeV, which was shown to include resonant states in S_{31} , D_{15} , F_{15} , and possibly other partial waves. More recently, by using partial-wave dispersion relations to pro-

vide a smooth energy dependence to the data, the CERN group succeeded in achieving smoothness in the resulting partial-wave phase shifts, which led to the "announcement" of the existence of nine new resonances of low elasticity.¹⁹

In this Letter we present a comparison between the elastic π^-p scattering data and the predictions of the phase-shift analyses of various groups. We wish to point out a discrepancy between the experimental data and the CERN dispersion-relation solution. We show below that this particular smooth solution is not a faithful representation of the experimental measurements of π^-p scattering. The effect on the determination of the pion-nucleon-resonance parameters is also briefly discussed.

The π^-p elastic scattering data previously available¹⁻³ are shown in Fig. 1 together with our own new data.²⁰ The strong structures around 1520 and 1688 MeV resulting from the formation of nucleon resonances are clearly visible. The two curves of the predicted cross sections of the phase-shift-analysis solutions are calculated with the absorption parameters and phase shifts (η 's and δ 's) given by the various groups^{21,22} using the expression

$$\sigma_{el} = 4\pi\lambda^2 \sum_l (J + \frac{1}{2}) |T_l^\pm|^2,$$

where

$$T_l^\pm = \frac{\eta_l^\pm \exp^{2i\delta_l^\pm} - 1}{2i}, \text{ and } J = l \pm \frac{1}{2}.$$

The solid curve in Fig. 1 is the π^-p elastic cross section predicted by the CERN phase-shift solution using experimental data as input together with constraints from partial-wave dispersion relations (referred to as CERN-EXPT hereafter), whereas the dashed curve is that predicted by the CERN dispersion-relation fit (referred to as CERN-TH hereafter, also referred to as CERN I²³ in literature).²⁴ It is clearly evident that though the CERN-EXPT solution describes the data well, there are marked discrepancies between the CERN-TH predictions and the experimental data. The CERN-TH solution is unable to reproduce the sharp peaks around 1520 and 1688 MeV.

To investigate further the nature of these discrepancies, we compared the predicted π^-p differential cross sections with those measured in our experiment. Here the differential cross sec-

FIG. 1. π^-p elastic scattering cross-section measurements previously available and those from this experiment. Solid and dashed curves correspond to the predicted π^-p elastic cross section of CERN-EXPT and CERN-TH phase shifts, respectively. The arrows indicate the six energies chosen for differential cross-section comparison as shown in Fig. 2.

tions are calculated from the usual expression²⁵

$$d\sigma/d\Omega = |f(\theta)|^2 + |g(\theta)|^2,$$

where

$$f(\theta) = \frac{1}{k} \sum_l [(l+1)T_l^+ + lT_l^-] P_l(\cos\theta),$$

$$g(\theta) = \frac{i}{k} \sin\theta \sum_l (T_l^+ - T_l^-) \frac{dP_l(\cos\theta)}{d\cos\theta}.$$

We show in Figs. 2(a)-2(f) our measured differential cross sections at six of our energies.²⁰

These six energies are indicated by the arrows in Fig. 1. The data agree very well with previous measurements.¹⁻³ The solid and dashed curves again correspond to the predictions of CERN-EXPT and CERN-TH, respectively. The discrepancies in both the forward and backward regions between the CERN-TH predictions and the data are quite obvious.

We have also compared the same data with the phase-shift solutions of the Saclay,¹³ Berkeley,²¹ and Glasgow groups.²² While the solutions from the first two groups do not possess the smoothness of the CERN-TH solution, they do have the virtue that their predicted cross sections do show reasonable agreement with the data. The Glasgow solution also employs an energy-dependent fit and shows qualitative agreement with π^-p elastic scattering data.²⁴

The total π^-p scattering cross sections predict-

ed by these phase-shift solutions may be obtained most conveniently from the forward scattering amplitude by using the optical theorem,

$$\sigma_{\text{tot}} = (4\pi/k) \text{Im} f(0^\circ).$$

We have compared these with the recent precision measurements of π^-p total cross section by

FIG. 2. π^-p differential cross sections measured in this experiment at six sample energies. Those predicted by CERN-EXPT and CERN-TH are shown as solid and dashed curves for comparison.

Carter *et al.*²⁶ Again, the CERN-TH solution fails to agree with the data in exactly the same fashion as illustrated in Fig. 1.

