Grand Unification & the Early Universe

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Reasons for Grand Unified Theories

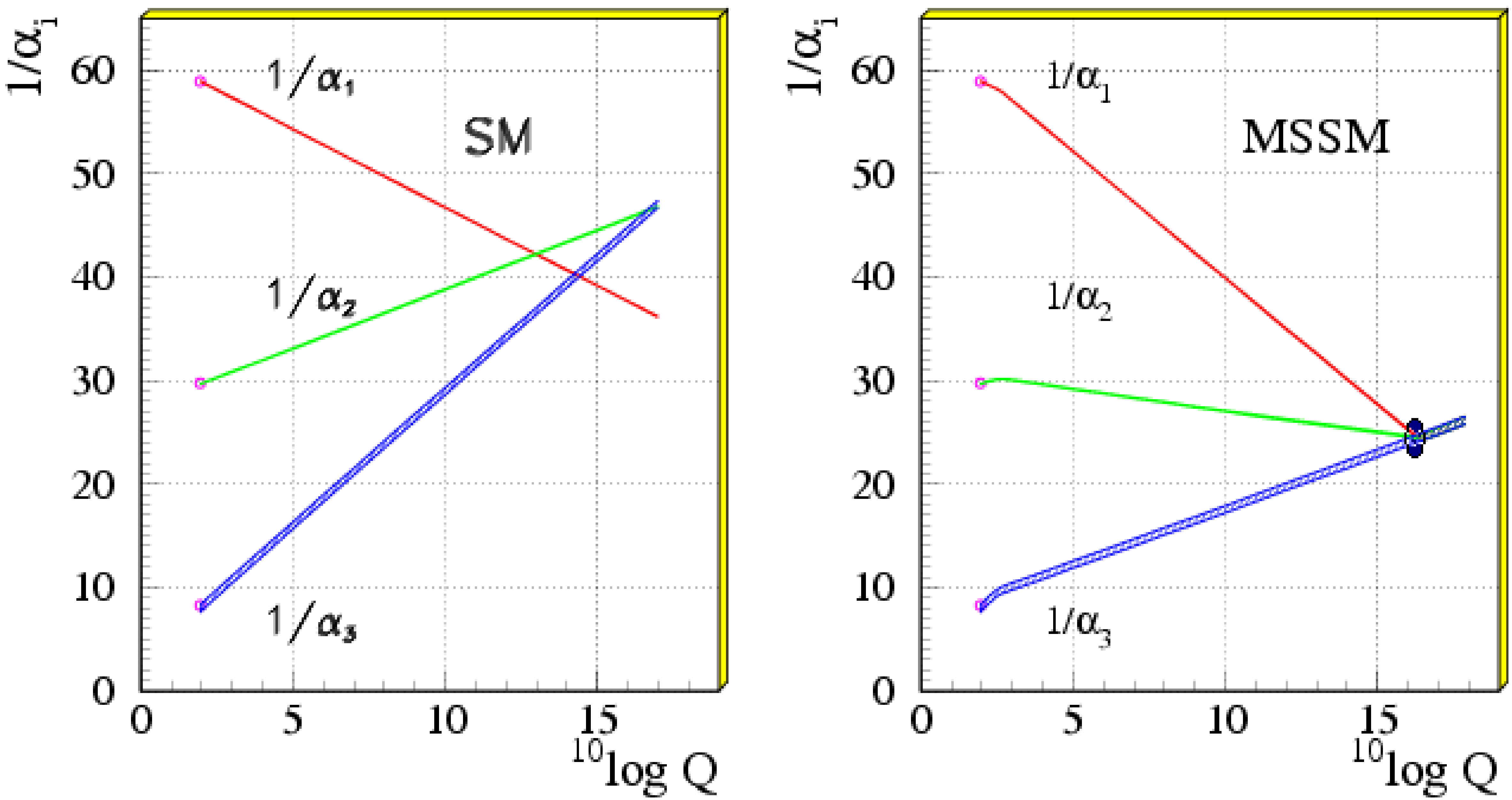
Forces and matter of the Standard Model naturally fit into the 'unification' (GUT) group $SU(5)$ [Georgi, Glashow '74], or larger extensions,

$$SU(5) = \begin{pmatrix} SU(3) & * \\ * & SU(2) \end{pmatrix} ,$$

with quarks and leptons represented by **10**, $\mathbf{5}^*$ and $(\mathbf{1} = \nu^{\mathbf{c}})$,

$$\begin{pmatrix} 0 & u^c & u^c & u & d \\ & 0 & u^c & u & d \\ & & 0 & u & d \\ & & & 0 & e^c \\ & & & & 0 \end{pmatrix} \oplus \begin{pmatrix} d^c \\ d^c \\ d^c \\ \nu \\ e \end{pmatrix} \oplus (\nu^c) \; .$$

Hope: extrapolation of SM possible up to the GUT energy scale $\Lambda_{\mathrm{GUT}} \sim 10^{15}$ GeV where weak, electromagnetic and strong forces have equal strength.



