Monte Carlo solution of DGLAP evolution equation and extraction of TMD densities

• Ola Lelek [2] Francesco Hautmann [1] Hannes Jung [2]

[1] University of Oxford

[2] Deutsches Elektronen-Synchrotron (DESY)

01.03.2016

DPG-Frühjahrstagung, Hamburg

Table of Contents

Introduction to the method

Sudakov formalism

Evolution equation for parton density

$$t\frac{\partial f(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\int \frac{dz}{z}P(z)_+ f\left(\frac{x}{z},t\right) =$$

$$= \frac{\alpha_s}{2\pi}\int \frac{dz}{z}P(z) f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi} f(x,t)\int dz P(z), \qquad (1)$$

where $\int_0^1 f(x)g(x)_+ dx = \int_0^1 f(x)g(x)dx - \int_0^1 f(1)g(x)dx$.

Introducing *Sudakov form factor*

$$\Delta_s(t,t_0) \equiv \Delta_s(t) = exp\Big(-\int_x^{z_{max}} dz \int_{t_0}^{t} \frac{\alpha_s}{2\pi}\frac{dt'}{t'}P(z)\Big) \qquad (2)$$

we can rewrite (1)

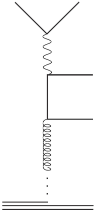
$$t\frac{\partial f(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\int \frac{dz}{z}P(z) f(\frac{x}{z},t) + f(x,t)\frac{t}{\Delta_s(t)}\frac{\partial \Delta_s(t)}{\partial t}. \qquad (3)$$

Sudakov formalism

Evolution equation for parton density

$$t\frac{\partial f(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\int \frac{dz}{z}P(z)_+ f\left(\frac{x}{z},t\right) = $$
$$= \frac{\alpha_s}{2\pi}\int \frac{dz}{z}P(z) f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi} f(x,t)\int dz P(z), \quad (1)$$

where $\int_0^1 f(x)g(x)_+dx = \int_0^1 f(x)g(x)dx - \int_0^1 f(1)g(x)dx$.



Introducing *Sudakov form factor*

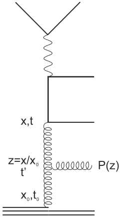
$$\Delta_s(t,t_0) \equiv \Delta_s(t) = exp\Big(-\int_x^{z_{max}} dz \int_{t_0}^{t} \frac{\alpha_s}{2\pi}\frac{dt'}{t'}P(z)\Big) \quad (2)$$

we can rewrite (1)

$$t\frac{\partial f(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\int \frac{dz}{z}P(z) f(\frac{x}{z},t) + f(x,t)\frac{t}{\Delta_s(t)}\frac{\partial \Delta_s(t)}{\partial t}. \quad (3)$$

After integration

$$f(x,t) = f(x,t_0)\Delta_s(t) + \frac{\alpha_s}{2\pi}\int \frac{dt'}{t'}\frac{\Delta_s(t)}{\Delta_s(t')}\int \frac{dz}{z}P(z) f(\frac{x}{z},t'). \quad (4)$$



Sudakov: probability of evolving from t_0 to t without any resolvable branching.
iterative solution:

$$f(x,t) = \lim_{n\to\infty} f_n(x,t) = \lim_{n\to\infty}\sum_n \frac{1}{n!}log^n(\frac{t}{t_0})A^n \otimes \Delta_s(t) f(\frac{x}{z},t_0), \quad (5)$$

where $A = \frac{\alpha_s}{2\pi}\int \frac{dz}{z}P(z)$.

MC solution of the evolution equation

MC solution:

First branching: evolve from t_0 to t' obtained from $\Delta_s(t')$:

$$R_1 = \Delta_s(t'), \tag{6}$$

where R_1 is a random number in the interval $(0, 1)$.
If $t' > t$ evolution is stopped without any branching. If $t' < t$ branching is generated according to $P(z)$

$$\int_{z_{min}}^{z} dz' P(z') = R_2 \int_{z_{min}}^{z_{max}} dz' P(z') \tag{7}$$

and the evolution continues.
Second branching: evolve from t' to t'' generated according to $\Delta_s(t'', t')$. If $t'' > t$ evolution is stopped only with one branching. If $t'' < t$ branching is generated according to $P(z)$ and the evolution continues...etc.

MC solution of the evolution equation

MC solution:

First branching: evolve from t_0 to t' obtained from $\Delta_s(t')$:

$$R_1 = \Delta_s(t'), \tag{6}$$

where R_1 is a random number in the interval $(0, 1)$.
If $t' > t$ evolution is stopped without any branching. If $t' < t$ branching is generated according to $P(z)$

$$\int_{z_{min}}^{z} dz' P(z') = R_2 \int_{z_{min}}^{z_{max}} dz' P(z') \tag{7}$$

and the evolution continues.
Second branching: evolve from t' to t'' generated according to $\Delta_s(t'', t')$. If $t'' > t$ evolution is stopped only with one branching. If $t'' < t$ branching is generated according to $P(z)$ and the evolution continues...etc.

