Bootstrapping the $6d$ $\mathcal{N} = (2,0)$ Theory

Madalena Lemos

Mar. 14 2016
10th Nordic String Theory Meeting

Based on:
C. Beem, L. Rastelli and B. van Rees – 1507.05637

Outline

1. **The (Super)conformal Bootstrap Program**
2. $6d$ $(2,0)$ **bootstrap**
3. $6d$ $(2,0)$ **bootstrap – Results**
   - Bounding the central charge
   - Bounding Operator Dimensions
4. **Outlook**

Outline

The (Super)conformal Bootstrap Program

What is the space of consistent (S)CFTs?

The (Super)conformal Bootstrap Program

What is the space of consistent (S)CFTs?

$\rightarrow$ Maximally supersymmetric theories: well known list of theories

The (Super)conformal Bootstrap Program

What is the space of consistent (S)CFTs?

$\rightarrow$ Maximally supersymmetric theories: well known list of theories

$\hookrightarrow$ Can we prove there are no exotic theories?

The (Super)conformal Bootstrap Program

What is the space of consistent (S)CFTs?

$\longrightarrow$ Maximally supersymmetric theories: well known list of theories

$\hookrightarrow$ Can we prove there are no exotic theories?

Can we bootstrap specific theories?

The (Super)conformal Bootstrap Program

What is the space of consistent (S)CFTs?

$\rightarrow$ Maximally supersymmetric theories: well known list of theories

$\hookrightarrow$ Can we prove there are no exotic theories?

Can we bootstrap specific theories?

$\rightarrow$ Particularly helpful if theory is uniquely fixed by a set of discrete data

$\rightarrow$ Only tool available for finite N non-Lagrangian theories, such as the $6d$ $(2,0)$ theory

The (Super)conformal Bootstrap Program

What is the space of consistent (S)CFTs?

$\rightarrow$ Maximally supersymmetric theories: well known list of theories

$\hookrightarrow$ Can we prove there are no exotic theories?

Can we bootstrap specific theories?

$\rightarrow$ Particularly helpful if theory is uniquely fixed by a set of discrete data

$\rightarrow$ Only tool available for finite N non-Lagrangian theories, such as the $6d$ $(2,0)$ theory

$\hookrightarrow$ Can we solve the A_1 theory?

Conformal Bootstrap

Conformal field theory defined by

Set of local operators and their correlation functions

Conformal Bootstrap

Conformal field theory defined by

Set of local operators and their correlation functions

Operator Product Expansion

$\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_k \lambda_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_k} c(x, \partial_x)\mathcal{O}_k(0)$

Conformal Bootstrap

Conformal field theory defined by

Set of local operators and their correlation functions

Operator Product Expansion

$\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_k \lambda_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_k} c(x, \partial_x)\mathcal{O}_k(0)$

$\rightarrow$ Finite radius of convergence

Conformal Bootstrap

Conformal field theory defined by

Set of local operators and their correlation functions

Operator Product Expansion

$\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_k \lambda_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_k} c(x, \partial_x)\mathcal{O}_k(0)$

- $\rightarrow$ Finite radius of convergence
- $\rightarrow$ $n-$point function by recursive use of the OPE until $\langle \mathbb{1} \rangle = 1$

Conformal Bootstrap

Conformal field theory defined by

Set of local operators and their correlation functions

Operator Product Expansion

$\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_k \lambda_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_k} c(x, \partial_x)\mathcal{O}_k(0)$

- $\rightarrow$ Finite radius of convergence
- $\rightarrow$ $n-$point function by recursive use of the OPE until $\langle \mathbb{1} \rangle = 1$

CFT data

$\{\mathcal{O}_{\Delta,\ell,\ldots}(x)\}$ and $\{\lambda_{\mathcal{O}_i\mathcal{O}_j\mathcal{O}_k}\}$

Conformal Bootstrap

Conformal field theory defined by

Set of local operators and their correlation functions

Operator Product Expansion

$\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_k \lambda_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_k} c(x, \partial_x)\mathcal{O}_k(0)$

$\rightarrow$ Finite radius of convergence

$\rightarrow$ $n-$point function by recursive use of the OPE until $\langle \mathbb{1} \rangle = 1$

CFT data

$\{\mathcal{O}_{\Delta,\ell,\ldots}(x)\}$ and $\{\lambda_{\mathcal{O}_i\mathcal{O}_j\mathcal{O}_k}\}$

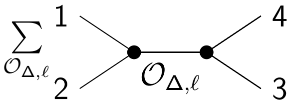
CFT data strongly constrained

- Unitarity
- Associativity of the operator product algebra

Conformal Bootstrap

Crossing Symmetry

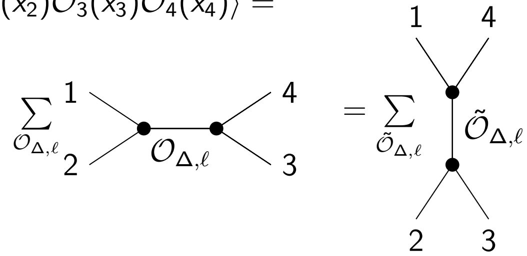
$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle =$



Conformal Bootstrap

Crossing Symmetry

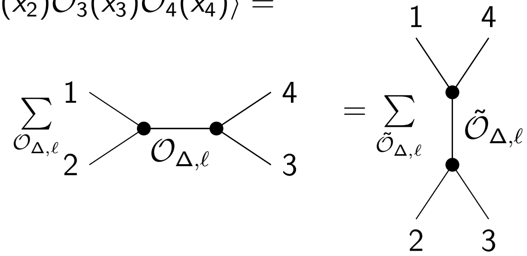
$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle =$



Conformal Bootstrap

Crossing Symmetry

$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle =$



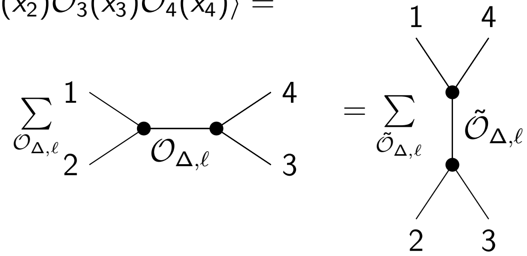
$$\frac{1}{x_{12}^{2\Delta_{\mathcal{O}}} x_{34}^{2\Delta_{\mathcal{O}}}} \sum_{\mathcal{O}_{\Delta,\ell}} \lambda_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_{\Delta,\ell}} \lambda_{\mathcal{O}_3\mathcal{O}_4\mathcal{O}_{\Delta,\ell}} g_{\Delta,\ell}(u,v) =$$

where $\Delta_{\mathcal{O}_i} = \Delta_{\mathcal{O}}$, $u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z\bar{z}$, $v = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2} = (1-z)(1-\bar{z})$