Effective interactions strengths (couplings α_i) depend on energy and momentum transfer; unification at GUT scale in SM and supersymmetric extension; hint for supersymmetry? Search at LHC, DM, ... !!

Grand Unification & Cosmology

- `First' model of inflation [Guth, '80]: inflation driven by false GUT vacuum energy (problem with subsequent phase transition ...)

- GUT baryogenesis: heavy leptoquark decay (gauge couplings too large, not enough departure from thermal equilibrium ... [Kolb, Wolfram '80])

- Potential problem: formation of topological defects in GUT phase transition, monopoles domain walls, strings, ...

- Theoretical developments: new ways of GUT symmetry breaking, Higgs-type phase transitions only for Abelian subgroups (only cosmic strings; SUSY GUTs and extra dimensions: new inflaton & dark matter candidates; `sphalerons', allows for leptogenesis, natural for GUT scale heavy neutrinos

- Attractive framework: hybrid inflation with with B-L breaking (difference of baryon and lepton number)[Linde; Dvali, et al; Binetruy, Dvali; Halyo; all '94]; potential observables: strings, gravity waves, CMB (large field inflation), ...

Seesaw & Leptogenesis

Lepton number can be violated by masses and couplings of heavy Majorana neutrinos to light neutrinos and charged leptons; after electroweak symmetry breaking, 3 light and 3 heavy neutrinos (seesaw mechanism):

$$N \simeq \nu_R + \nu_R^c \,, \qquad \nu \simeq V_\nu^T \nu_L + \nu_L^c V_\nu^*$$

$$m_N \simeq M \,, \qquad m_\nu \simeq -V_\nu^T m_D^T \frac{1}{M} m_D V_\nu$$

For hierarchical right-handed neutrinos, after electroweak symmetry breaking, light neutrino masses naturally related to the GUT scale:

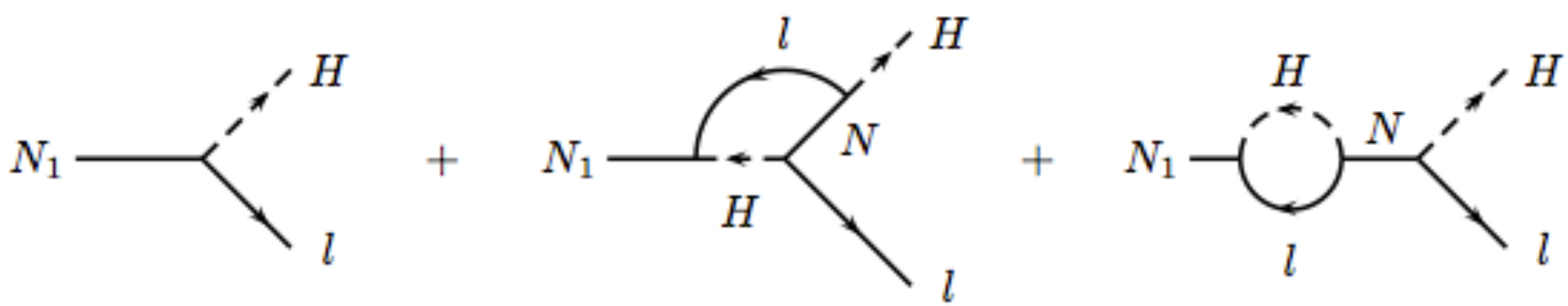
$$M_3 \sim \Lambda_{\rm GUT} \sim 10^{15}\ \mathrm{GeV} \,, \quad m_3 \sim \frac{v^2}{M_3} \sim\ 0.01\ \mathrm{eV}$$

suggests that neutrino physics probes the mass scale of grand unification (but there are also other examples ...)