Observation:

$$\frac{\partial}{\partial t'} \frac{\Delta_s(t)}{\Delta_s(t')} = \frac{\alpha_s}{2\pi} \frac{\Delta_s(t)}{\Delta_s(t')} \frac{1}{t'} \int_x^{z_{max}} dz P(z) \tag{8}$$

rewrite (4)

$$f(x,t) = f_0(x,t)\Delta_s(t) + \int_x^1 \frac{dz'}{z'} \int_{t_0}^{t} d\Delta_s(t,t') P(z') f_0(\tfrac{x}{z'}, t') (\int_x^{z_{max}} dz P(z))^{-1} \tag{9}$$

Momentum weighted parton densities & momentum sum rule

To include all flavours in the evolution & use the Sudakov formalism we need to switch from $f(x, t)$ to $xf(x, t)$ & use momentum sum rule:

$$\sum_a \int_0^1 zP_{ab}(\alpha_s, z)dz = 0. \tag{10}$$

Momentum weighted parton densities & momentum sum rule

To include all flavours in the evolution & use the Sudakov formalism we need to switch from $f(x,t)$ to $xf(x,t)$ & use momentum sum rule:

$$\sum_a \int_0^1 zP_{ab}(\alpha_s, z)dz = 0. \tag{10}$$

---

example for gluon:

P - regularized splitting function (with *plus prescription*), $\widehat{P}$ - unregularized splitting function (without *plus prescription*)

$$t\frac{\partial xg(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\left[P_{gq}(z)xq\left(\frac{x}{z},t\right) + P_{gg}(z)xg\left(\frac{x}{z},t\right)\right] \tag{11}$$

Momentum weighted parton densities & momentum sum rule

To include all flavours in the evolution & use the Sudakov formalism we need to switch from $f(x,t)$ to $xf(x,t)$ & use momentum sum rule:

$$\sum_a \int_0^1 zP_{ab}(\alpha_s, z)dz = 0. \tag{10}$$

---

example for gluon:
P - regularized splitting function (with *plus prescription*), $\widehat{P}$ - unregularized splitting function (without *plus prescription*)

$$t\frac{\partial xg(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\left[P_{gq}(z)xq\left(\frac{x}{z},t\right) + P_{gg}(z)xg\left(\frac{x}{z},t\right)\right] - \frac{\alpha_s}{2\pi}xg(x,t)\underbrace{\int_0^1 dzz\left[\sum_i P_{q_i g} + P_{gg}\right]}_{0} \tag{11}$$

Momentum weighted parton densities & momentum sum rule

To include all flavours in the evolution & use the Sudakov formalism we need to switch from $f(x,t)$ to $xf(x,t)$ & use momentum sum rule:

$$\sum_a \int_0^1 z P_{ab}(\alpha_s, z) dz = 0. \tag{10}$$

---

example for gluon:

P - regularized splitting function (with *plus prescription*), $\widehat{P}$ - unregularized splitting function (without *plus prescription*)

$$t\frac{\partial xg(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\left[P_{gq}(z)xq\left(\frac{x}{z},t\right) + P_{gg}(z)xg\left(\frac{x}{z},t\right)\right] - \underbrace{\frac{\alpha_s}{2\pi}xg(x,t)\int_0^1 dzz\left[\sum_i P_{q_i g} + P_{gg}\right]}_{0} \rightarrow$$

$$\rightarrow \frac{\alpha_s}{2\pi}\int_x^{zmax} \frac{dz}{z}\left(\widehat{P}_{gq}(z)xq\left(\frac{x}{z},t\right) + \widehat{P}_{gg}(z)xg\left(\frac{x}{z},t\right)\right) - \frac{\alpha_s}{2\pi}xg(x,t)\int_x^{zmax} dzz\left(2n_f\widehat{P}_{qg} + z\widehat{P}_{gg}\right) \tag{11}$$

Momentum weighted parton densities & momentum sum rule

To include all flavours in the evolution & use the Sudakov formalism we need to switch from $f(x,t)$ to $xf(x,t)$ & use momentum sum rule:

$$\sum_a \int_0^1 zP_{ab}(\alpha_s, z)dz = 0. \quad (10)$$

example for gluon:
P - regularized splitting function (with *plus prescription*), $\widehat{P}$ - unregularized splitting function (without *plus prescription*)

$$t\frac{\partial xg(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\left[P_{gq}(z)xq\left(\frac{x}{z},t\right) + P_{gg}(z)xg\left(\frac{x}{z},t\right)\right] - \frac{\alpha_s}{2\pi}xg(x,t)\underbrace{\int_0^1 dzz\left[\sum_i P_{q_i g} + P_{gg}\right]}_{0} \rightarrow$$

$$\rightarrow \frac{\alpha_s}{2\pi}\int_x^{z_{max}} \frac{dz}{z}\left(\widehat{P}_{gq}(z)xq\left(\frac{x}{z},t\right) + \widehat{P}_{gg}(z)xg\left(\frac{x}{z},t\right)\right) - \frac{\alpha_s}{2\pi}xg(x,t)\int_x^{z_{max}} dzz\left(2n_f\widehat{P}_{qg} + z\widehat{P}_{gg}\right) \quad (11)$$

If we define now Sudakov form factor as:

$$\Delta_s(t,t_0)_i \equiv \Delta_s(t)_i = \exp\left(-\int_x^{z_{max}} dz\int_{t_0}^t \frac{\alpha_s}{2\pi}\frac{dt'}{t'}z\sum_j \widehat{P}(z)_{ji}\right) \quad (12)$$

where i, j are q or g,
we obtain equation of the same form as eq. (3), just multiplied from both sides by x and with $zP(z)$ in $\Delta_s(t)_i$ $\rightarrow$ it can be solved in analogical way.