Conformal Bootstrap

Crossing Symmetry

$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle =$



$$\frac{1}{x_{12}^{2\Delta_{\mathcal{O}}} x_{34}^{2\Delta_{\mathcal{O}}}} \sum_{\mathcal{O}_{\Delta,\ell}} \lambda_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_{\Delta,\ell}} \lambda_{\mathcal{O}_3\mathcal{O}_4\mathcal{O}_{\Delta,\ell}} g_{\Delta,\ell}(u,v) =$$

$$\frac{1}{x_{14}^{2\Delta_{\mathcal{O}}} x_{23}^{2\Delta_{\mathcal{O}}}} \sum_{\tilde{\mathcal{O}}_{\Delta,\ell}} \lambda_{\mathcal{O}_1\mathcal{O}_4\tilde{\mathcal{O}}_{\Delta,\ell}} \lambda_{\mathcal{O}_2\mathcal{O}_3\tilde{\mathcal{O}}_{\Delta,\ell}} g_{\Delta,\ell}(v,u)$$

where $\Delta_{\mathcal{O}_i} = \Delta_{\mathcal{O}}$, $u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z\bar{z}$, $v = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2} = (1-z)(1-\bar{z})$

Conformal Bootstrap

→ Solve crossing equations for *all* four-point functions

Conformal Bootstrap

$\rightarrow$ Solve crossing equations for *all* four-point functions

[Rattazzi Rychkov Tonni Vichi]

- Solving $\Rightarrow$ constraining

Conformal Bootstrap

$\rightarrow$ Solve crossing equations for *all* four-point functions

[Rattazzi Rychkov Tonni Vichi]

- Solving $\Rightarrow$ constraining
  - $\rightarrow$ Guess for the spectrum

Conformal Bootstrap

$\rightarrow$ Solve crossing equations for *all* four-point functions

[Rattazzi Rychkov Tonni Vichi]

- Solving $\Rightarrow$ constraining
  - $\rightarrow$ Guess for the spectrum
    - $\hookrightarrow$ there's a large gap in the spectrum

Conformal Bootstrap

$\rightarrow$ Solve crossing equations for *all* four-point functions

[Rattazzi Rychkov Tonni Vichi]

- Solving $\Rightarrow$ constraining
  - $\rightarrow$ Guess for the spectrum
    - $\hookrightarrow$ there's a large gap in the spectrum
  - $\rightarrow$ Can it ever define a consistent CFT?

Conformal Bootstrap

$\rightarrow$ Solve crossing equations for *all* four-point functions

[Rattazzi Rychkov Tonni Vichi]

- Solving $\Rightarrow$ constraining
  - $\rightarrow$ Guess for the spectrum
    - $\hookrightarrow$ there's a large gap in the spectrum
  - $\rightarrow$ Can it ever define a consistent CFT?

Sum rule: identical scalars ϕ

Conformal Bootstrap

$\rightarrow$ Solve crossing equations for *all* four-point functions

[Rattazzi Rychkov Tonni Vichi]

- Solving $\Rightarrow$ constraining
  - $\rightarrow$ Guess for the spectrum
    - $\hookrightarrow$ there's a large gap in the spectrum
  - $\rightarrow$ Can it ever define a consistent CFT?

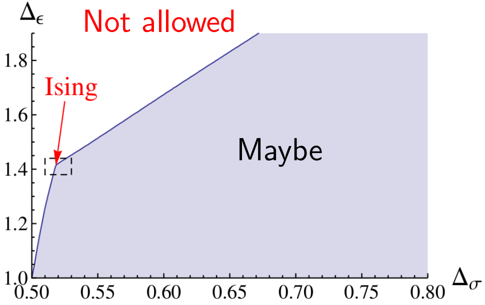
Sum rule: identical scalars ϕ

$\rightarrow$ Identity operator $\lambda_{\mathcal{O}\mathcal{O}\mathbb{1}} = 1$

$$\boxed{1 = \sum_{\substack{\mathcal{O}_{\Delta_\ell} \neq \mathbb{1} \\ \mathcal{O} \in \phi\phi}} \lambda^2_{\phi\phi\mathcal{O}} \underbrace{\frac{u^{\Delta_\phi} g_{\Delta,\ell}(v,u) - v^{\Delta_\phi} g_{\Delta,\ell}(u,v)}{v^{\Delta_\phi} - u^{\Delta_\phi}}}_{F_{\Delta,\ell}}}$$

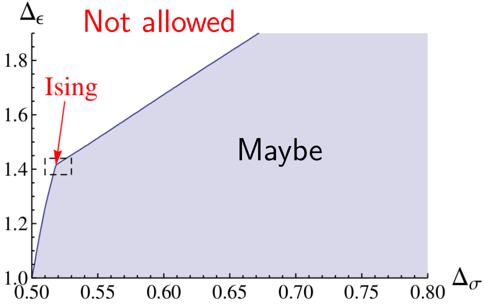
$3d$ Ising Model

[El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi, PRD 86 025022]



3d Ising Model

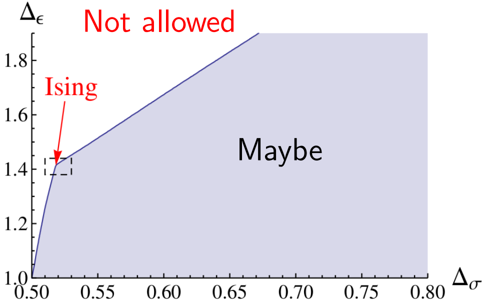
[El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi, PRD 86 025022]



$\rightarrow$ Saturated by 3d Ising model

$3d$ Ising Model

[El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi, PRD 86 025022]



$\longrightarrow$ Saturated by $3d$ Ising model

$\longrightarrow$ $3d$ Ising lives at “kink”

Outline

$6d$ $(2,0)$ **theories**

$\rightarrow$ Largest dimension for superconformal theories

$\rightarrow$ Maximally supersymmetric (16 $\mathcal{Q}$'s and 16 $\mathcal{S}$'s)

$6d$ $(2,0)$ theories

→ Largest dimension for superconformal theories

→ Maximally supersymmetric (16 $\mathcal{Q}$'s and 16 $\mathcal{S}$'s)

Free tensor multiplet

$6d\ (2,0)$ theories

$\rightarrow$ Largest dimension for superconformal theories

$\rightarrow$ Maximally supersymmetric (16 $\mathcal{Q}$'s and 16 $\mathcal{S}$'s)

Free tensor multiplet

$$\phi^I\,, \qquad \Delta = 2\,, \qquad \qquad \mathbf{5} \text{ of } SO(5)_R$$

$6d$ $(2,0)$ theories

- $\rightarrow$ Largest dimension for superconformal theories
- $\rightarrow$ Maximally supersymmetric (16 $\mathcal{Q}$'s and 16 $\mathcal{S}$'s)

Free tensor multiplet

$$\phi^I\,, \qquad \Delta = 2\,, \qquad \mathbf{5} \text{ of } SO(5)_R$$
$$\lambda_{a,A}\,, \qquad \Delta = 5/2\,, \qquad \mathbf{4} \text{ of } SO(5)_R$$

$6d$ $(2,0)$ theories

→ Largest dimension for superconformal theories
→ Maximally supersymmetric (16 $\mathcal{Q}$'s and 16 $\mathcal{S}$'s)