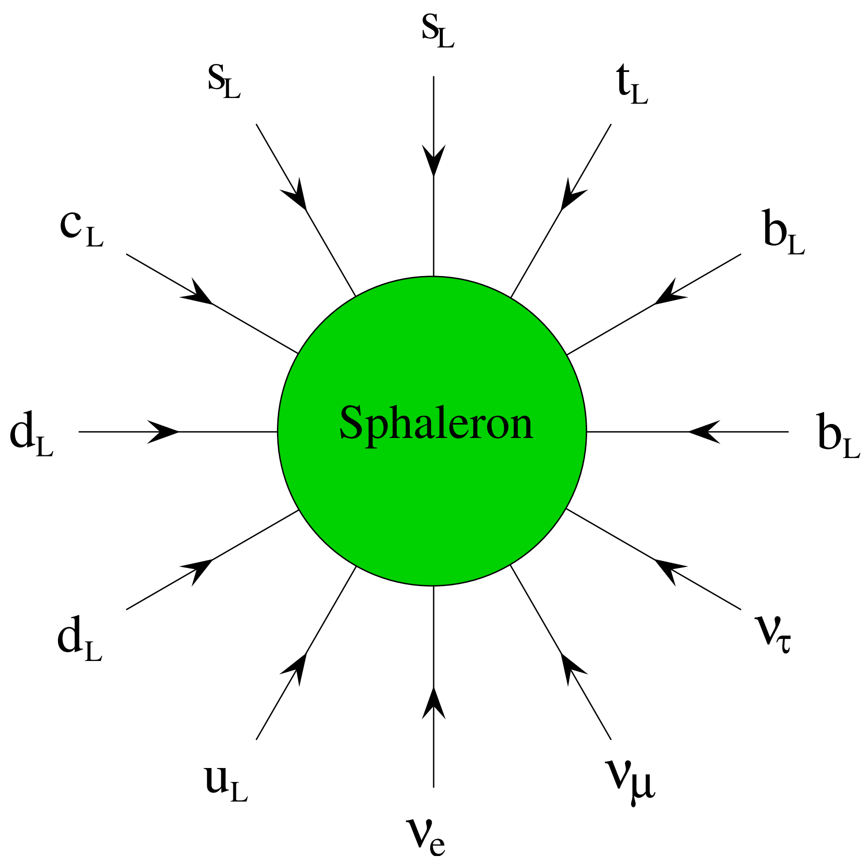
CP violating heavy neutrino decays:

$$\varepsilon_1 = \frac{\Gamma\left(N_1 \to H + l_L\right) - \Gamma\left(N_1 \to H^\dagger + l_L^\dagger\right)}{\Gamma\left(N_1 \to H + l_L\right) + \Gamma\left(N_1 \to H^\dagger + l_L^\dagger\right)}$$

$$\simeq \; -\frac{3}{16\pi}\frac{M_1}{(hh^\dagger)_{11} v_F^2} \mathrm{Im}\left(h^* m_\nu h^\dagger\right)_{11}$$



Leptogenesis relates successfully neutrino masses and matter-antimatter asymmetry; CP asymmetry is small: suppressed by small Yukawa couplings, and loop effect (quantum interference); natural explanation of very small observed baryon asymmetry



Rough estimate for CP asymmetry (hierarchical heavy neutrinos) and baryon asymmetry:

$$\varepsilon_1 \sim 0.1 \frac{M_1 m_3}{v_F^2} \sim 0.1 \frac{M_1}{M_3} \sim 10^{-5} \dots 10^{-6}$$

$$\eta_B \simeq -\frac{c_s}{f} N_{B-L} \simeq 10^{-2} \varepsilon_1 \kappa_f$$

Neutrino masses suggest that leptogenesis is process close to thermal equilibrium, i.e. $\Gamma_1 \sim H|_{T=M_1}$; in terms of neutrino masses:

$$\widetilde{m} = \frac{(m_D m_D^\dagger)_{11}}{M_1} \sim m_* = \frac{16\pi^{5/2}}{3\sqrt{5}} g_*^{1/2} \frac{v_F^2}{M_{\mathrm{P}}} \simeq 10^{-3}\ \mathrm{eV}$$

confirmed by solution of Boltzmann equations; good quantitative understanding; significant progress in QFT description

Leptogenesis & Supersymmetry

[WB, Domcke, Kamada, Schmitz '13, '14]

Leptogenesis & gravitinos: for (thermal) leptogenesis and typical superparticle masses, thermal production yields observed amount of dark matter:

$$\Omega_{3/2}h^2 = C \left(\frac{T_R}{10^{10}\,\mathrm{GeV}}\right)\left(\frac{100\,\mathrm{GeV}}{m_{3/2}}\right)\left(\frac{m_{\tilde{g}}}{1\,\mathrm{TeV}}\right)^2 \ , \quad C \sim 0.5$$

$\Omega_{3/2}h^2 \sim 0.1$ is natural value; but why is reheating temperature close to minimal LG temperature?

