Planck Collaboration: The Planck mission

Fig. 19. The temperature angular power spectrum of the primary CMB from Planck x showing a precise measurement of seven acoustic peaks that are well fit by a simple six parameter CDM theoretical model the model plotted is the one labelled [Planck + WP + high L] in Planck Collaboration XVI The shaded area around the best fit curve represents cosmic variance including the sky cut used The error bars on individual points also include cosmic variance The horizontal axis is logarithmic up to $\mu K^2$ and linear beyond. The vertical scale is $C_l/2$.

Fig. 20. The temperature angular power spectrum of the CMB estimated from the SMIC Planck map The model plotted is the one labelled [Planck + WP + high L] in Planck Collaboration XVI The shaded area around the best fit curve represents cosmic variance including the sky cut used The error bars on individual points do not include cosmic variance The horizontal axis is logarithmic up to $\mu K^2$ and linear beyond. The vertical scale is $C_l/2$.

8.1.1. Main catalogue

The Planck Catalogue of Compact Sources (PCCS) Planck Collaboration XXVIII is a list of compact sources detected by Planck over the entire sky and which therefore contains both Galactic and extragalactic objects. No polarization information is provided for the sources at this time. The PCCS differs from the ERCSC in its extraction philosophy: more effort has been made on the completeness of the catalogue without reducing notably the reliability of the detected sources, whereas the ERCSC was built in the spirit of releasing a reliable catalogue suitable for quick follow-up in particular with the short-lived Herschel telescope. The greater amount of data, different selection process and the improvements in the calibration and map-making processing references help the PCCS to improve the performance in depth and numbers with respect to the previous ERCSC.

The sources were extracted from the Planck frequency maps (Sect. 7) which include data acquired over more than two sky coverages. This implies that the flux densities of most of the sources are an average of three or more different observations over a period of 16.6 months. The Mexican Hat Wavelet algorithm (LopezyCaniego et al. 2007) has been selected as the baseline method for the production of the PCCS. However, one additional method, MTXF (GonzálezNuevo et al. 2017) was implemented in order to support the validation and characterization of the PCCS.

The source selection for the PCCS is made on the basis of Signal-to-Noise Ratio (SNR). However, the properties of the background in the Planck maps vary substantially depending on frequency and part of the sky. Up to 219 GHz, the CMB is the

Non-equilibrium random matrix theory and inflation

Alexander Westphal
in collaboration with: Francisco Pedro
arXiv:1606.07768, 1609.xxxxx
horizon problem of the hot big bang

today: 13.7 Gyr

$\tau = 0$

big bang

$\sim 400 \text{ kyr}$

$10^5$ causally disconnected patches, but $\Delta T/T \sim 10^{-5}$!

CMB
today: 13.7 Gyr

\[ \tau = 0 \]

\[ \tau < 0 \]

CMB

\[ a = - \frac{1}{H_\tau} \sim e^{Ht} \]
idea ...

today: 13.7 Gyr

$H^{-1} \sim \tau^q \sim t$

$H^{-1} \sim \text{const.}$

$\tau = 0$

$\tau < 0$

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$\text{big bang}$

$a = -\frac{1}{H\tau} \sim e^{Ht}$
slow-roll inflation ...

(Guth, Linde, Albrecht, Steinhardt ‘80s)

\[ V(\phi) = \frac{m^2}{2} \phi^2 \]

\[ 3H \dot{\phi} \simeq -V', \quad H^2 \simeq \frac{1}{3}V \]

\[
\epsilon = \frac{V'^2}{2V^2} \ll 1, \quad \eta = \frac{V''}{V} \ll 1
\]
slow-roll inflation...

\[ V(\phi) = \frac{m^2}{2}\phi^2 \]

\[ m \sim 10^{13} \text{ GeV} \]

---

\[ \Delta T / T \sim 10^{-5} \]

\[ \epsilon = \frac{V''}{2V^2} \ll 1 \quad , \quad \eta = \frac{V''}{V} \ll 1 \]

---

[picture from lecture notes: Linde '07]
slow-roll inflation ...

\[ V(\phi) = \frac{m^2}{2} \phi^2 \]

Angular scale

\[ D_\ell[\mu K^2] \]

Multipole moment, \( \ell \)
slow-roll inflation ...

\[ V(\phi) = \frac{m^2}{2} \phi^2 \]

Angular scale

\[ n_s = 1 - 6\epsilon + 2\eta \]
\[ \simeq 0.9655 \pm 0.0062 \]
slow-roll inflation ...