Free tensor multiplet

$$
\begin{aligned}
\phi^I\,, \quad & \Delta = 2\,, && \mathbf{5}\text{ of } SO(5)_R \\
\lambda_{a,A}\,, \quad & \Delta = 5/2\,, && \mathbf{4}\text{ of } SO(5)_R \\
\omega_{[ab]}\,, \quad & \Delta = 3\,, && \text{self-dual field strength of 2-form}
\end{aligned}
$$

$6d$ $(2,0)$ theories

→ Largest dimension for superconformal theories
→ Maximally supersymmetric (16 $\mathcal{Q}$'s and 16 $\mathcal{S}$'s)

Free tensor multiplet

$\phi^I\,,$	$\Delta = 2\,,$	$\mathbf{5}$ of $SO(5)_R$
$\lambda_{a,A}\,,$	$\Delta = 5/2\,,$	$\mathbf{4}$ of $SO(5)_R$
$\omega_{[ab]}\,,$	$\Delta = 3\,,$	self-dual field strength of 2-form

Interacting Theory

→ No Lagrangian description

$6d$ $(2,0)$ theories

→ Largest dimension for superconformal theories

→ Maximally supersymmetric (16 $\mathcal{Q}$'s and 16 $\mathcal{S}$'s)

Free tensor multiplet

$\phi^I\,,$	$\Delta = 2\,,$	$\mathbf{5}$ of $SO(5)_R$
$\lambda_{a,A}\,,$	$\Delta = 5/2\,,$	$\mathbf{4}$ of $SO(5)_R$
$\omega_{[ab]}\,,$	$\Delta = 3\,,$	self-dual field strength of 2-form

Interacting Theory

→ No Lagrangian description

→ Isolated fixed points - no marginal deformations

$6d$ $(2,0)$ theories

→ Largest dimension for superconformal theories

→ Maximally supersymmetric (16 $\mathcal{Q}$'s and 16 $\mathcal{S}$'s)

Free tensor multiplet

$$\begin{aligned} &\phi^I\,, && \Delta = 2\,, && \mathbf{5} \text{ of } SO(5)_R \\ &\lambda_{a,A}\,, && \Delta = 5/2\,, && \mathbf{4} \text{ of } SO(5)_R \\ &\omega_{[ab]}\,, && \Delta = 3\,, && \text{self-dual field strength of 2-form} \end{aligned}$$

Interacting Theory

→ No Lagrangian description

→ Isolated fixed points - no marginal deformations

→ Existence and properties from string and M-theory

$6d$ $(2,0)$ theories

Interacting Theory

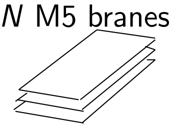
$\rightarrow$ ADE classification

$6d$ $(2,0)$ theories

Interacting Theory

$\rightarrow$ ADE classification

A_{N-1} theory



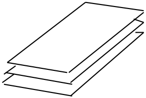
$6d$ $(2,0)$ theories

Interacting Theory

$\rightarrow$ ADE classification

A_{N-1} theory

N M5 branes



$\Downarrow$

$(2,0)$ A_{N-1} theory $+$ free tensor

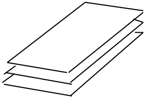
$\rightarrow$ Large N limit: SUGRA on $AdS_7 \times S^4$

$6d$ $(2,0)$ theories

Interacting Theory

$\rightarrow$ ADE classification

A_{N-1} theory



$\Downarrow$

$(2,0)$ A_{N-1} theory $+$ free tensor

$\rightarrow$ Large N limit: SUGRA on $AdS_7 \times S^4$

$\rightarrow$ Many lower dimensional theories obtained from its compactification

$6d$ $(2,0)$ theories

Bootstrap

$\rightarrow$ Just another (very special) CFT!

$6d$ $(2,0)$ theories

Bootstrap

$\rightarrow$ Just another (very special) CFT!
Defined by a local operator algebra

$6d$ $(2,0)$ theories

Bootstrap

- $\rightarrow$ Just another (very special) CFT!
  Defined by a local operator algebra
- $\rightarrow$ Guaranteed conformal primary: $T_{\mu\nu}$

$6d$ $(2,0)$ **theories**

Bootstrap

- $\rightarrow$ Just another (very special) CFT!
  Defined by a local operator algebra
- $\rightarrow$ Guaranteed conformal primary: $T_{\mu\nu}$
- $\rightarrow$ Superconformal descendant

$6d$ $(2,0)$ theories

Bootstrap

$\rightarrow$ Just another (very special) CFT!
Defined by a local operator algebra

$\rightarrow$ Guaranteed conformal primary: $T_{\mu\nu}$

$\rightarrow$ Superconformal descendant

Stress tensor multiplet

- Superconformal primary: Φ^{IJ}

$6d$ $(2,0)$ theories

Bootstrap

- $\rightarrow$ Just another (very special) CFT!
  Defined by a local operator algebra
- $\rightarrow$ Guaranteed conformal primary: $T_{\mu\nu}$
- $\rightarrow$ Superconformal descendant

Stress tensor multiplet

- Superconformal primary: Φ^{IJ}
- dimension 4 scalar

$6d$ $(2,0)$ theories

Bootstrap

$\rightarrow$ Just another (very special) CFT!
Defined by a local operator algebra

$\rightarrow$ Guaranteed conformal primary: $T_{\mu\nu}$

$\rightarrow$ Superconformal descendant

Stress tensor multiplet

- Superconformal primary: Φ^{IJ}
- dimension 4 scalar
- **14** of $SO(5)_R$

$6d$ $(2,0)$ theories

Bootstrap

- $\rightarrow$ Just another (very special) CFT!
  Defined by a local operator algebra
- $\rightarrow$ Guaranteed conformal primary: $T_{\mu\nu}$
- $\rightarrow$ Superconformal descendant

Stress tensor multiplet

- Superconformal primary: Φ^{IJ} $\underbrace{(= \phi^{(I}\phi^{J)} - \frac{1}{5}\delta^{IJ}\phi^2)}_{\text{free tensor}}$
- dimension 4 scalar
- **14** of $SO(5)_R$
- $\frac{1}{2}$ BPS

Crossing Equation

- Conformal blocks $\rightsquigarrow$ superconformal blocks

$6d$ $(2,0)$ theories

Crossing Equation

- Conformal blocks $\rightsquigarrow$ superconformal blocks
- Identity $\rightsquigarrow$ protected contributions

$6d$ $(2,0)$ theories

Crossing Equation

- Conformal blocks $\rightsquigarrow$ superconformal blocks
- Identity $\rightsquigarrow$ protected contributions

$$\boxed{\sum \lambda^2_{\Delta,\ell}\left(\underbrace{\mathcal{G}_{\Delta,\ell}(z,\bar{z})}_{\text{superblock}}-\mathcal{G}_{\Delta,\ell}(1-z,1-\bar{z})\right)=\mathcal{F}^{\text{short}}_c}$$