Simple observation: heavy neutrino decay width (for typical LG parameters)

$$\Gamma_{N_1} = \frac{\widetilde{m}_1}{8\pi}\left(\frac{M_1}{v_\mathrm{F}}\right)^2 \sim 10^3\ \mathrm{GeV}\ , \quad \widetilde{m}_1 \sim 0.01\ \mathrm{eV}\ , \quad M_1 \sim 10^{10}\ \mathrm{GeV}$$

yields reheating temperature (for gas of decaying heavy neutrinos)

$$T_R \sim 0.2 \cdot \sqrt{\Gamma^0_{N_1} M_P} \sim 10^{10}\ \mathrm{GeV}$$

wanted for gravitino DM. *Intriguing hint or misleading coincidence?*

Spontaneous B-L breaking & false vacuum decay

Supersymmetric SM with right-handed neutrinos:

$$W_M = h^u_{ij} \mathbf{10}_i \mathbf{10}_j H_u + h^d_{ij} \mathbf{5}^*_i \mathbf{10}_j H_d + h^\nu_{ij} \mathbf{5}^*_i n^c_j H_u + h^n_i n^c_i n^c_i S_1$$

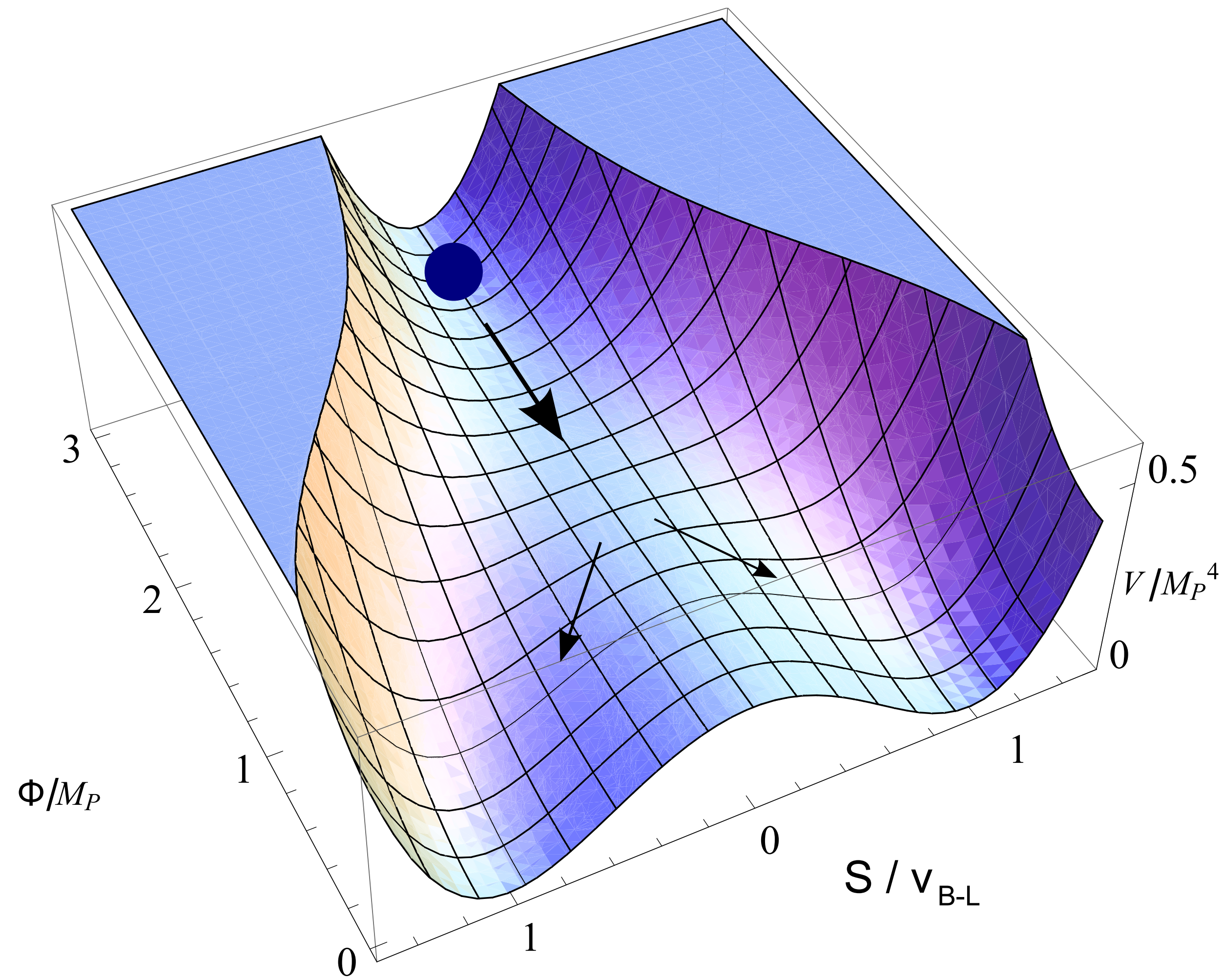
in SU(5) notation: $\mathbf{10} \supset (q, u^c, e^c),\ \mathbf{5}^* \supset (d^c, l),\ n^c \supset (\nu^c)$; B-L breaking:

$$W_{B-L} = \lambda \Phi \left(\frac{1}{2} v^2_{B-L} - S_1 S_2 \right)$$

$\langle S_{1,2} \rangle = v_{B-L}/\sqrt{2}$ yields heavy neutrino masses.

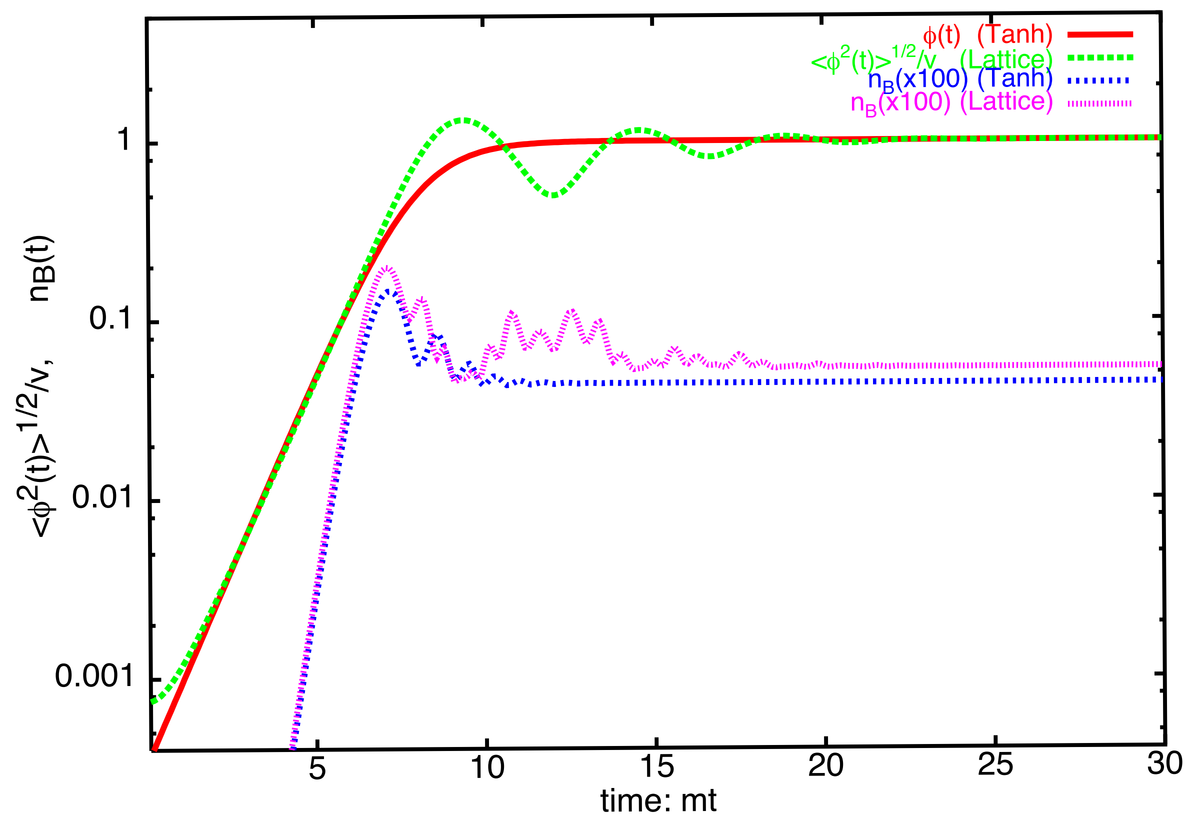
Lagrangian is determined by low energy physics: quark, lepton, neutrino masses etc, but it *contains all ingredients wanted in cosmology*: inflation, leptogenesis, dark matter, ..., all related! [F-term hybrid inflation: Copeland et al. '94, Dvali et al. '94].