In smoothing the partial-wave phase shifts, by imposing theoretical considerations, one hopes to eliminate the wobbles and structures which were generated purely due to inadequate or inconsistent input data. In principle, one of the most important merits of this smoothing is that one can not only determine the resonance parameters more accurately, but also identify the very inelastic resonances with more certainty. However, as also pointed out by others,^{13,27} it is often extremely difficult to determine the significance of the fine structures observed in the partial-wave phase shifts and to determine which of these structures needs to be smoothed out. Therefore, it is very important that the final smooth solution faithfully describes the experimentally well-measured quantities such as the total and differential cross sections. In the case of CERN-EXPT, the procedure has been successful in that the phase-shift parameters are considerably smoother than those obtained by other energy-independent analyses. On the other hand, the CERN-TH solution, from which the published resonance parameters were obtained, fails to agree with the rapidly varying π^-p measured cross section, as illustrated above.²⁸ This disagreement is reflected in the nucleon resonance parameters determined from this solution. Since the predicted resonance peaks in the cross section appear to be lower and wider than observed in the data, the elasticity is underestimated and the resonance width overestimated. This effect can be seen in Fig. 3, where the absorption parameter η , the phase shift δ , and the elastic partial-wave cross sections are shown for three resonating waves $-D_{13}$, D_{15} , F_{15} —as examples. We see that the apparently good agreement of the two solutions for η and δ is extremely deceptive since the cross sections are very sensitive functions of both parameters. The underestimation of the strength of the coupling to the elastic channel, and the overestimation of the width, are apparent in each case for the CERN-TH solution.²⁹ It is easily seen that these lead to the discrepancy between the predicted elastic cross section and the data as illustrated in Fig. 1.

In summary, we find that the smooth phase-shift solution of CERN, obtained from the partial-wave dispersion relations CERN-TH or CERN I, is not a good representation of the measured π^-p elastic scattering data.³⁰ Thus, while we agree with the need for obtaining smooth partial-wave

FIG. 3. (a)-(c) Partial-wave absorption parameter η for D_{13} , D_{15} , and F_{15} . (d)-(f) Partial-wave phase shift δ for D_{13} , D_{15} , and F_{15} . (g)-(i) Partial-wave π - p elastic cross section for D_{13} , D_{15} , and F_{15} . Those of CERN-EXPT are shown as dots connected by solid lines, and those of CERN-TH are shown as dashed curves. The solid line represents both when they agree.

phase shifts, we emphasize that one must be careful in carrying this out so as not to produce misleading results.

We wish to thank F. Gilman and R. G. Moorhouse for valuable comments and C. Lovelace for useful criticism.

*Work supported in part by the U. S. Atomic Energy Commission.

†Present address: Deutsches Elektronen Synchrotron, DESY, Hamburg, Germany.

¹P. J. Duke, D. P. Jones, M. A. R. Kemp, P. G. Murphy, J. D. Prentice, and J. J. Thresher, *Phys. Rev.* **149**, 1077 (1966).

²J. A. Helland, T. J. Devlin, D. E. Hagge, M. J. Longo, B. J. Moyer, and C. D. Wood, *Phys. Rev.* **134**, B1062 (1964); J. A. Helland, C. D. Wood, T. J. Devlin, D. E. Hagge, M. J. Longo, B. J. Moyer, and V. Perez-Mendez, *Phys. Rev.* **134**, B1079 (1964).

³P. M. Ogden, D. E. Hagge, J. A. Helland, M. Banner, J. F. Detoeuf, and J. Teiger, *Phys. Rev.* **137**, B1115 (1965).

⁴F. Bulos, R. E. Lanou, A. E. Pifer, A. M. Shapiro, M. Widgoff, R. Panvini, A. E. Brenner, C. A. Bordner, M. E. Law, E. E. Ronat, K. Strauch, J. Szyman-ski, P. Bastien, B. B. Brabson, Y. Eisenberg, B. T. Feld, V. K. Fischer, I. A. Pless, L. Rosenson, R. K. Yamamoto, G. Calvelli, L. Guerriero, G. A. Saladin, T. Tomasin, L. Ventura, G. Voci, and F. Waldner,

Phys. Rev. Letters **13**, 558 (1964); L. Guerriero, *Proc. Roy. Soc. (London)*, Ser. A **289**, 471 (1966).