$$V(\phi) = \frac{m^2}{2} \phi^2$$

Angular scale

$$n_s = 1 - 6\epsilon + 2\eta \approx 0.9655 \pm 0.0062$$

$$r = \frac{\Delta_T^2}{\Delta_S^2} = 16\epsilon < 0.09 \text{ (95\%)}$$
slow-roll inflation ...

\[ V(\phi) = \frac{m^2}{2} \phi^2 \]

Angular scale

\[ n_s = 1 - 6\epsilon + 2\eta \]
\[ \simeq 0.9655 \pm 0.0062 \]
\[ r = \frac{\Delta_T^2}{\Delta_S^2} = 16\epsilon < 0.07 \ (95\%) \]
• string theory: many vacua - recipes w/ many ingredients …

non-perturbative effects

• add need for control (weak coupling / large volume):
  ➡️ we are often forced under lamp posts
• string theory: many vacua - recipes w/ many ingredients …

non-perturbative effects

• add need for control (weak coupling / large volume):
  ➣ we are often forced under lamp posts

branes

fluxes

moduli & axions: massless scalars!!

extra dimensions: geometry & topology
varieties of string inflation ...

- tensor-to-scalar ratio linked to field range:
  \[
  \frac{\Delta \phi(N_e)}{M_P} \gtrsim \frac{N_e}{50} \sqrt{\frac{r}{0.01}}, \quad r = \frac{P_T}{P_S} \quad [\text{Lyth '97}]
  \]

- \( r \ll O(1/N_e^2) \) models:
  \[
  \Delta \phi \ll O(M_P) \quad \Rightarrow
  \]

- \( r = O(1/N_e^2) \) models:
  \[
  \Delta \phi \sim O(M_P) \quad \Rightarrow
  \]

- \( r = O(1/N_e) \) models:
  \[
  \Delta \phi \sim \sqrt{N_e M_P} \gg M_P \quad \Rightarrow
  \]
varieties of string inflation ...

• $r \ll O(1/N_e^2)$ models:

\[ \Delta \phi \ll O(M_P) \implies \]

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warped D-brane inflation & DBI; varieties of Kähler moduli inflation

fibre inflation in LARGE volume scenarios (LVS)

low-/ suppression generic:
[Cicoli, Downes, Dutta, Pedro & AW]
[Kallosh, Linde & AW]
varieties of string inflation ...

- $r \ll O(1/N_e^2)$ models:
  \[ \Delta \phi \ll O(M_P) \Rightarrow \]
  warped D-brane inflation & DBI; varieties of Kähler moduli inflation

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  fibre inflation in LARGE volume scenarios (LVS)

- $r = O(1/N_e)$ models:
  \[ \Delta \phi \sim \sqrt{N_e M_P} \gg M_P \Rightarrow \]
  axion monodromy inflation
  2-axion inflation
  N-flation

\[ n_s \approx 0.97 \]
\[ r \approx 0.006 \]
**small-field string inflation ...**

- **Brane-Antibrane** Dvali & Tye; Alexander; Dvali, Shafi & Solganik; Burgess, Majumdar, Nolte, Quevedo, Rajesh & Zhang.

- **D3-D7** Dasgupta, Herdeiro, Hirano & Kallosh; Hsu, Kallosh & Prokushkin; Hsu & Kallosh; Aspinwall & Kallosh; Haack, Kallosh, Krause, Linde, Lüst & Zagermann; ...

- **warped brane-antibrane** Kachru, Kallosh, Linde, Maldacena, McAllister & Trivedi; Firouzjahi & Tye; Burgess, Cline, Stoica & Quevedo; Iizuka & Trivedi; Krause & Pajer; Baumann, Dymarsky, Klebanov, McAllister & Steinhardt; Baumann, Dymarsky, Kachru, Klebanov & McAllister; ...

- **DBI** Silverstein & Tong; Alishahiha, Silverstein & Tong; Chen; Chen; Shiu & Underwood; Leblond & Shandera; ...

- **Racetrack** Blanco-Pillado, Burgess, Cline, Escoda, Gomez-Reino, Kallosh, Linde & Quevedo; Linde & AW; ...

- **Kähler moduli** Conlon & Quevedo; AW; Bond, Kofman, Prokushkin & Vaudrevange; Ben-Dayan, Jing, AW & Zarate ...