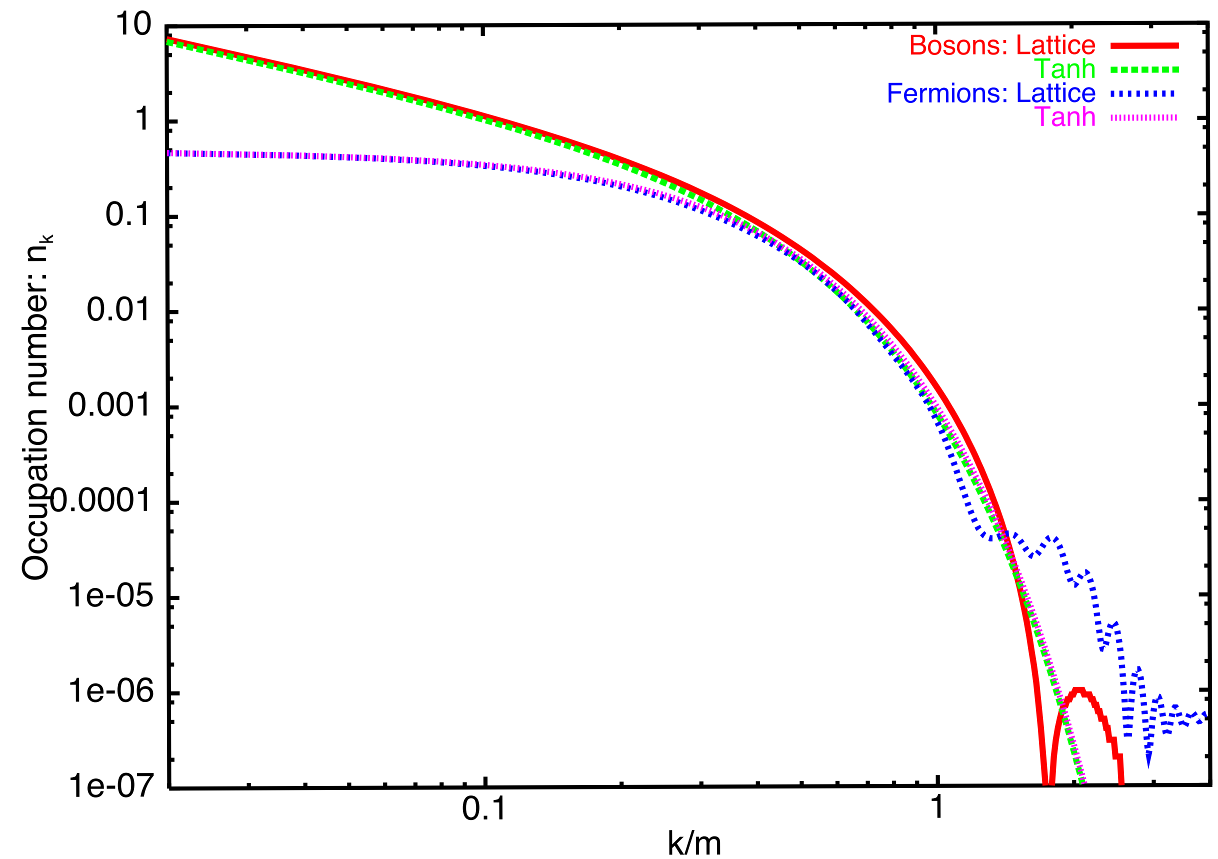
Technically: Abelian Higgs model in unitary gauge; inflation ends with phase transition ("tachyonic preheating", "spinodal decomposition")