⁵C. B. Chiu, R. D. Eandi, A. C. Helmholtz, R. W. Kenny, B. J. Moyer, J. A. Poirier, W. B. Richards, R. J. Cence, V. J. Peterson, N. K. Sehgal, and V. J. Stenger, *Phys. Rev.* **156**, 1415 (1967); C. B. Chiu, University of California Lawrence Radiation Laboratory Report No. UCRL-16209, 1965 (unpublished).

⁶P. Bareyre, C. Bricman, M. J. Longo, G. Valladas, G. Villet, G. Bizard, J. Duchon, J. M. Fontaine, J. P. Patry, J. Seguinot, and J. Yonnet, *Phys. Rev. Letters* **14**, 198, 878 (1965).

⁷O. Chamberlain, M. J. Hansroul, C. H. Johnson, P. D. Grannis, L. E. Holloway, L. Valentin, P. R. Robrish, and H. M. Steiner, *Phys. Rev. Letters* **17**, 975 (1966); M. Hansroul, University of California Lawrence Radiation Laboratory Report No. UCRL-17263, 1967 (unpublished).

⁸S. Suwa, A. Yokosawa, N. E. Booth, R. J. Esterling, and R. E. Hill, *Phys. Rev. Letters* **15**, 560 (1965); R. J. Esterling, R. E. Hill, N. E. Booth, S. Suwa, and A. Yokosawa, Enrico Fermi Institute for Nuclear Studies Report No. EFINS 66-29, 1966 (unpublished).

⁹H. H. Atkinson, C. R. Cox, P. J. Duke, K. S. Heard, D. P. Jones, M. A. R. Kemp, P. G. Murphy, J. D. Prentice, and J. J. Thresher, *Proc. Roy. Soc. (London)*, Ser. A **289**, 449 (1966), and *Phys. Rev. Letters* **15**, 468 (1965).

¹⁰R. D. Eandi, T. J. Devlin, R. W. Kenny, P. G. McManigal, and B. J. Moyer, *Phys. Rev.* **136**, B536, B1187 (1964).

¹¹C. R. Cox, K. S. Heard, J. C. Sleeman, P. J. Duke, R. E. Hill, W. R. Holley, D. P. Jones, J. C. Shoemaker, J. J. Thresher, and J. B. Warren, Rutherford High-Energy Laboratory Report No. RHEL/M137 (unpublished).

¹²For a review on πp phase-shift analyses, see A. Donnachie, in *Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, September, 1968*, edited by J. Prentki and J. Steinberger (CERN Scientific Information Service, Geneva, Switzerland, 1968), p. 137.

¹³P. Bareyre, C. Bricman, A. V. Stirling, and G. Villet, *Phys. Letters* **18**, 342 (1965); P. Bareyre, C. Bricman, and G. Villet, *Phys. Rev.* **165**, 1730 (1968).

¹⁴B. H. Bransden, P. J. O'Donnell, and R. G. Moorhouse, *Phys. Letters* **11**, 339 (1964), and *Phys. Rev.* **139**, B1566 (1965), and *Proc. Roy. Soc. (London)*, Ser. A **289**, 538 (1966), and *Phys. Letters* **19**, 420 (1965).

¹⁵P. Auvil, A. Donnachie, A. T. Lea, and C. Lovelace, *Phys. Letters* **12**, 76 (1964), and **19**, 148 (1965).

¹⁶L. D. Roper, *Phys. Rev. Letters* **12**, 340 (1964); L. D. Roper, R. M. Wright, and B. T. Feld, *Phys. Rev.* **138**, B190 (1965); L. D. Roper and R. M. Wright, *Phys. Rev.* **138**, B921 (1965).

¹⁷R. F. Cence, *Phys. Letters* **20**, 306 (1966).

¹⁸C. H. Johnson, University of California Lawrence Radiation Laboratory Report No. UCRL-17683, 1967 (unpublished).

¹⁹A. Donnachie, R. G. Kirsopp, and C. Lovelace, *Phys. Letters* **26B**, 161 (1968); C. Lovelace, in *Proceed-*

ings of the International Conference on Elementary Particles, Heidelberg, Germany, 1967, edited by H. Filthuth (North-Holland Publishing Company, Amsterdam, The Netherlands, 1967), p. 79, and in Proceedings of the Conference on πN Scattering, Irvine, California, December, 1967 (unpublished), and CERN Report No. TH-839, 1967 (unpublished).

