We consider hybrid inflation for small couplings of the inflaton to matter such that the critical value of the inflaton field exceeds the Planck mass. It has recently been shown that inflation then continues at subcritical inflaton field values where quantum fluctuations generate an effective inflaton mass. The effective inflaton potential interpolates between a quadratic potential at small field values and a plateau at large field values. An analysis of the allowed parameter space leads to predictions for the scalar spectral index $n_s$ and the tensor-to-scalar ratio $r$ similar to those of natural inflation. Using the ranges for $n_s$ and $r$ favored by the Planck data, we find that the energy scale of the plateau is constrained to the interval $(1.6-2.4) \times 10^{16}$ GeV, which includes the energy scale of gauge coupling unification in the supersymmetric standard model. The tensor-to-scalar ratio is predicted to satisfy the lower bound $r > 0.049$ for 60 $e$-folds before the end of inflation.

\begin{equation}
    n_s = 0.9603 \pm 0.0073, \quad r < 0.11(95\% \text{CL}),
\end{equation}

we find that $M_{\text{inf}}$ has to be close to the energy scale $M_{\text{GUT}}$ of grand unification. Furthermore, we obtain a lower bound on the tensor-to-scalar ratio, $r > 0.049(0.085)$ for 60(50) $e$-folds before the end of inflation, which is in reach of upcoming experiments.

\section{II. Subcritical Hybrid Inflation}

The framework of D-term hybrid inflation in supergravity is defined by a Kähler potential, a superpotential and a D-term scalar potential [11,12,15,16],

\begin{equation}
    K = \frac{1}{2} (\Phi + \Phi^\dagger)^2 + |S_+|^2 + |S_-|^2, \quad (2)
\end{equation}

\begin{equation}
    W = \lambda \Phi S_+ S_- \quad (3)
\end{equation}

\begin{equation}
    V_D = \frac{g^2}{2} (|S_+|^2 - |S_-|^2 - \xi)^2 \quad (4)
\end{equation}

The “waterfall fields” $S_{\pm}$ carry the $U(1)$ charges $\pm 1$, and the inflaton is contained in the gauge singlet $\Phi$. The Kähler potential is invariant under the shift $\text{Im}(\Phi) \to \text{Im}(\Phi) + \alpha$ where $\alpha$ is a real constant; i.e., it is independent of the constant part of $\varphi \equiv \sqrt{2} \text{Im}(\Phi)$, which is identified as the inflaton field. The gauge coupling $g = O(1)$, and $\lambda$ is a Yukawa coupling, which may be much smaller than $g$. The only dimensionful parameter is the FI term $\xi$ that sets the energy scale of inflation.\footnotemark

\footnotetext[1]{Note that FI terms in supergravity are a subtle issue [17–20]. For recent discussions and references on field-dependent and field-independent FI terms, see Refs. [21,22]. In the following we shall treat $\xi$ as a constant.
Standard hybrid inflation takes place at inflaton field values $\varphi$ larger than the critical value $\varphi_c = (g/\lambda)\sqrt{2\xi}$. Here the waterfall fields $S_{\pm}$ have a positive mass squared and are stabilized at the origin. Classically, the potential is independent of modulus and phase of the gauge-singlet $\Phi$. The flatness in $|\Phi|$ is lifted by quantum corrections.

For subcritical field values $|\Phi| < \sqrt{2}\varphi_c$, the complex scalar $S_{-}$ remains stabilized at the origin, whereas $S_{+}$ acquires a tachyonic instability. The sum of F- and D-terms yields for the scalar potential as function of $\varphi$ and $s = \sqrt{2}|S_{+}|$

$$V(\varphi, s) = V_F(\varphi, s) + V_D(s)$$

$$= \frac{\lambda^2}{4} s^2 \varphi^2 + \frac{g^2}{8} (s^2 - 2\xi^2) + O(s^{2n}\varphi^2), \quad n \geq 2.$$  \hfill (5)

Note that, due to the shift symmetry of the Kähler potential, the Planck suppressed terms are also only quadratic in $\varphi$. The scalar potential contains higher powers in $\text{Re}(\Phi)$, which we have neglected since they are not important for inflation.

Following Ref. [16], we solve the classical equations of motion for homogeneous fields, corresponding to the scalar potential (5),

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{\lambda^2}{2} s^2 \varphi = 0,$$

$$\ddot{s} + 3H\dot{s} - \left(\frac{g^2}{2} - \frac{\lambda^2}{2} \varphi^2\right)s + \frac{g^2}{2} s^3 = 0. \quad (6)$$

The initial conditions for the waterfall field are obtained by considering the tachyonic growth of its quantum fluctuations [23–26] close to the critical point $\varphi_c$,

$$\langle s^2(t) \rangle = \int_0^{k_s(t)} dk \frac{k^2}{2\pi} e^{-3H_ct}|s_k(t)|^2. \quad (7)$$