Evolution in the code

In this presentation: MC results obtained with uPDFevolv code for gluons, valence and sea quarks for:

- DGLAP evolution,
- $xf(x,t)$,
- LO in $P(z)$,
- 1-loop-α_s.

https://updfevolv.hepforge.org/

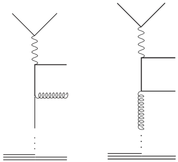
Evolution in the code

In this presentation: MC results obtained with uPDFevolv code for gluons, valence and sea quarks for:

- DGLAP evolution,
- $xf(x,t)$,
- LO in $P(z)$,
- 1-loop-α_s.

https://updfevolv.hepforge.org/

We consider ep collisions in which we can measure different pdfs:



Forward evolution: final parton is not specified when the evolution begins.

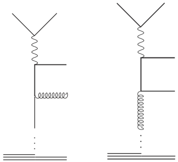
Evolution in the code

In this presentation: MC results obtained with ▸ uPDFevolv code for gluons, valence and sea quarks for:

- DGLAP evolution,
- $xf(x,t)$,
- LO in $P(z)$,
- 1-loop-α_s.

https://updfevolv.hepforge.org/

We consider ep collisions in which we can measure different pdfs:



Forward evolution: final parton is not specified when the evolution begins.

Four different situations:

- gluon at the beginning and at the end,
- quark (valence or sea) at the beginning and gluon at the end
- quark (valence or sea) at the beginning and quark (valence or sea) at the end and
- gluon at the beginning and sea quark at the end.

Valence quark at the end can come only from valence quark at the beginning.

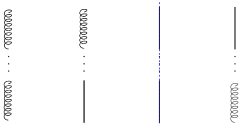


Figure : K_{gg}, K_{gq}, K_{qq},K_{qg}

Integrated PDFs from MC solution

Ordering dependence- z_{max} origin

Some of the splitting functions are divergent for $z \to 1$.

To avoid divergences:

$$\frac{\partial xf(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\frac{1}{t}\int_x^1 dzP(z)\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_x^1 dzP(z) \approx$$
$$\approx \frac{\alpha_s}{2\pi}\frac{1}{t}\int_x^{z_{max}} dzP(z)\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_x^{z_{max}} dzP(z). \qquad (13)$$

Ordering dependence- z_{max} origin

Some of the splitting functions are divergent for $z \to 1$.

To avoid divergences:

$$\frac{\partial xf(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\frac{1}{t}\int_x^1 dzP(z)\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_x^1 dzP(z) \approx$$
$$\approx \frac{\alpha_s}{2\pi}\frac{1}{t}\int_x^{z_{max}} dzP(z)\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_x^{z_{max}} dzP(z). \tag{13}$$

it can be shown that terms $\int_{zmax}^{1}$ skipped in the integral in eq. (13) are of order $\mathcal{O}(1-z_{max})$ multiplied by $xf(x,t)$ or $x\frac{df(x,t)}{dt}$

Ordering dependence- z_{max} origin

Some of the splitting functions are divergent for $z \to 1$.

To avoid divergences:

$$\begin{aligned}\frac{\partial xf(x,t)}{\partial t} &= \frac{\alpha_s}{2\pi}\frac{1}{t}\int_x^1 dzP(z)\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_x^1 dzP(z) \approx \\ &\approx \frac{\alpha_s}{2\pi}\frac{1}{t}\int_x^{z_{max}} dzP(z)\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_x^{z_{max}} dzP(z).\end{aligned} \tag{13}$$

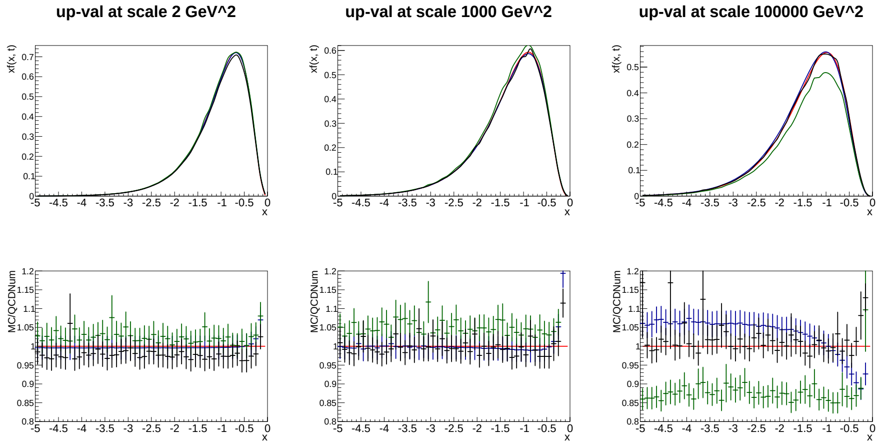
it can be shown that terms $\int_{zmax}^1$ skipped in the integral in eq. (13) are of order $\mathcal{O}(1-z_{max})$ multiplied by $xf(x,t)$ or $x\frac{df(x,t)}{dt}$

Different choices of z_{max}:

- z_{max} - fixed
- z_{max} - can change dynamically with the scale, for example: angular ordering: $z_{max} = \left(\frac{Q_0}{Q}\right)^2$

up-val quarks

$2GeV^2$:QCDNum, angular ordering, $z_{max} = 0.99$ $z_{max} = 0.97$
$1000GeV^2$:QCDNum, angular ordering, $z_{max} = 0.999$ $z_{max} = 0.997$
$100000GeV^2$:QCDNum, angular ordering, $z_{max} = 0.999$ $z_{max} = 0.9999$



MC results close to the QCDNum results. Effect on z_{max} observed. z_{max} value giving the results the closest to QCDNum depends on scale and shape of the distribution (terms of order $\mathcal{O}(1 - z_{max})$ multiplied by $xf(x, t)$ or $x\frac{df(x,t)}{dt}$ skipped).