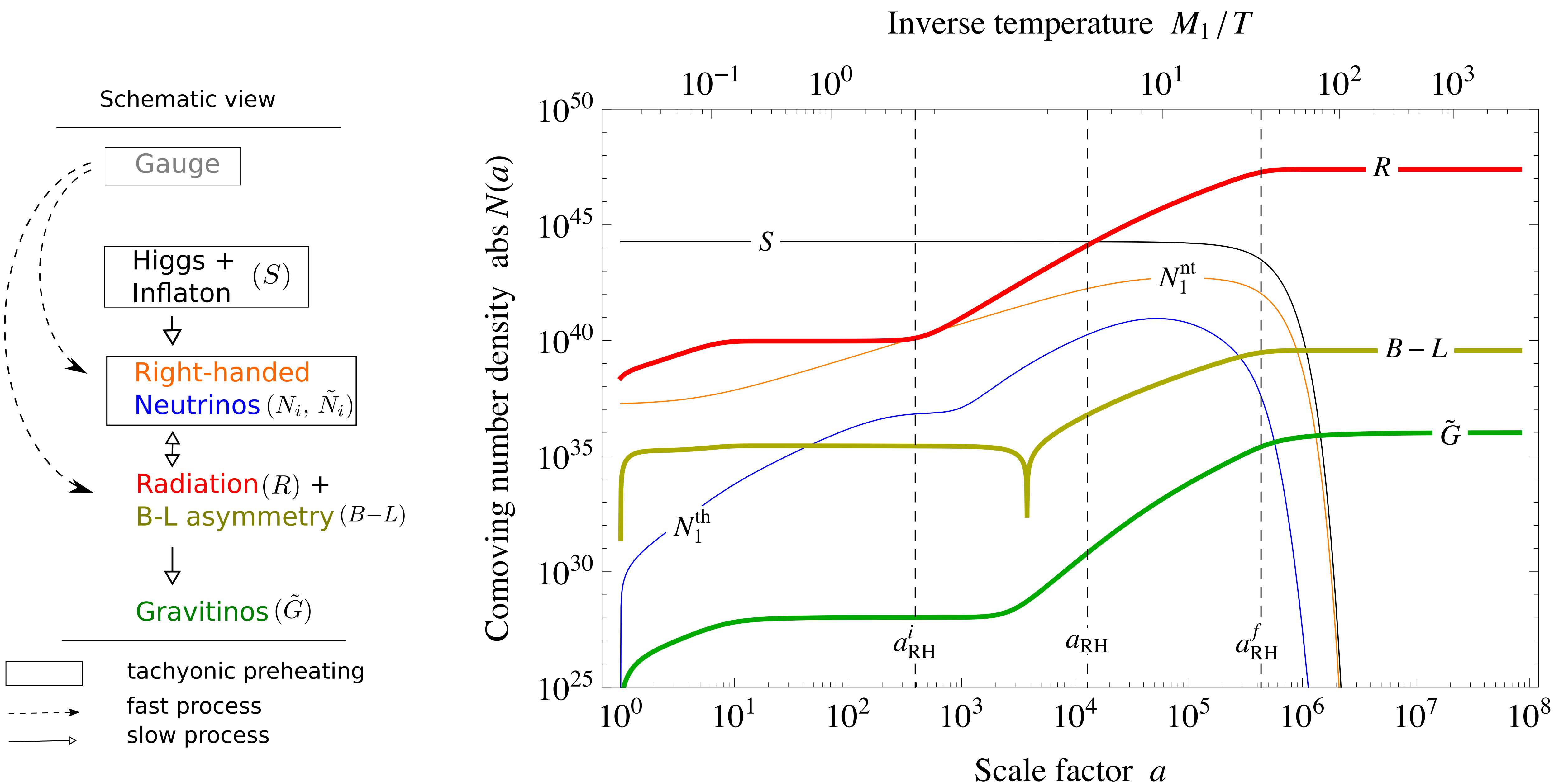
time-dependent masses of B-L Higgs, inflaton, heavy neutrinos ... (bosons and fermions):

$$m_\sigma^2 = \frac{1}{2}\lambda(3v^2(t) - v_{B-L}^2)\ , \quad m_\phi^2 = \lambda v^2(t)\ , \quad M_i^2 = (h_i^n)^2 v^2(t)\ \ldots$$





Rapid transition from false to true vacuum by fluctuations of 'waterfall' B-L Higgs field; production of low momentum Higgs bosons (contain most energy), also other bosons and fermions coupled to B-L Higgs field [Garcia-Bellido, Morales '02], production of cosmic strings: initial conditions for reheating



Transition from end of inflation to hot early universe (typical parameters), calculated by means of Boltzmann equations:

$$E\left(\frac{\partial}{\partial t} - Hp\frac{\partial}{\partial p}\right) f_X(t,p) = \sum_{i'j'..}\sum_{ij..} C_X(Xi'j'.. \leftrightarrow ij..)$$