Here $s_k(t)$ are the momentum modes of the field operator in an exponentially expanding, spatially flat background with Hubble parameter $H_c = H(\varphi_c)$ and a time-dependent inflaton field $\varphi(t) = \varphi_c + \dot{\varphi}_c t$ [24],

$$\ddot{s}_k + \left(k^2 e^{-2H_c t} - \frac{9}{4} H_c^2 - D^2 t\right)s_k = 0. \quad (8)$$

The integration in Eq. (7) extends over all soft momentum modes below $k_s(t)$ where the time-dependent mass operator for $s_k(t)$ in the brackets of Eq. (8) vanishes. At a decoherence time $t_{\text{dec}} \approx \frac{3 \ln(2R_{\text{dec}}/A)}{4}^{2/3} / D$, where $R_{\text{dec}} \approx 100$ and $D = (\sqrt{2\xi} g_l |\varphi_l|/\lambda)^{1/2}$, the waterfall field becomes classical. Matching the variance and classical field near the decoherence time, $s(t) \equiv \langle s^2(t) \rangle^{1/2}$, one obtains $s$ and $\dot{s}$ at $t = t_{\text{dec}}$. As shown in Ref. [16], the classical waterfall field reaches the local, inflaton-dependent minimum soon after the decoherence time,

$$s_{\text{min}}^2(\varphi) = 2\xi - (\lambda^2 / g^2) \varphi^2. \quad (9)$$

and, together with the inflaton field, it reaches the global minimum after a large number of $e$-folds.

On the inflationary trajectory, the inflaton potential takes a simple form,

$$V_{\text{inf}}(\varphi) = V(\varphi, s_{\text{min}}(\varphi)) = g^2 \xi \frac{\varphi^2}{\varphi_c^2} \left(1 - \frac{1}{2} \frac{\varphi^2}{\varphi_c^2}\right), \quad \varphi \leq \varphi_c. \quad (10)$$

For small $\varphi$, the potential is quadratic, and as $\varphi$ approaches $\varphi_c$, the potential reaches the plateau $g^2 \xi^2 / 2$. Figure 1 shows the potential for a certain choice of parameters. As we shall see in the following sections, in the relevant parameter range, the predictions for $n_s$ and $r$ only depend on the potential (10). The initial conditions, in particular the initial value of $\varphi$ and the tachyonic growth of the waterfall field, only affect the total number of $e$-folds and the formation of cosmic strings.

### III. COSMOLOGICAL OBSERVABLES

In this section we analyze the implications of the constraints on the cosmological observables $n_s$ and $r$ by the Planck data on the parameters of the inflaton potential (10). Obviously, the potential only depends on two parameters, which can be chosen as

![Figure 1](color online). Effective inflaton potential in subcritical hybrid inflation (solid line) normalized to $M^4_{\text{GUT}}$, with $M_{\text{GUT}} = 2 \times 10^{16}$ GeV. For reference, a quadratic potential is shown (dashed line). $\varphi_r$, $\varphi_f$, and $\varphi_t$ are the inflaton field values at the beginning of the waterfall transition, beginning and end of the last $50 \ e$-folds of inflation. Parameters: $\lambda = 7 \times 10^{-4}$, $M_{\text{inf}} = 1.95 \times 10^{16}$ GeV. (See also Fig. 3 in Ref. [16]).
where we have used \( q_r^2 = 2\sqrt{2}M_{	ext{inf}}^2/\lambda^2 \). Then the energy density of the plateau is given by \( V_{\text{inf}}(\phi_e) = M_{\text{inf}}^4 \).

Scalar spectral index and tensor-to-scalar ratio are conveniently expressed in terms of the slow-roll parameters

\[
e(\phi) = \frac{1}{2}\left(\frac{V_{\text{inf}}}{V_{\text{inf}}'}\right)^2, \quad \eta(\phi) = \frac{V_{\text{inf}}''}{V_{\text{inf}}},
\]

where the superscript “prime” denotes the derivative with respect to \( \phi \) and we have set the Planck mass \( M_{\text{pl}} = 1 \).

Inflation ends at \( \phi = \phi_f \) which is defined by \( \max\{e(\phi_f),|\eta(\phi_f)|\} = 1 \). The number of e-folds between \( t_s \) and \( t_f \) can then be expressed as

\[
N_e = \int_{t_s}^{t_f} dtH = \int_{\phi_f}^{\phi_e} dp\frac{1}{\sqrt{2e(\phi)}},
\]

where \( \phi_e = \phi(t_s) \). Solving this equation, one obtains \( \phi_e \) in terms of \( N_e \),

\[
\phi_e^2 = 4N_e + 2 - \sum_{n \geq 1} \frac{a_n}{q_{cn}},
\]

where \( a_1 = 4(N_e^2 + N_e + 1), \quad a_2 = (2/3)(2N_e^2 - 3)(2N_e + 3), \quad a_3 = -(4/3)(N_e^2 + 2N_e^2 + 6N_e^2 - 3), \ldots \). The first three terms in the expansion (14) yield \( \phi_e \) to sufficient accuracy. Together with the standard expressions for \( n_s \) and \( r \),

\[
n_s = 1 + 2\eta_e - 6\epsilon_e, \quad r = 16\epsilon_e,
\]

where \( \epsilon_e = e(\phi_e) \) and \( \eta_e = \eta(\phi_e) \), this yields \( n_s \) and \( r \) for a given number of e-folds \( N_e \).