²⁰A. D. Brody, A. Kernan, D. W. G. S. Leith, B. S. Levi, A. Minten, B. C. Shen, P. Berge, D. Herndon, R. Longacre, L. Price, A. H. Rosenfeld, and P. Söding, Stanford Linear Accelerator Center Report No. SLAC-PUB-531 (unpublished); for a detailed description of the experiment, elastic-scattering data, and the comparison with various phase-shift solutions and models, see A. D. Brody *et al.*, Stanford Linear Accelerator Center Report No. SLAC-PUB-584 (to be published).

²¹The phase-shift solutions of CERN-EXPT, CERN-TH, Saclay, and Berkeley were obtained at the Conference on πN Scattering at Irvine, California, 1967 through the courtesy of C. Johnson and H. Steiner.

²²A. T. Davies and R. G. Moorhouse, in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, September, 1968 (unpublished). The Glasgow solutions were communicated to us by R. G. Moorhouse.

²³CERN-TH is referred to as CERN I since the resonance parameters quoted by Donnachie (Ref. 12 above) and the Particle Data Group [Rev. Mod. Phys. 41, 109 (1969)] come from this solution. For clarification of this point, see footnote on p. 87 of the first paper by Lovelace, Ref. 19.

²⁴The CERN-EXPT solution is obtained by an iterative fitting procedure in which information from par-

tial-wave dispersion relations in addition to the experimental data is used in the overall phase-shift analysis. The theoretical "data points" are weighted with large errors. CERN-TH is the dispersion-relation calculation using the parameters of their final fit. The fact that in the final iteration the two solutions are different indicates the limitations of their method. The predicted π^-p cross sections of Saclay and Berkeley agree well with that of CERN-EXPT. The comparisons of the data with the predictions of Saclay, Berkeley, and Glasgow are summarized in Stanford Linear Accelerator Center Report No. SLAC-PUB-582 (unpublished).

²⁵The Coulomb-scattering contribution amounts to less than 0.5% at the lowest energy considered here.

²⁶A. A. Carter, K. F. Riley, R. J. Tapper, D. V. Bugg, R. S. Gilmore, K. M. Knight, D. C. Salter, G. H. Stafford, E. J. N. Wilson, J. D. Davies, J. D. Dowell, P. M. Hattersley, R. J. Homer, and A. W. O'Dell, Phys. Rev. 168, 1457 (1968).

²⁷C. Johnson and H. Steiner, in Proceedings of the Conference on πN Scattering, Irvine, California, December, 1967 (unpublished).

²⁸No parameters derived from the CERN-EXPT solution have been published for either the well-established resonances or the nine new resonances.

²⁹For example, in the case of F_{15} , we estimate for CERN-TH $\Gamma = 190$ MeV, $x = 0.56$, and for CERN-EXPT, $\Gamma = 110$ MeV, $x = 0.65$ based on the elastic partial-wave cross sections.

³⁰Since completing this work it has been brought to our attention that the National Bureau of Standards group (J. Coyne, S. Meshkov, W. Parke, R. Ponzini, and H. Williams) had independently observed this discrepancy.

VANISHING OF TOTAL CROSS SECTIONS AT INFINITE ENERGY

Hung Cheng*

Department of Mathematics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

and

Tai Tsun Wu†

Gordon McKay Laboratory, Harvard University, Cambridge, Massachusetts 02138

(Received 1 May 1969)

We have examined the scattering amplitude of two scalar particles in scalar electrodynamics for arbitrary order of perturbation, and found uncanceled terms proportional to $s(\ln s)^n$, for n any positive integer. This suggests a branch point or an essential singularity at $J=1$, and that total cross sections vanish at infinite energy as some negative power of $\ln s$. This form fits very well the sum of the measured π^+p and π^-p cross sections between 8 and 22 BeV/c.

Previous work^{1,2} in quantum electrodynamics has dealt with all two-body elastic scattering amplitudes at high energies in the framework of perturbation theory. In particular, electron-electron scattering up to the fourth order, electron-

photon scattering up to the sixth order, and photon-photon scattering up to the eighth order have been treated. It was found that individual Feynman diagrams give terms proportional to $s(\ln s)^n$, with n a positive integer, in the limit $s \rightarrow \infty$ with