Crossing Equation

- Conformal blocks $\rightsquigarrow$ superconformal blocks
- Identity $\rightsquigarrow$ protected contributions

$$\boxed{\sum_{\substack{\Delta\geqslant\ell+6,\\ \ell\geqslant 0}} \lambda^2_{\Delta,\ell}\left(\underbrace{\mathcal{G}_{\Delta,\ell}(z,\bar{z})}_{\text{superblock}} - \mathcal{G}_{\Delta,\ell}(1-z,1-\bar{z})\right) = \mathcal{F}^{\text{short}}_{c}}$$

$\rightarrow$ Sum runs over long multiplets $\Delta > \ell + 6$, $\ell \geqslant 0$

$6d$ $(2,0)$ theories

Crossing Equation

- Conformal blocks $\rightsquigarrow$ superconformal blocks
- Identity $\rightsquigarrow$ protected contributions

$$\boxed{\sum_{\substack{\Delta \geqslant \ell+6,\\ \ell \geqslant 0}} \lambda^2_{\Delta,\ell} \left(\underbrace{\mathcal{G}_{\Delta,\ell}(z,\bar{z})}_{\text{superblock}} - \mathcal{G}_{\Delta,\ell}(1-z,1-\bar{z}) \right) = \mathcal{F}_c^{\text{short}}}$$

$\rightarrow$ Sum runs over long multiplets $\Delta > \ell + 6$, $\ell \geqslant 0$

$\rightarrow$ Short multiplet *at* the bound $\Delta = \ell + 6$, $\ell \geqslant 0$ not fixed

$6d$ $(2,0)$ theories

Crossing Equation

- Conformal blocks $\rightsquigarrow$ superconformal blocks
- Identity $\rightsquigarrow$ protected contributions

$$\boxed{\sum_{\substack{\Delta\geqslant\ell+6,\\ \ell\geqslant 0}} \lambda^2_{\Delta,\ell}\left(\underbrace{\mathcal{G}_{\Delta,\ell}(z,\bar{z})}_{\text{superblock}} - \mathcal{G}_{\Delta,\ell}(1-z,1-\bar{z})\right) = \mathcal{F}_c^{\text{short}}}$$

$\rightarrow$ Sum runs over long multiplets $\Delta > \ell + 6$, $\ell \geqslant 0$

$\rightarrow$ Short multiplet *at* the bound $\Delta = \ell + 6$, $\ell \geqslant 0$ not fixed

$\hookrightarrow$ Lightcone limit $\Rightarrow$ present for large ℓ

$6d$ $(2,0)$ theories

Crossing Equation

- Conformal blocks $\rightsquigarrow$ superconformal blocks
- Identity $\rightsquigarrow$ protected contributions

$$\boxed{\sum_{\substack{\Delta\geqslant\ell+6,\\ \ell\geqslant 0}} \lambda^2_{\Delta,\ell}\left(\underbrace{\mathcal{G}_{\Delta,\ell}(z,\bar{z})}_{\text{superblock}} - \mathcal{G}_{\Delta,\ell}(1-z,1-\bar{z})\right) = \mathcal{F}_c^{\text{short}}}$$

$\rightarrow$ Sum runs over long multiplets $\Delta > \ell + 6$, $\ell \geqslant 0$

$\rightarrow$ Short multiplet *at* the bound $\Delta = \ell + 6$, $\ell \geqslant 0$ not fixed

$\hookrightarrow$ Lightcone limit $\Rightarrow$ present for large ℓ

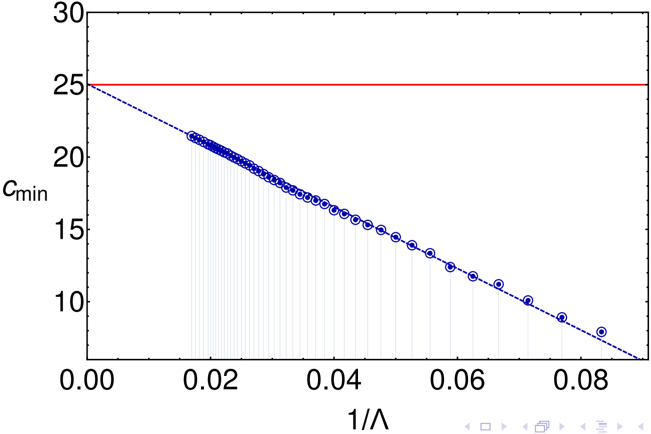
$\rightarrow$ In fixing $\mathcal{F}_c^{\text{short}}$ excluded free theory

Outline

Outline

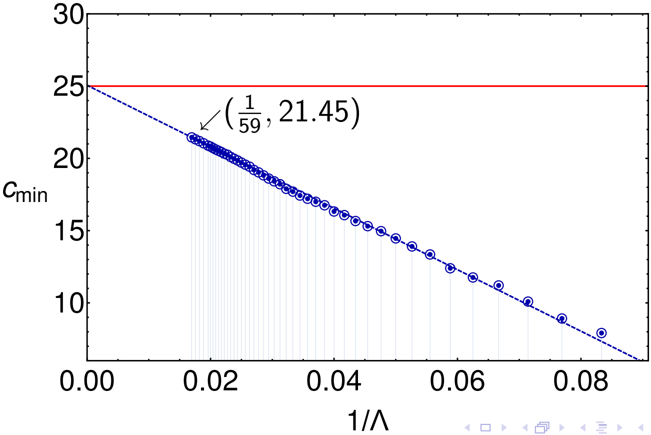
Bounding the Central charge

Minimum allowed c



Bounding the Central charge

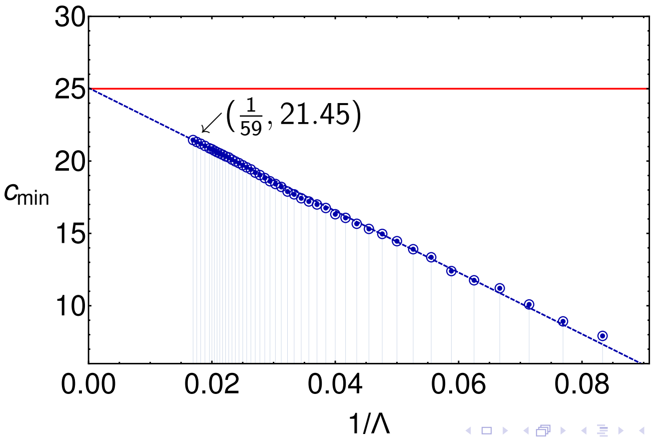
Minimum allowed c



Bounding the Central charge

Minimum allowed c

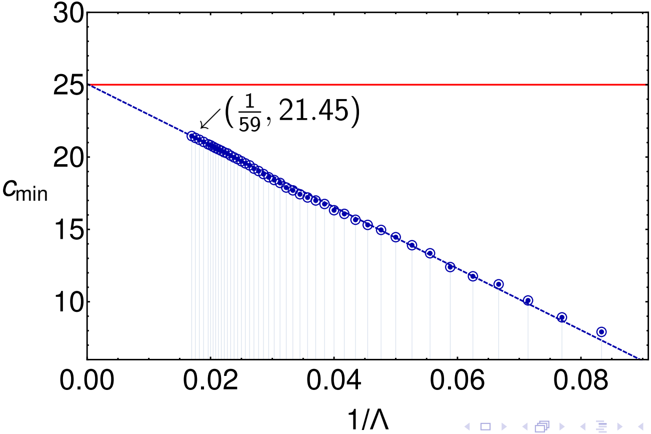
→ $c_{min} \to 25$: central charge of A_1 theory (two M5 branes)