Results for TMDs

k_T dependence

Why Transverse Momentum Dependent PDFs?
Important in studies on:

- nonperturbative information on hadron structure,
- QCD factorization theorems,
- resummation at all orders in the QCD coupling to many observables in high-energy hadronic collisions,
- multi-scale problems in hadronic collisions,
- ...

Important processes: Drell-Yan hadroproduction of electroweak gauge bosons, Higgs poduction ...

k_T dependence

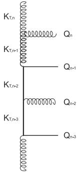
Why Transverse Momentum Dependent PDFs?
Important in studies on:

- nonperturbative information on hadron structure,
- QCD factorization theorems,
- resummation at all orders in the QCD coupling to many observables in high-energy hadronic collisions,
- multi-scale problems in hadronic collisions,
- ...

Important processes: Drell-Yan hadroproduction of electroweak gauge bosons, Higgs poduction ...

MC solution: for every branching Q is generated and Q_x and Q_y are calculated

$\rightarrow$ The information about k_T is available for every branching.



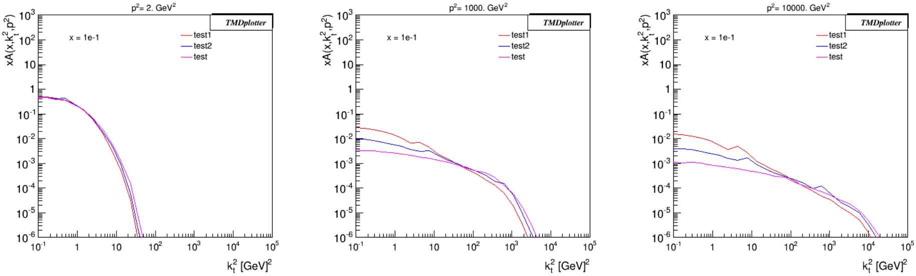
k_T contains the whole history of the evolution:

$$\vec{k}_{T,n} = \vec{k}_{T,n-1} + \vec{Q}_{T,n-1}.$$

up-val

$z_{max} = 0.99$, $z_{max} = 0.999$, $z_{max} = 0.9999$

x = 0.1

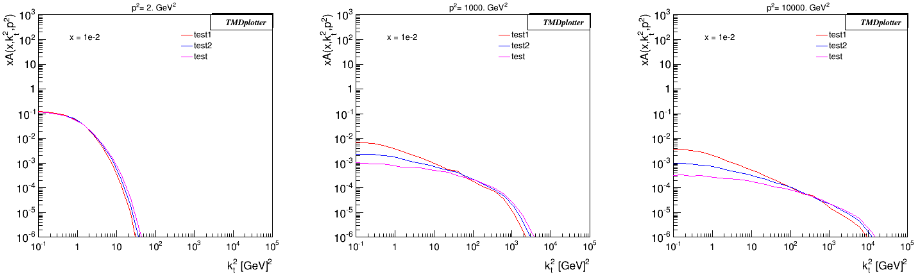


- Initial scale: intrinsic k_t distribution.
- Different choices of z_{max} lead to different uTMDs, especially different large k_T tails.

up-val

$z_{max} = 0.99$, $z_{max} = 0.999$, $z_{max} = 0.9999$

x = 0.01

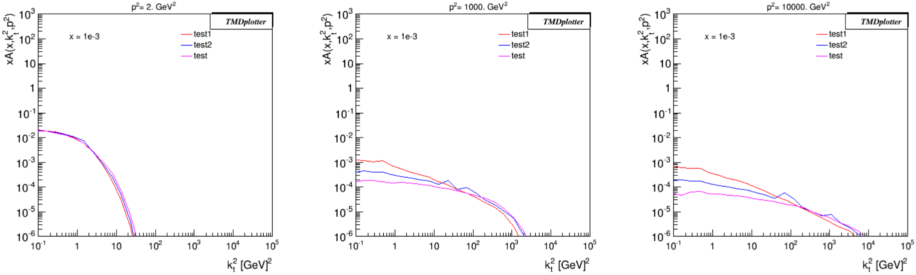


- Initial scale: intrinsic k_t distribution.
- Different choices of z_{max} lead to different uTMDs, especially different large k_T tails.

up-val

$z_{max} = 0.99$, $z_{max} = 0.999$, $z_{max} = 0.9999$

x = 0.001



- Initial scale: intrinsic k_t distribution.
- Different choices of z_{max} lead to different uTMDs, especially different large k_T tails.

Summary

Summary

New approach to solve coupled gluon and quark DGLAP evolution equation with MC method was shown.

Advantages:

- evolution for gluons, valence and sea quarks implemented,
- reproduce analytical solution (results consistent with QCDNum17),
- applicable for TMDs evolution,
- direct usage in PS matched calculation.

Prospects:

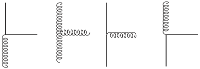
- implementation into xFitter: first full TMD distributions,
- include NLO in $P(z)$,
- development of full TMD MC - CASCADE.

Thank you!