Finally, a crucial observable is the amplitude of the scalar power spectrum \( A_s \),

\[
A_s = \frac{V_{\text{inf}}(\phi_e)}{24\pi^2 \epsilon_e},
\]

which is determined as \( A_s = 2.196^{+0.051}_{-0.060} \times 10^{-9} \) at 68% C.L. \cite{27} from the combined data sets of the Planck and WMAP collaborations. Imposing the central value of \( A_s \) as constraint yields a line in the \( \lambda - M_{\text{inf}} \) plane for a given number of e-folds. The result is shown in Fig. 2 for \( N_e = 60 \) and 50. The shaded region is consistent with the constraints (1) of the Planck data on \( n_s \) and \( r \). The energy scale of the plateau is rather precisely determined,

\[
1.6(1.9) \leq \left(\frac{M_{\text{inf}}}{10^{16} \text{ GeV}}\right) \leq 2.4(2.2),
\]

for \( N_e = 60(50) \). It is very remarkable how accurately the energy scale \( M_{\text{inf}} \) of the plateau agrees with the energy scale \( M_{\text{GUT}} \) of gauge coupling unification in the supersymmetric standard model. For comparison, Fig. 2 also shows the allowed region in the \( \lambda - \phi_e \) plane. The allowed values of \( \phi_e \), and also \( \phi_f \), are super-Planckian, similar to chaotic inflation.\(^2\)

Varying \( \lambda \) yields a line also in the \( r - n_s \) plane for a given number of e-folds. In Fig. 3 the result is compared with various constraints from CMB data and the prediction of natural inflation \cite{2}. As one can see, subcritical hybrid inflation and natural inflation \cite{5,28} yield qualitatively similar predictions. This is not surprising, given the similarity of the potential (10) to a cosine potential.\(^3\)

\(^2\)Note that for \( \lambda \gtrsim 10^{-4} \) the treatment of the initial tachyonic growth of the waterfall field is consistent, while \( \lambda \lesssim 10^{-3} \) is small enough to allow for 60 e-folds below the critical point \cite{16}.

\(^3\)A similar potential can be obtained in chaotic inflation with nonminimal coupling to gravity \cite{29}. For a recent discussion of universality classes for models of inflation, see Ref. \cite{30}.
The interpretation, however, is very different. In natural inflation the band is obtained by varying a super-Planckian axion decay constant or, as in aligned two-axion models [31], the ratio of sub-Planckian decay constants. On the contrary, in subcritical hybrid inflation, different points of the band correspond to different values of the ratio of a small Yukawa coupling and a gauge coupling \( \lambda \). The lower bound from the Planck data on the spectral index \( s \) which implies the lower bound on the tensor-to-scalar ratio \( r \) for 60(50) e-folds is blown up to \( 1/H_0 \), the size of the present Universe. We thus obtain the lower bound on the average cosmic string distance

\[
d_{cs}(t_0) > e^{380} \frac{1}{H_0}. \tag{20}
\]

Hence, cosmic strings are unobservable in subcritical hybrid inflation for parameters consistent with the Planck data.

IV. CONCLUSIONS

We have studied subcritical hybrid inflation, which occurs in supersymmetric D-term inflation for small couplings of the inflaton to matter. The effective inflaton potential interpolates between a quadratic potential at small field values and a plateau at large field values. It is characterized by two parameters, the energy scale of the plateau and the critical value of the inflaton field, at which the plateau is reached.

The model can accommodate the Planck data very well, and it is striking how accurately the energy scale \( M_{\text{inf}} \) of inflation agrees with the scale \( M_{\text{GUT}} \) of gauge coupling unification in the supersymmetric standard model. This reopens the question on the possible connection between grand unification and inflation.

The predictions for the scalar spectral index and tensor-to-scalar ratio are qualitatively similar to those from natural inflation. Quantitatively, however, the predicted values for the tensor-to-scalar ratio are larger, and one obtains the lower bounds \( r > 0.049(0.085) \) for 60(50) e-folds before the end of inflation, which is in reach of upcoming experiments.

Acknowledgements

We are grateful to Valerie Domcke and Kai Schmitz for valuable discussions and support. We also thank Rose Lerner, Alexander Westphal, and Clemens Wieck for helpful comments on the manuscript. This work has been supported in part by the German Science Foundation within the Collaborative Research Center 676 “Particles, Strings and the Early Universe.”
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PHYSICAL REVIEW D 91, 081302(R) (2015)