Bounding the Central charge

Minimum allowed c

- $\rightarrow$ $c_{min} \rightarrow 25$: central charge of A_1 theory (two M5 branes)
- $\rightarrow$ OPE coefficient of $\frac{1}{4}$ BPS $\mathcal{D}[0,4]$ operator is zero at c_{min}



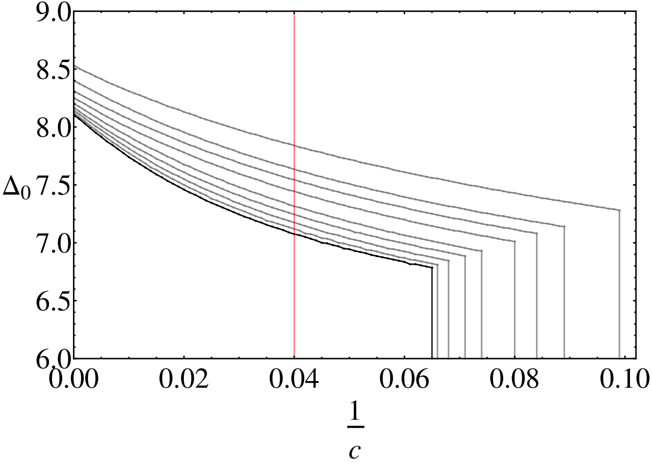
Outline

Bounding operator dimensions

→ Assume all shorts are there

→ Unitarity bound $\Delta_0 > 6$

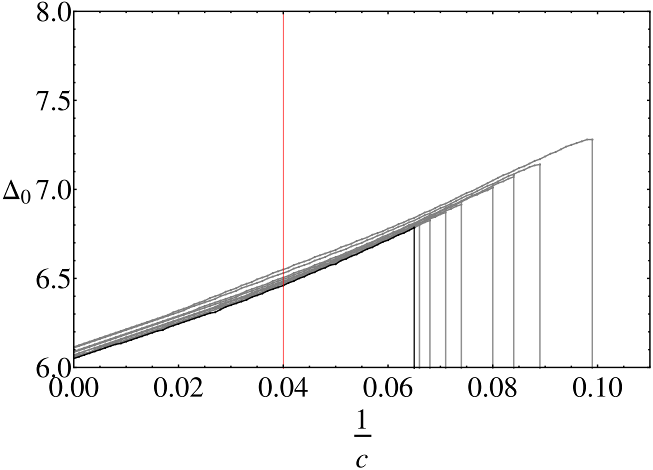
First unprotected long scalar bound



Bounding operator dimensions

- $\rightarrow$ Assume all but the $\mathcal{D}[0,4]$ shorts are there
- $\rightarrow$ Unitarity bound $\Delta_0 > 6$

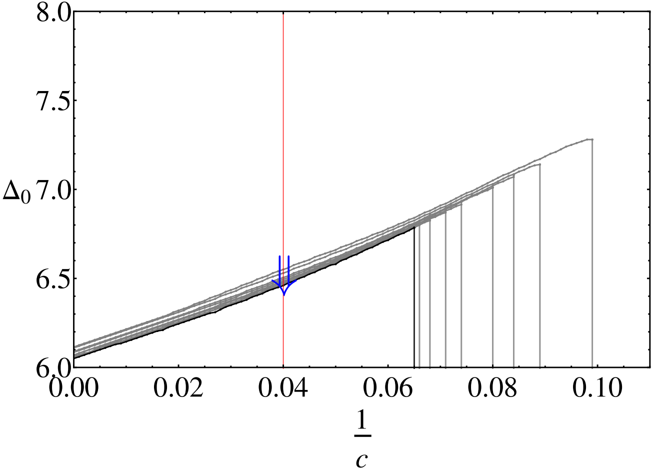
First unprotected long scalar bound



Bounding operator dimensions

- → Assume all but the $\mathcal{D}[0,4]$ shorts are there
- → Unitarity bound $\Delta_0 > 6$

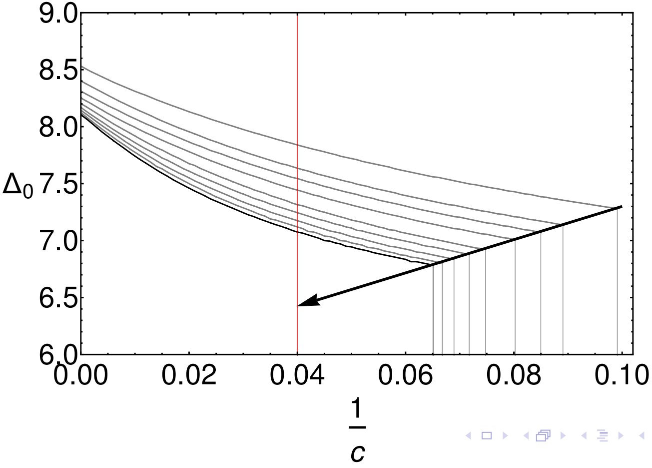
First unprotected long scalar bound



Bounding operator dimensions

- → Assume all shorts are there
- → Unitarity bound $\Delta_0 > 6$

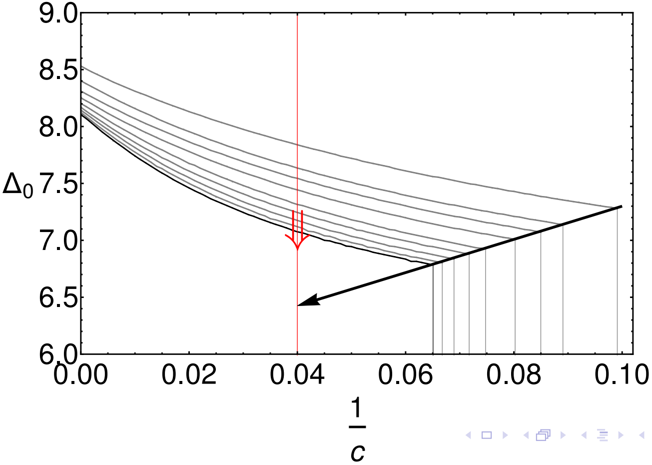
First unprotected long scalar bound



Bounding operator dimensions

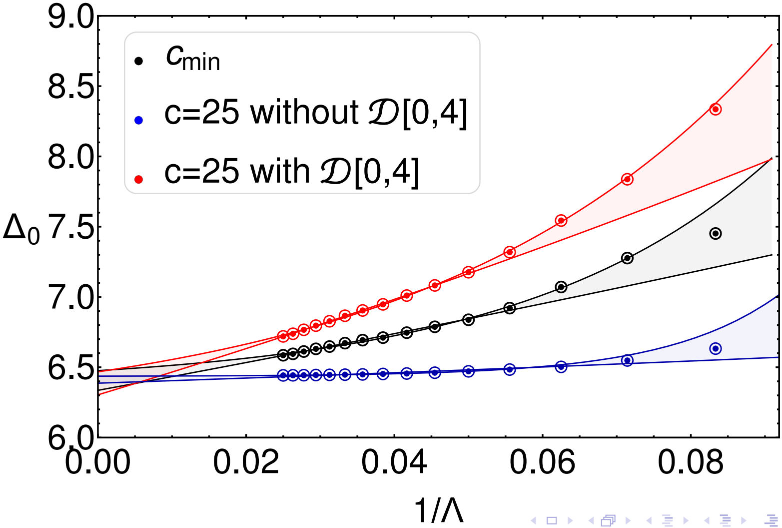
- → Assume all shorts are there
- → Unitarity bound $\Delta_0 > 6$