Back up

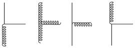
evolution in the code

Different kind of splittings can happen during the evolution process:

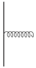


Commonly used flavour decomposition:

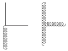
- see
  $u + \overline{u} + d + \overline{d} + s + \overline{s} = 2(\overline{u} + \overline{d} + \overline{s}) + u_v + d_v$

- valence
  $\overline{u} - u$
  $\overline{d} - d$

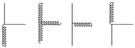
  

- gluon

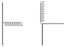
  

Flavour decomposition used in the method:

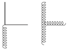
- see
  $2(\overline{u} + \overline{d} + \overline{s})$

- valence
  $\overline{u} - u$
  $\overline{d} - d$

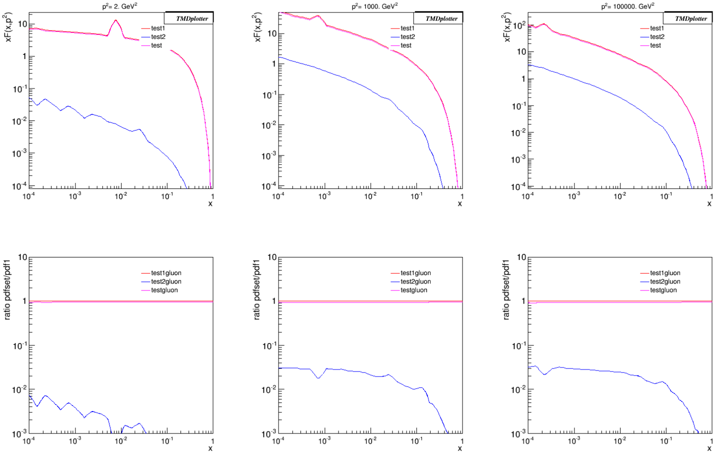
  

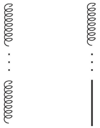
- gluon

Contribution from quark and gluon evolution

GLUON density: test1: merged quark and gluon evolution chain test2: quark evolution chain test: gluon evolution chain

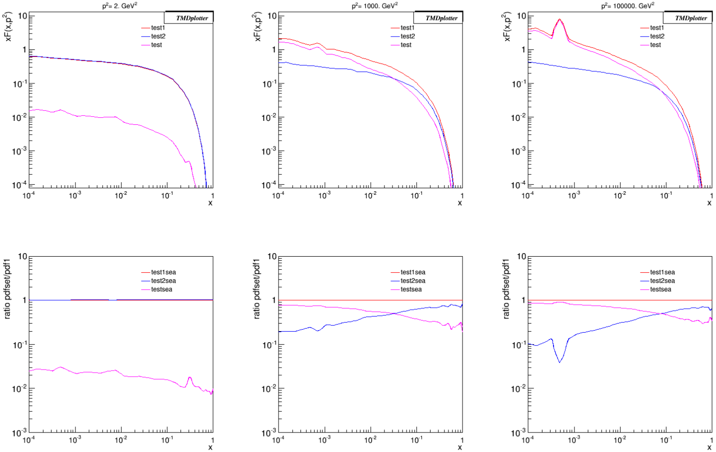


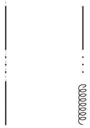


Contribution from gluon kernel dominates, contribution from quark kernel (10^{-2}) times smaller.

Contribution from quark and gluon evolution

SEA density: test1: merged quark and gluon evolution chain test2: quark evolution chain test: gluon evolution chain





At small x main contribution to sea quark density from gluon kernel, for large x quark kernel dominates.

Ordering dependence- z_{max} origin

Some of the splitting functions are divergent for $z \to 1$.

To avoid divergences:

$$\frac{\partial xf(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\frac{1}{t}\int_x^1 dzP(z)\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_x^1 dzP(z) \approx$$
$$\approx \frac{\alpha_s}{2\pi}\frac{1}{t}\int_x^{z_{max}} dzP(z)\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_x^{z_{max}} dzP(z). \qquad (14)$$

Ordering dependence- z_{max} origin

Some of the splitting functions are divergent for $z \to 1$.

To avoid divergences:

$$\begin{aligned}\frac{\partial xf(x,t)}{\partial t} &= \frac{\alpha_s}{2\pi}\frac{1}{t}\int_x^1 dzP(z)\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_x^1 dzP(z) \approx \\ &\approx \frac{\alpha_s}{2\pi}\frac{1}{t}\int_x^{z_{max}} dzP(z)\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_x^{z_{max}} dzP(z).\end{aligned} \tag{14}$$

Using the form of the splitting functions:

$$P_{ab}(\alpha_s, z) = A_{ab}(\alpha_s)\delta(1-z) + K_{ab}(\alpha_s)\frac{1}{(1-z)_+} + R_{ab}(\alpha_s) \tag{15}$$

and the expansion of $xf(\frac{x}{z},t)$

$$xf\left(\frac{x}{z},t\right) = xf(x,t) + x^2 f'(x,t)(1-z) + \mathcal{O}(1-z_{max}) \tag{16}$$

it can be shown that terms $\int_{zmax}^1$ skipped in the integral in eq. (14) are of order $\mathcal{O}(1-z_{max})$ multiplied by $xf(x,t)$ or $x\frac{df(x'\,t)}{dt}$

Ordering dependence- z_{max} origin

Some of the splitting functions are divergent for $z \to 1$.