**large-field string inflation ...**

- **Fibre inflation** (r < 0.01) Cicoli, Burgess & Quevedo / + de Alwis; Broy, Pedro & AW; Broy, Ciupke, Pedro & AW; Cicoli, Ciupke, de Alwis & Muia

- **Single-Axion inflation** with f > M_p Grimm; Blumenhagen & Plauschinn;

- **2-Axion inflation** Kim, Nilles & Peloso; Berg, Pajer & Sjors; Kappl, Krippendorf & Nilles; Long, McAllister & McGuirk; Tye & Wong; Ben-Dayan, Pedro & AW; Gao, Li & Shukla ...

- **N-flation** Dimopoulos, Kachru, McGreevy, Wacker; Easther & McAllister; Grimm; Cicoli, Dutta & Maharana; Choi, Kim & Yun; Bachlechner, Dias, Frazer & McAllister

- **axion monodromy** Silverstein & AW; McAllister, Silverstein & AW; Flauger, McAllister, Pajer, AW & Xu; Dong, Horn, Silverstein & AW; Shlaer; Gur-Ari; Palti & Weigand; Marchesano, Shiu & Uranga; Blumenhagen & Plauschinn; Hebecker, Kraus & Witkowski; Ibanez & Valenzuela; Kaloper, Lawrence & Sorbo; McAllister, Silverstein, AW & Wrase; Franco, Galloni, Retolaza & Uranga;
• **inflation in string theory** …

many \( (N) \) moduli and axions

\[ \text{e.g. axion monodromy} \ldots \]

• **field spaces:**

with structure: axions, approximate no-scale directions

\[ \text{fibre inflation} \ldots \]

[Burgess et al. ’08/’16; Cicoli et al. ’16]
[Broy, Ciupke, Pedro & AW ’15]
[Broy, Pedro & AW ’14]

[McAllister, Silverstein, AW ’08]
many since …
• inflation in string theory ...

many \((N)\) moduli and axions

e.g. axion monodromy ...

• field spaces:

with structure: axions, approximate no-scale directions

no structure: e.g. flux-stabilized moduli

• statistical description of the landscape:

\(\Rightarrow\) Random Matrix Theory of critical points

\[\text{[McAllister, Silverstein, AW '08]}
\[\text{many since ...}
\[\text{[Burgess et al. '08'/16; Cicoli et al.'16]}
\[\text{[Broy, Ciupke, Pedro & AW '15]}
\[\text{[Broy, Pedro & AW '14]}

\]
• describe system by large-N Gaussian random matrices:

much of the structure described by eigenvalue dynamics

likely eigenvalue configurations

Dyson Brownian Motion

equilibrium states of 1D gas of charged particles

*Dyson '62 & '63*

example: $N$ scalar fields coupled to Einstein gravity

\[ 
\mathcal{L} = \frac{1}{2} \partial^a \phi \partial^a \phi - \Lambda^4_v \sqrt{N} \left[ v_0 + v_a \phi^a + \frac{1}{2} v_{ab} \phi^a \phi^b \right] 
\]

Let $v_{ab}$ be a **Wigner matrix**

\[ W = \frac{A + A^\dagger}{2} \]

\[ A \sim \Omega(\mu, \sigma) \]
• relevant matrices for inflation - Wigner ensemble:
  ➔ approximates the tachyonic directions in both random SUGRA and non-SUGRA cases

$\rho(\lambda) = \frac{1}{\pi N \sigma^2} \sqrt{2N\sigma^2 - \lambda^2}$, $\sigma = \sqrt{\frac{2}{N}}$
• two relevant questions:

- how often do I get a certain eigenvalue distribution in a given equilibrium matrix ensemble (e.g. inflation) ?

  answered by: [Dean & Majumdar ’06/’08]

- given a non-equilibrium configuration — how fast do I relax to another configuration (e.g. exit from inflation) ?

  main result here!

many applications in systems with large-N correlation matrices:

  condensed matter, nuclear physics, string landscape, computational biology, quantitative finance, …
• eigenvalue probability distribution (pdf)  

\[ dP = C \exp \left\{ -\frac{\beta}{2\sigma^2} \text{Tr} M^2 \right\} dM_{ij} = Ce^{-\beta \mathcal{H}} \prod_{i=1}^{N} d\lambda_i \]

\[ \mathcal{H} = \frac{1}{2\sigma^2} \sum_{i=1}^{N} \lambda_i^2 - \sum_{i<j} \ln |\lambda_i - \lambda_j| \]