First unprotected long scalar bound



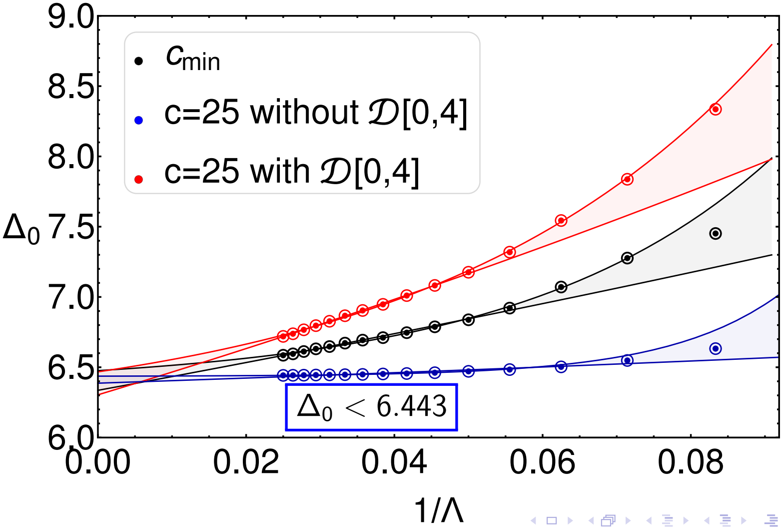
Bounding operator dimensions

Extrapolating first unprotected long scalar bound



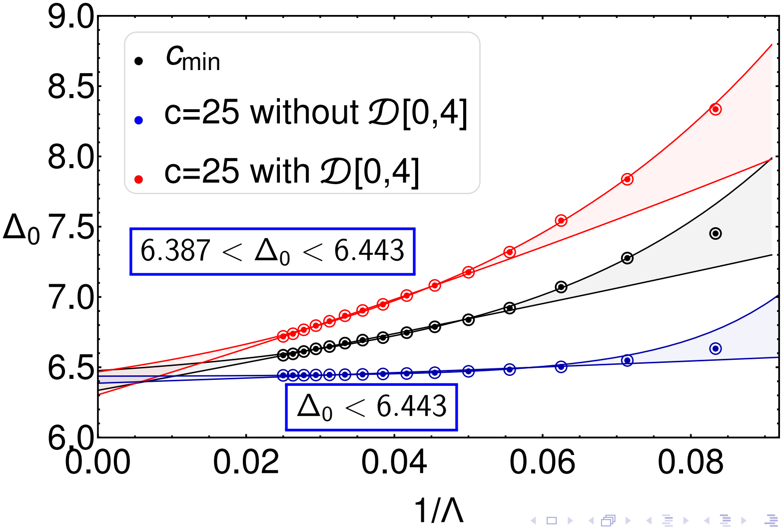
Bounding operator dimensions

Extrapolating first unprotected long scalar bound



Bounding operator dimensions

Extrapolating first unprotected long scalar bound



Outline

1. **The (Super)conformal Bootstrap Program**
2. $6d$ $(2,0)$ **bootstrap**
3. $6d$ $(2,0)$ **bootstrap – Results**
   - Bounding the central charge
   - Bounding Operator Dimensions
4. **Outlook**

Outlook

Determination of A_1 spectrum and OPE coefficients

$\rightarrow$ Precision numerics?

Outlook

Determination of A_1 spectrum and OPE coefficients

$\rightarrow$ Precision numerics?

Mixed correlators

$\rightarrow$ generators of $\frac{1}{2}BPS$ ring

Outlook

Determination of A_1 spectrum and OPE coefficients

$\rightarrow$ Precision numerics?

Mixed correlators

$\rightarrow$ generators of $\frac{1}{2}BPS$ ring

Analytic approach?

Outlook

Determination of A_1 spectrum and OPE coefficients

$\rightarrow$ Precision numerics?

Mixed correlators

$\rightarrow$ generators of $\frac{1}{2}BPS$ ring

Analytic approach?

$\rightarrow$ Light-cone can teach about $\ell \rightarrow \infty$

Thank you!

Backup slides

Outline

Numerics

Crossing equation

Bounding OPE coefficients

Conformal Bootstrap

Sum rule

- $\rightarrow$ Guess for the spectrum
- $\rightarrow$ Can it ever define a consistent CFT?

Conformal Bootstrap

Sum rule

$\rightarrow$ Guess for the spectrum

$\rightarrow$ Can it ever define a consistent CFT?

- Find Functional Ψ such that

  $\hookrightarrow$ $\psi \cdot 1 < 0$ $(\mathbb{1})$

  $\hookrightarrow$ $\psi \cdot F_{\Delta,\ell}(u, v) \geq 0$ for all $\{\Delta, \ell\}$ in spectrum

Conformal Bootstrap

Sum rule

- $\rightarrow$ Guess for the spectrum
- $\rightarrow$ Can it ever define a consistent CFT?
- ▶ Find Functional Ψ such that
  - $\hookrightarrow$ $\psi \cdot 1 < 0$ ($\mathbb{1}$)
  - $\hookrightarrow$ $\psi \cdot F_{\Delta,\ell}(u, v) \geq 0$ for all $\{\Delta, \ell\}$ in spectrum
- $\rightarrow$ Spectrum is inconsistent $\Rightarrow$ rule out CFT

Conformal Bootstrap

Sum rule

- $\rightarrow$ Guess for the spectrum
- $\rightarrow$ Can it ever define a consistent CFT?
  - Find Functional Ψ such that
    - $\hookrightarrow$ $\psi \cdot 1 < 0$ ($\mathbb{1}$)
    - $\hookrightarrow$ $\psi \cdot F_{\Delta,\ell}(u,v) \geq 0$ for all $\{\Delta, \ell\}$ in spectrum
- $\rightarrow$ Spectrum is inconsistent $\Rightarrow$ rule out CFT
  - Truncate

$$\psi = \sum_{m,n}^{m,n \leqslant \Lambda} a_{mn} \partial_z^m \partial_{\bar{z}}^n \big|_{z=\bar{z}=\frac{1}{2}}$$