To avoid divergences:

$$\frac{\partial xf(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\frac{1}{t}\int_x^1 dz P(z)\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_x^1 dz P(z) \approx$$
$$\approx \frac{\alpha_s}{2\pi}\frac{1}{t}\int_x^{z_{max}} dz P(z)\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_x^{z_{max}} dz P(z). \tag{14}$$

Using the form of the splitting functions:

$$P_{ab}(\alpha_s, z) = A_{ab}(\alpha_s)\delta(1-z) + K_{ab}(\alpha_s)\frac{1}{(1-z)_+} + R_{ab}(\alpha_s) \tag{15}$$

and the expansion of $xf(\frac{x}{z},t)$

$$xf\left(\frac{x}{z},t\right) = xf(x,t) + x^2 f'(x,t)(1-z) + \mathcal{O}(1-z_{max}) \tag{16}$$

it can be shown that terms $\int_{z_{max}}^1$ skipped in the integral in eq. (14) are of order $\mathcal{O}(1-z_{max})$ multiplied by $xf(x,t)$ or $x\frac{df(x' t)}{dt}$
example:

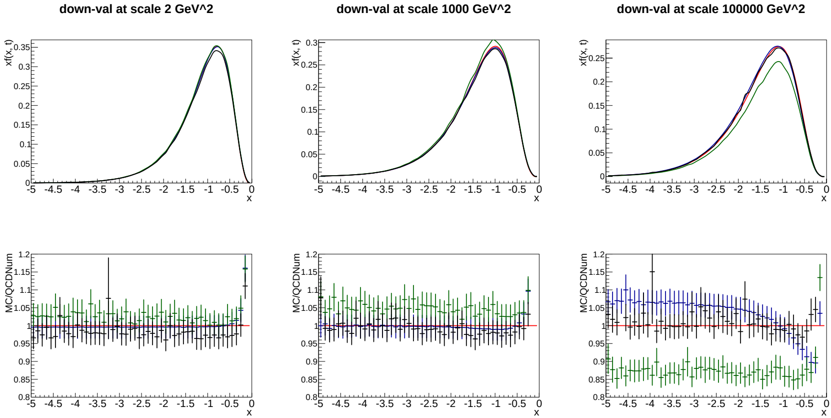
$$\frac{\alpha_s}{2\pi}\frac{1}{t}\int_{z_{max}}^1 dz K_{ab}(\alpha_s)\frac{1}{1-z}\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_{z_{max}}^1 dz K_{ab}(\alpha_s)\frac{1}{1-z} =$$
$$= \frac{\alpha_s}{2\pi}\frac{1}{t}K_{ab}(\alpha_s)\left[x^2 f'(x,t)(1-z_{max}) + \mathcal{O}(1-z_{max})\right] \tag{17}$$

down-val quarks

$2GeV^2$:QCDNum, angular ordering, $z_{max} = 0.99$ $z_{max} = 0.97$

$1000GeV^2$:QCDNum, angular ordering, $z_{max} = 0.999$ $z_{max} = 0.997$

$100000GeV^2$:QCDNum, angular ordering, $z_{max} = 0.999$ $z_{max} = 0.9999$



MC results close to the QCDNum results.

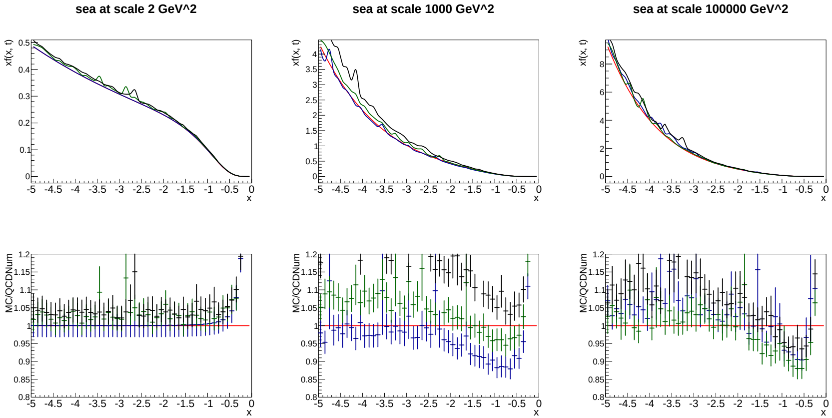
Effect on z_{max} observed. z_{max} value giving the results the closest to QCDNum depends on scale and shape of the distribution.

sea quarks

$2GeV^2$:QCDNum, angular ordering, $z_{max} = 0.99$ $z_{max} = 0.995$

$1000GeV^2$:QCDNum, $z_{max} = 0.97$, $z_{max} = 0.98$ $z_{max} = 0.99$

$100000GeV^2$:QCDNum, $z_{max} = 0.996$, $z_{max} = 0.995$ $z_{max} = 0.997$



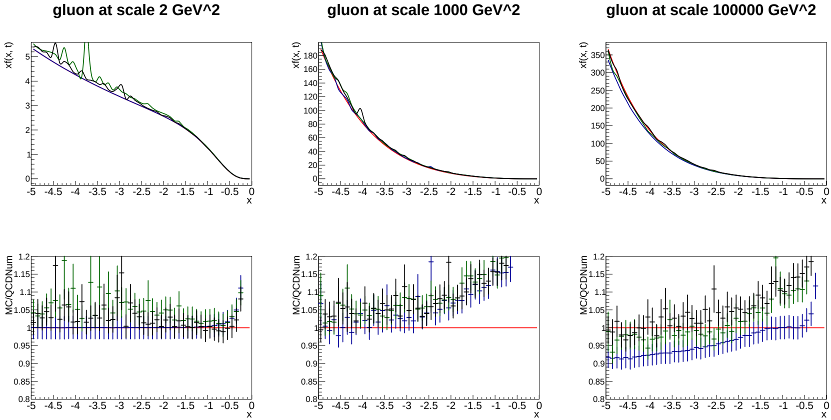
MC results close to the QCDNum results.
Effect on z_{max} observed. z_{max} value giving the results the closest to QCDNum depends on scale and shape of the distribution.