• compute probabilities by integrating pdf:

\[ P(\forall \lambda > \zeta) = \int_{\zeta}^{\infty} dP \]
• probabilities can be computed:
  ▶ numerically
  ▶ analytically via saddle point approximation

\[ P(\forall \lambda > \zeta) = \exp \left( -\beta N^2 \Phi(\zeta) \right) \]
• Move away from the static ensemble

\[ M_{ij}(s + \delta s) = M_{ij}(s) + \delta M_{ij} \]

\[ \langle \delta M_{ij} \rangle = -M_{ij} \frac{\delta s}{\sigma^2 f} \]

\[ \langle (\delta M_{ij})^2 \rangle = (1 + \delta_{ij}) \frac{\delta s}{\beta f} \]

s coordinate along the background trajectory

• “Time” dependence: Dyson Brownian Motion (DBM)

Dyson Brownian Motion

Eigenvalue Brownian motion

Matrix Brownian motion

[Dyson '63]
[Marsh et al. '13]
[Dias et al. '16]
[Freivogel et al. '16]

“Time” dependence introduced by postulating Brownian motion

Applied to the string landscape by

Locally reconstruct \( V(\{i\}) \)

and study inflationary dynamics

```
\[ s \quad \text{coordinate along the background trajectory} \]

\[ M_{ij}(s + \delta s) = M_{ij}(s) + \delta M_{ij} \]

\[ \langle \delta M_{ij} \rangle = -M_{ij} \frac{\delta s}{\sigma^2 f} \]

\[ \langle (\delta M_{ij})^2 \rangle = (1 + \delta_{ij}) \frac{\delta s}{\beta f} \]

```

eigenvalues relax in 1 corr. length toward Wigner distrib.

fluctuating toward a minimum extremely suppressed
• DBM described by Fokker/Planck eq. - solution: [Dyson ’62 & ’63]

\[
P(M(s), M_0) = \int dP
\]

\[
dP = C \exp \left\{ -\frac{\beta}{2\sigma^2(1-q^2)} \text{Tr}[(M - qM_0)^2] \right\} dM_{ij}
\]

Hamiltonian:

\[
\mathcal{H} = \frac{1}{2\sigma^2(1-q^2)} \sum_{i=1}^{N} \left( \lambda_i^2 - 2q\lambda_i M_0^{ii} \right) - \sum_{i<j} \ln |\lambda_i - \lambda_j|
\]

linear potential for eigenvalues: memory of init. conds.!
• Estimate exit probability:

  ● Numerical evolution of matrices
  ● Analytical saddle point integration of $dP$

    $\frac{1}{N} \sum_{i=1}^{N} M_{0}^{ii} = \frac{1}{N} \text{Tr}[M_{0}] \equiv \langle \lambda_{M_{0}} \rangle$.  

    mean eigenvalue of $M_{0}$

[Dean & Majumdar ’06/’08]

for time dependent case - our result [Pedro & AW ’16]

• Each eigenvalue subject to different linear potential

To get a qualitative description approximate:

$M_{0}^{ii} \rightarrow m \equiv \frac{1}{N} \sum_{i=1}^{N} M_{0}^{ii}$

$\frac{1}{N} \text{Tr}[M_{0}] \equiv \langle \lambda_{M_{0}} \rangle$.  

mean eigenvalue of $M_{0}$
• time-dependent rate function:

$$
\Psi(\tilde{\zeta}) = \frac{1}{108\tilde{a}^2} \left\{ 36\tilde{a}\tilde{\zeta}^2 - \tilde{\zeta}^4 + (15\tilde{a}\tilde{\zeta} + \tilde{\zeta}^3)\sqrt{6\tilde{a} + \tilde{\zeta}^2} + 27\tilde{a}^2 \left[ \ln(72\tilde{a}) - 2 \ln(2(\sqrt{6\tilde{a} - \tilde{\zeta} - \bar{\zeta})) \right] \right\}
$$

\[\tilde{a} \equiv 2(1 - q^2)\]
\[b \equiv -2qm\]
\[\bar{\zeta} \equiv \zeta + b/2\]
What is the probability of: \( M(s) : \forall \lambda > \zeta \)

\[
P(M(s), M_0) = \exp \left[ -\beta N^2 \Psi(s, \zeta) + \mathcal{O}(N) \right]
\]
Apply static D&M result to: small-field landscape how many minima vs inflationary patches?