Conformal Bootstrap

Sum rule

- $\rightarrow$ Guess for the spectrum
- $\rightarrow$ Can it ever define a consistent CFT?
  - ▶ Find Functional Ψ such that
    - $\hookrightarrow$ $\psi \cdot 1 < 0$ $(\mathbb{1})$
    - $\hookrightarrow$ $\psi \cdot F_{\Delta,\ell}(u,v) \geq 0$ for all $\{\Delta, \ell\}$ in spectrum
- $\rightarrow$ Spectrum is inconsistent $\Rightarrow$ rule out CFT
  - ▶ Truncate

$$\psi = \sum_{m,n}^{m,n \leqslant \Lambda} a_{mn} \partial_z^m \partial_{\bar{z}}^n \big|_{z=\bar{z}=\frac{1}{2}}$$

- $\rightarrow$ Increase $\Lambda \Rightarrow$ bounds get stronger

Conformal Bootstrap

Sum rule

- $\rightarrow$ Guess for the spectrum
- $\rightarrow$ Can it ever define a consistent CFT?
  - Find Functional Ψ such that
    - $\hookrightarrow$ $\psi \cdot 1 < 0$ ($\mathbb{1}$)
    - $\hookrightarrow$ $\psi \cdot F_{\Delta,\ell}(u,v) \geq 0$ for all $\{\Delta, \ell\}$ in spectrum
- $\rightarrow$ Spectrum is inconsistent $\Rightarrow$ rule out CFT
  - Truncate

$$\psi = \sum_{m,n}^{m,n\leqslant\Lambda} a_{mn} \partial_z^m \partial_{\bar{z}}^n \big|_{z=\bar{z}=\frac{1}{2}}$$

- $\rightarrow$ Increase $\Lambda \Rightarrow$ bounds get stronger
- $\rightarrow$ Always true bounds

Outline

$6d\ (2,0)$ **theories**

- $SO(5)_R$ structure

$6d\ (2,0)$ **theories**

- $SO(5)_R$ structure

$$\langle \Phi^{I_1 J_1}(x_1)\Phi^{I_2 J_2}(x_2)\Phi^{I_3 J_3}(x_3)\Phi^{I_4 J_4}(x_4)\rangle = \frac{1}{x_{12}^8 x_{34}^8} \sum_{R\in \mathbf{14}\times\mathbf{14}} A_R(u,v) P^{I_1\dots J_4}$$

$6d$ $(2,0)$ theories

- $SO(5)_R$ structure

$$\langle \Phi^{I_1 J_1}(x_1)\Phi^{I_2 J_2}(x_2)\Phi^{I_3 J_3}(x_3)\Phi^{I_4 J_4}(x_4)\rangle = \frac{1}{x_{12}^8 x_{34}^8} \sum_{R\in \mathbf{14}\times\mathbf{14}} A_R(u,v) P^{I_1\dots J_4}$$

- 6 channels in $\mathbf{14}\times\mathbf{14} \Rightarrow$ 6 coupled crossing equations

$6d$ $(2,0)$ **theories**

- $SO(5)_R$ structure

$$\langle \Phi^{I_1J_1}(x_1)\Phi^{I_2J_2}(x_2)\Phi^{I_3J_3}(x_3)\Phi^{I_4J_4}(x_4)\rangle = \frac{1}{x_{12}^8 x_{34}^8}\sum_{R\in\mathbf{14}\times\mathbf{14}} A_R(u,v)P^{I_1\ldots J_4}$$

- 6 channels in $\mathbf{14}\times\mathbf{14}$ $\Rightarrow$ 6 coupled crossing equations

$$A_R(u,v) = \sum_{\Delta,\ell} \boxed{\lambda^2_{R\,,\Delta,\ell}} \; g_{\Delta,\ell}(u,v)$$

$$\downarrow$$

$6d$ $(2,0)$ theories

- $SO(5)_R$ structure

$$\langle \Phi^{I_1 J_1}(x_1)\Phi^{I_2 J_2}(x_2)\Phi^{I_3 J_3}(x_3)\Phi^{I_4 J_4}(x_4)\rangle = \frac{1}{x_{12}^8 x_{34}^8} \sum_{R \in \mathbf{14}\times\mathbf{14}} A_R(u,v) P^{I_1 \dots J_4}$$

- 6 channels in $\mathbf{14} \times \mathbf{14} \Rightarrow$ 6 coupled crossing equations

$$A_R(u,v) = \sum_{\Delta,\ell} \boxed{\lambda^2_{R\,,\Delta,\ell}} \, g_{\Delta,\ell}(u,v)$$

superconformal descendants
fixed in terms of primary

$6d$ $(2,0)$ theories

- $SO(5)_R$ structure

$$\langle \Phi^{I_1J_1}(x_1)\Phi^{I_2J_2}(x_2)\Phi^{I_3J_3}(x_3)\Phi^{I_4J_4}(x_4)\rangle = \frac{1}{x_{12}^8 x_{34}^8}\sum_{R\in\mathbf{14}\times\mathbf{14}} A_R(u,v)P^{I_1\dots J_4}$$

- 6 channels in $\mathbf{14}\times\mathbf{14}\Rightarrow$ 6 coupled crossing equations

$$A_R(u,v) = \sum_{\Delta,\ell} \underset{\substack{\downarrow\\ \text{superconformal descendants}\\ \text{fixed in terms of primary}}}{\boxed{\lambda^2_{R,\Delta,\ell}}} \; g_{\Delta,\ell}(u,v)$$

- Conformal blocks $\rightsquigarrow$ superconformal blocks

$6d$ $(2,0)$ theories

- $SO(5)_R$ structure

$$\langle \Phi^{I_1 J_1}(x_1)\Phi^{I_2 J_2}(x_2)\Phi^{I_3 J_3}(x_3)\Phi^{I_4 J_4}(x_4)\rangle = \frac{1}{x_{12}^8 x_{34}^8} \sum_{R\in \mathbf{14}\times\mathbf{14}} A_R(u,v) P^{I_1 \dots J_4}$$

- 6 channels in $\mathbf{14}\times\mathbf{14} \Rightarrow$ 6 coupled crossing equations

$$A_R(u,v) = \sum_{\Delta,\ell} \underbrace{\boxed{\lambda^2_{R\,,\Delta,\ell}}}_{\substack{\text{superconformal descendants} \\ \text{fixed in terms of primary}}} g_{\Delta,\ell}(u,v)$$

- Conformal blocks $\rightsquigarrow$ superconformal blocks
- Identity $\rightsquigarrow$ protected contributions

$6d$ $(2,0)$ theories

Superconformal Ward Identities [Dolan Gallot Sokatchev]

$6d$ $(2,0)$ theories

Superconformal Ward Identities [Dolan Gallot Sokatchev]

- *All* $A_R(u,v)$ fixed in terms of two functions:

$$a(u,v) \text{ and } h(z)$$

$6d$ $(2,0)$ theories

Superconformal Ward Identities [Dolan Gallot Sokatchev]

- *All* $A_R(u,v)$ fixed in terms of two functions:

$$a(u,v) \text{ and } h(z)$$

Crossing Equation

- Can solve *analytically* for $h(z)$ in terms of c

$6d$ $(2,0)$ theories

Superconformal Ward Identities [Dolan Gallot Sokatchev]