gluon

$2 GeV^2$:QCDNum, angular ordering, $z_{max} = 0.9$ $z_{max} = 0.87$

$1000 GeV^2$:QCDNum, $z_{max} = 0.99$, $z_{max} = 0.995$ $z_{max} = 0.993$

$100000 GeV^2$:QCDNum, angular ordering, $z_{max} = 0.995$ $z_{max} = 0.997$

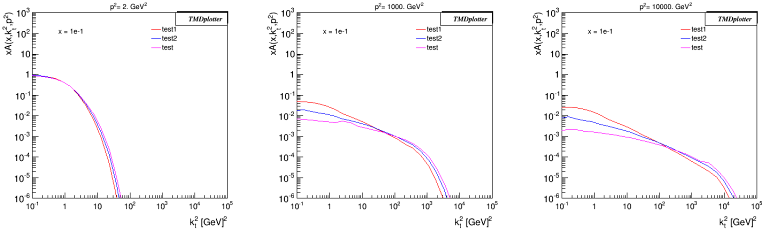


MC results close to the QCDNum results.

Effect on z_{max} observed. z_{max} value giving the results the closest to QCDNum depends on scale and shape of the distribution.

down-val

$z_{max} = 0.99$, $z_{max} = 0.999$, $z_{max} = 0.9999$

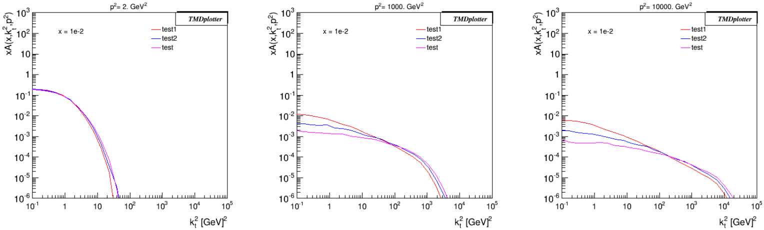


$x = 0.1$

- Initial scale: intrinsic k_t distribution.
- Different choices of z_{max} lead to different uTMDs, especially different large k_T tails.

down-val

$z_{max} = 0.99$, $z_{max} = 0.999$, $z_{max} = 0.9999$

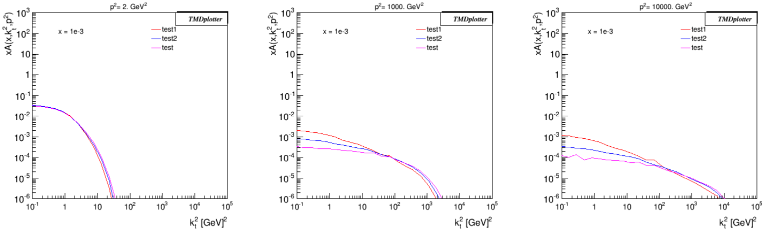


x = 0.01

- Initial scale: intrinsic k_t distribution.
- Different choices of z_{max} lead to different uTMDs, especially different large k_T tails .

down-val

$z_{max} = 0.99$, $z_{max} = 0.999$, $z_{max} = 0.9999$

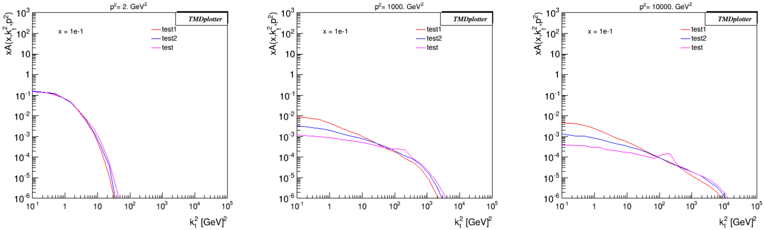


x = 0.001

- Initial scale: intrinsic k_t distribution.
- Different choices of z_{max} lead to different uTMDs, especially different large k_T tails .

sea

$z_{max} = 0.99$, $z_{max} = 0.999$, $z_{max} = 0.9999$

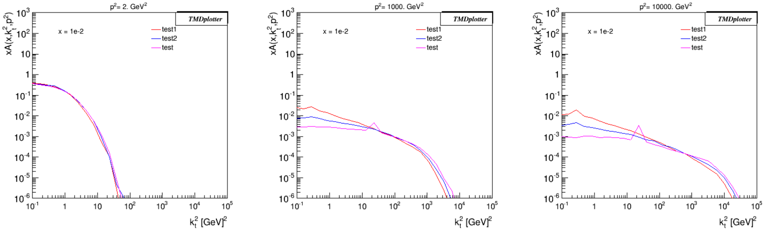


x = 0.1

- Initial scale: intrinsic k_t distribution.
- Different choices of z_{max} lead to different uTMDs, especially different large k_T tails .