$$\frac{P(\text{inf})}{P(\text{min})} \sim e^{-\beta N^2 \{\Phi(-\eta) - \Phi(\eta)\}} = e^{-\beta N^2 \Delta} \quad \Delta = \frac{4}{3\sqrt{3}} \eta + O(\eta^2)$$
• Inflationary patches MUCH more abundant than minima:

How are they connected in the landscape?

\[ \{ \phi_i \} \quad \text{inf.} \]

\[ \{ \phi_i + \delta \phi_i \} \quad \text{min.} \]

Want to find the exit probability
• Apply analytical relaxation method to: small-field landscape

Good agreement after one correlation length: $q < e^{-1} \approx 0.35$

$M_{\inf} \gg \Lambda$ : interested in small $q$ regime
• Transition probability is exponentially suppressed

\[
\frac{N_{\text{inf}}}{N_{\text{min}}} \quad \text{vs} \quad \frac{N_{\text{inf} \wedge \text{min}}}{N_{\text{min}}}
\]

? Who wins then?  

Conditional probability:

\[
P(\text{min}|\text{inf}) = \frac{P(\text{min} \cap \text{inf})}{P(\text{inf})}
\]

Anthropically relevant trajectories:

\[
P(\text{min} \cap \text{inf}) = P(\text{inf}) \cdot P(\text{min}|\text{inf})
\]

- initial condition
- evolution
• **Globally:** \( q \ll 1 \land |\eta| \ll 1 \)

\[
P(\min \cap \inf) \simeq \exp \left[ -\beta N^2 \left\{ \frac{\ln 3}{4} - \frac{2}{3\sqrt{3}}(\eta + mq) \right\} \right]
\]

\( m \simeq 0.7 \)

Transition probability overcomes number count

In small-field landscapes … Our History is highly unlikely !!!!
The landscape ‘Drake equations’ of tensor modes

\[
\nu_{\text{small}} \sim N_{\text{manifolds}} \times N_{\text{cr.p.}} \times f_{\text{dS-min.}} \times P_{\text{small}}(\text{min} \cap \text{inf})
\]

\[
\nu_{\text{large}} \sim N_{\text{manifolds}} \times N_{\text{cr.p.}} \times f_{\text{dS-min.}} \times P_{\text{large}}(\text{min} \cap \text{inf})
\]

\[\ll 1\]
• The landscape ‘Drake equations’ of tensor modes

\[ \nu_{\text{small}} \sim N_{\text{manifolds}} \times N_{\text{cr.p.}} \times f_{dS-\text{min.}} \times P_{\text{small}}(\text{min } \cap \text{ inf}) \approx 1 \]

\[ \nu_{\text{large}} \sim N_{\text{manifolds}} \times N_{\text{cr.p.}} \times f_{dS-\text{min.}} \times P_{\text{large}}(\text{min } \cap \text{ inf}) \]

large-field models: minimum built-in!

\[ P_{\text{large}}(\text{min} | \text{inf}) = 1 \]

\[ \Rightarrow P_{\text{large}}(\text{min } \cap \text{ inf}) \sim 1 \]

maybe string landscape wants large/ish \( r \) by preferring structure?
compute prior probability for e-folds $N_e$

$$V = V_0 \left( 1 - \frac{\eta_0}{2} \phi^2 - \frac{1}{p \Delta \phi^p} \phi^p + \ldots \right)$$

$$P(N_e) = \int d\sqrt{H} \, d\Delta \phi \, d\eta \, \delta \left( N_e - \frac{1}{\eta} \right) \delta \left( \frac{\delta \rho}{\rho} - f(\sqrt{H}, \Delta \phi, \eta) \right)$$

$$\sim \frac{1}{N_e^{4+2/(p-2)}} e^{\mathcal{O}(1) \frac{N^2}{N_e}}$$

so $N < 10$ to get $\Omega_K < 0.001$!!
where do we go from here ...

small-field models

less-accidental small-field saddle points (e.g. Kahler moduli) or plateaus with structure

accidental small-field saddle points (e.g. complex structure moduli)

large-field models

axion monodromy

unwinding inflation ... others/unknown ??
where do we go from here ...

- small-field models
- large-field models

less-*accidental* small-field saddle points (e.g. Kahler moduli) or plateaus with structure

*accidental* small-field saddle points (e.g. complex structure moduli)

*axion monodromy*

unwinding inflation ... others/unknown ??

covered here ...

exit also often built-in ... $P = ??$

maybe $P_{\text{large}} / P_{\text{small}} \gg 1$ ?

$P = ??$