- *All* $A_R(u,v)$ fixed in terms of two functions:

$$a(u,v) \text{ and } h(z)$$

Crossing Equation

- Can solve *analytically* for $h(z)$ in terms of c

$$\boxed{\sum_{\substack{\Delta \geqslant \ell+6, \\ \ell \geqslant 0}} \lambda^2_{\Delta,\ell} \left(z\bar{z} a_{\Delta,\ell}(z,\bar{z}) - (z \to (1-z), \bar{z} \to (1-\bar{z})) \right) = \underbrace{\mathcal{F}_c^{\text{short}}}_{a^{\text{short}},\ h(z)}}$$

$6d\ (2,0)$ theories

Superconformal Ward Identities [Dolan Gallot Sokatchev]

- *All* $A_R(u,v)$ fixed in terms of two functions:

$$a(u,v) \text{ and } h(z)$$

Crossing Equation

- Can solve *analytically* for $h(z)$ in terms of c

$$\boxed{\sum_{\substack{\Delta\geqslant\ell+6,\\ \ell\geqslant 0}} \lambda^2_{\Delta,\ell}\left(z\bar{z}a_{\Delta,\ell}(z,\bar{z}) - (z\to(1-z),\bar{z}\to(1-\bar{z}))\right) = \underbrace{\mathcal{F}_c^{\text{short}}}_{a^{\text{short}},\ h(z)}}$$

$\rightarrow$ Sum runs over long multiplets $\Delta > \ell + 6$, $\ell \geqslant 0$

$6d$ $(2,0)$ theories

Superconformal Ward Identities [Dolan Gallot Sokatchev]

- *All* $A_R(u,v)$ fixed in terms of two functions:

$$a(u,v) \text{ and } h(z)$$

Crossing Equation

- Can solve *analytically* for $h(z)$ in terms of c

$$\boxed{\sum_{\substack{\Delta \geqslant \ell+6, \\ \ell \geqslant 0}} \lambda^2_{\Delta,\ell} \left(z\bar{z} a_{\Delta,\ell}(z,\bar{z}) - (z \to (1-z), \bar{z} \to (1-\bar{z})) \right) = \underbrace{\mathcal{F}_c^{\text{short}}}_{a^{\text{short}},\ h(z)}}$$

$\rightarrow$ Sum runs over long multiplets $\Delta > \ell + 6$, $\ell \geqslant 0$

$\rightarrow$ Short multiplet *at* the bound $\Delta = \ell + 6$, $\ell \geqslant 0$ not fixed

$6d$ $(2,0)$ theories

Superconformal Ward Identities [Dolan Gallot Sokatchev]

- *All* $A_R(u,v)$ fixed in terms of two functions:

$$a(u,v) \text{ and } h(z)$$

Crossing Equation

- Can solve *analytically* for $h(z)$ in terms of c

$$\boxed{\sum_{\substack{\Delta \geqslant \ell+6,\\ \ell \geqslant 0}} \lambda^2_{\Delta,\ell}\left(z\bar{z}a_{\Delta,\ell}(z,\bar{z}) - (z \to (1-z), \bar{z} \to (1-\bar{z}))\right) = \underbrace{\mathcal{F}_c^{\text{short}}}_{a^{\text{short}},\ h(z)}}$$

$\rightarrow$ Sum runs over long multiplets $\Delta > \ell + 6$, $\ell \geqslant 0$

$\rightarrow$ Short multiplet *at* the bound $\Delta = \ell + 6$, $\ell \geqslant 0$ not fixed

$\hookrightarrow$ Lightcone limit $\Rightarrow$ present for large ℓ

$6d$ $(2,0)$ theories

Superconformal Ward Identities [Dolan Gallot Sokatchev]

- *All* $A_R(u,v)$ fixed in terms of two functions:

$$a(u,v) \text{ and } h(z)$$

Crossing Equation

- Can solve *analytically* for $h(z)$ in terms of c

$$\boxed{\sum_{\substack{\Delta\geqslant\ell+6,\\ \ell\geqslant 0}} \lambda^2_{\Delta,\ell}\left(z\bar{z}a_{\Delta,\ell}(z,\bar{z}) - (z\to(1-z),\bar{z}\to(1-\bar{z}))\right) = \underbrace{\mathcal{F}_c^{\text{short}}}_{a^{\text{short}},\ h(z)}}$$

$\rightarrow$ Sum runs over long multiplets $\Delta > \ell+6$, $\ell \geqslant 0$

$\rightarrow$ Short multiplet *at* the bound $\Delta = \ell + 6$, $\ell \geqslant 0$ not fixed

$\hookrightarrow$ Lightcone limit $\Rightarrow$ present for large ℓ

$\rightarrow$ Excluded free theory!

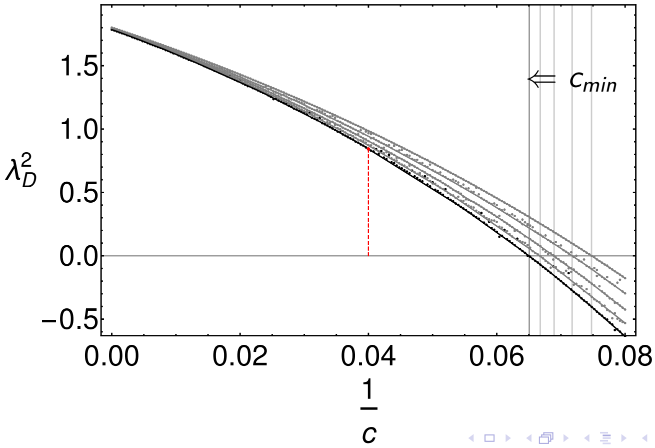
Outline

Numerics
Crossing equation
Bounding OPE coefficients

Bounding OPE coefficients

The $\mathcal{D}[0,4]$ OPE coefficient

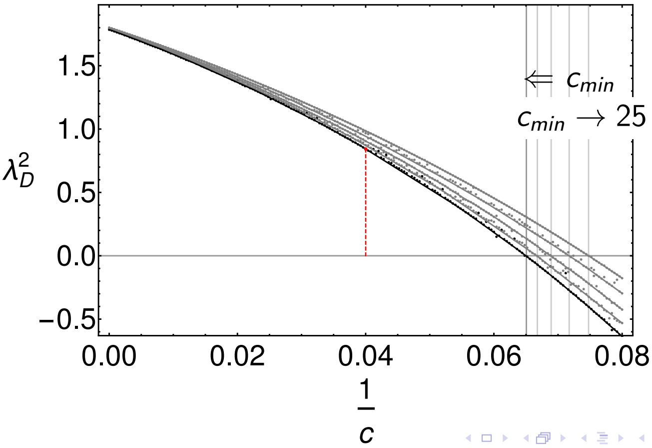
$\rightarrow$ Absent at c_{min}



Bounding OPE coefficients

The $\mathcal{D}[0,4]$ OPE coefficient

$\rightarrow$ Absent at c_{min}



Bounding OPE coefficients

The $\mathcal{D}[0,4]$ OPE coefficient

- $\rightarrow$ Absent at c_{min}
- $\rightarrow$ Expected to be absent from $\frac{1}{4}$ BPS partition function of A_1 [Batthacharyya Minwalla]