sea

$z_{max} = 0.99$, $z_{max} = 0.999$, $z_{max} = 0.9999$

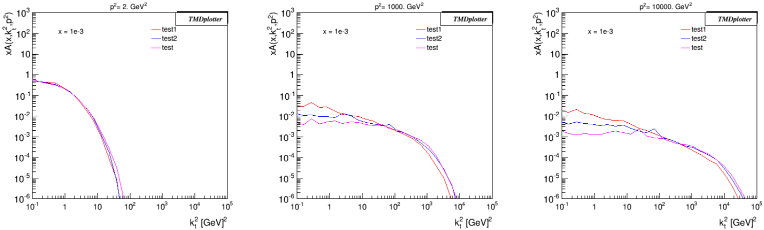


x = 0.01

- Initial scale: intrinsic k_t distribution.
- Different choices of z_{max} lead to different uTMDs, especially different large k_T tails .

sea

$z_{max} = 0.99$, $z_{max} = 0.999$, $z_{max} = 0.9999$

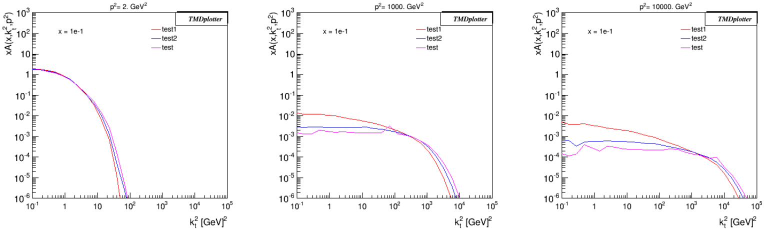


x = 0.001

- Initial scale: intrinsic k_t distribution.
- Different choices of z_{max} lead to different uTMDs, especially different large k_T tails .

gluon

$z_{max} = 0.99$, $z_{max} = 0.999$, $z_{max} = 0.9999$

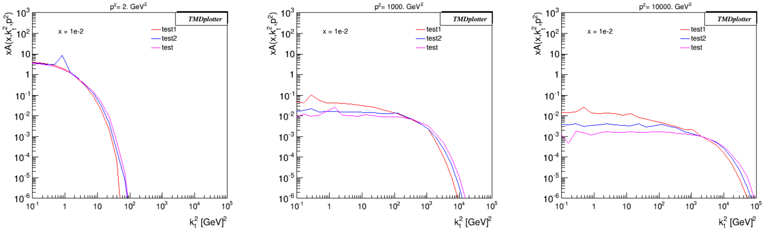


x = 0.1

- Initial scale: intrinsic k_t distribution.
- Different choices of z_{max} lead to different uTMDs, especially different large k_T tails.

gluon

$z_{max} = 0.99$, $z_{max} = 0.999$, $z_{max} = 0.9999$

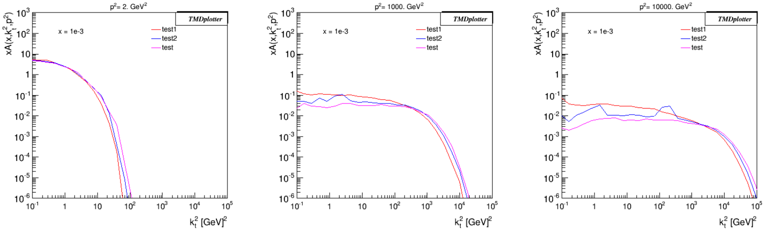


$x = 0.01$

- Initial scale: intrinsic k_t distribution.
- Different choices of z_{max} lead to different uTMDs, especially different large k_T tails.

gluon

$z_{max} = 0.99$, $z_{max} = 0.999$, $z_{max} = 0.9999$



x = 0.001

- Initial scale: intrinsic k_t distribution.
- Different choices of z_{max} lead to different uTMDs, especially different large k_T.

Fitting method

Evolution kernels

Two different evolution kernels are defined:

- initial quark (valence or sea) $\longrightarrow$ quark grid,
- initial gluon $\longrightarrow$ gluon grid.

Kernels for evolution initiated by gluons and quarks are calculated separately only once per run of the code and combined at the end $\longrightarrow$ fitting procedure is fast.

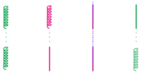


Figure : K_{gg}, K_{gq}, K_{qq},K_{qg}: gluon grid, quark grid

Evolution kernels

Two different evolution kernels are defined:

- initial quark (valence or sea)$\rightarrow$ quark grid,
- initial gluon $\rightarrow$ gluon grid.

Kernels for evolution initiated by gluons and quarks are calculated separately only once per run of the code and combined at the end $\rightarrow$ fitting procedure is fast.

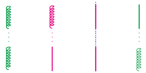


Figure : K_{gg}, K_{gq}, K_{qq},K_{qg}: gluon grid, quark grid

To get the final pdf: evolution kernel is folded with starting distribution

$$xf(x,t) = x\int dx_0 \int dz f_0(x_0) K(z,t)\delta(zx_0 - x) \tag{18}$$

For example for gluon:

$$xf(x,t)_g = x\int dx_0 \int dz \left(f_{0g}(x_0)K_{gg} + f_{0q}(x_0)K_{gq}\right)\delta(zx_0 - x), \tag{19}$$

and for up-valence quark:

$$xf(x,t)_{q,up} = x\int dx_0 \int dz \left(f_{0,up}(x_0)K_{qq}\right)\delta(zx_0 - x). \tag{20}$$