yields correct baryon asymmetry and dark matter abundance

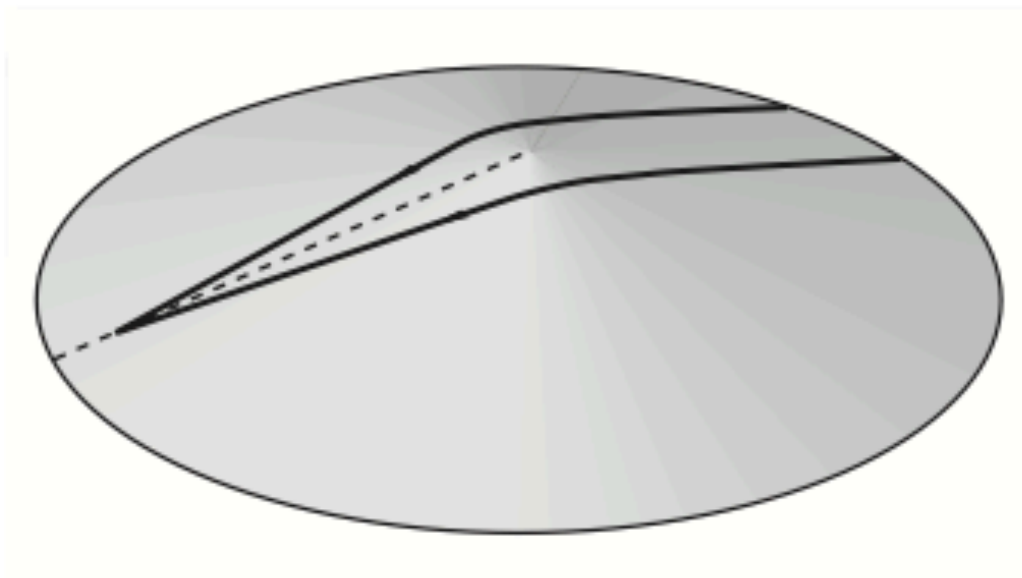


Fig. 1. The spacetime around a cosmic string is conical, as if a narrow wedge were removed from a flat sheet and the edges identified. For this reason cosmic strings can create double images of distant objects. Strings moving across the line of sight will cause line-like discontinuities in the CMB radiation.

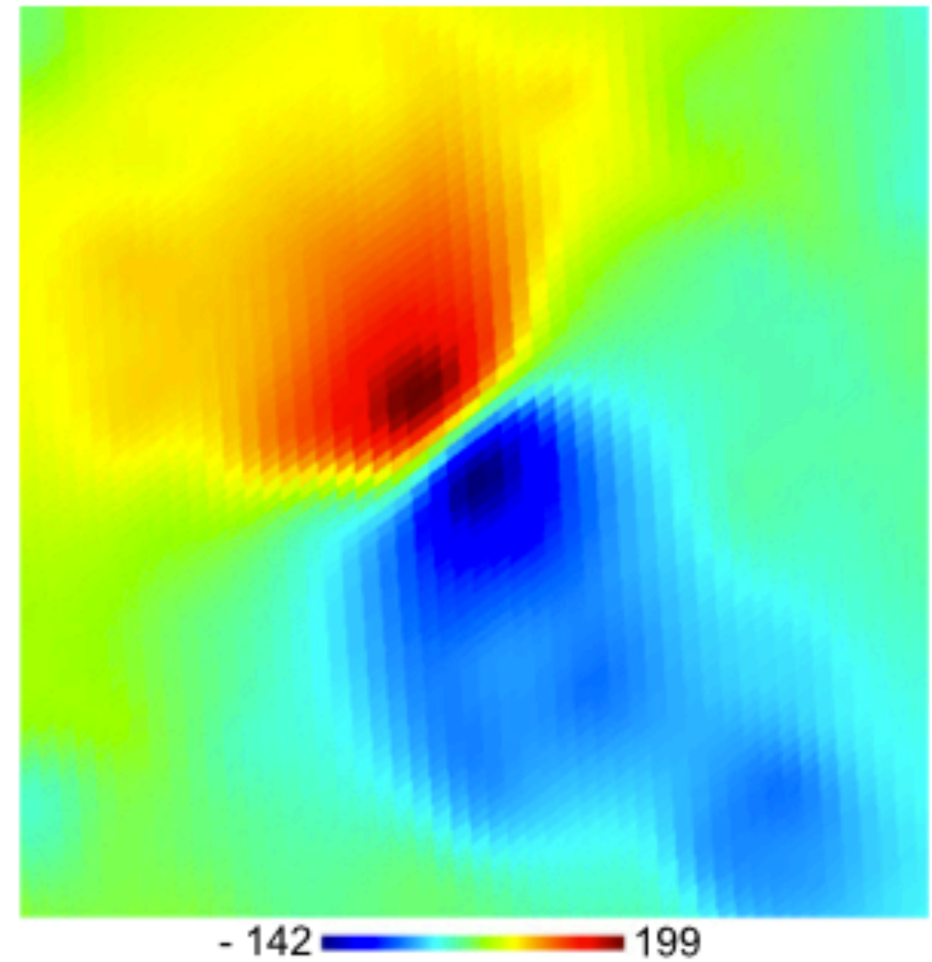


Fig. 2. Characteristic CMB temperature discontinuity created by a cosmic string. Here, the simulated Nambu-Goto string has produced a cusp, a small region on the string that approaches the speed of light, which has generated a localised CMB signal.

from two sets of abelian-Higgs simulations are given (Moore et al. 2002; Bevis et al. 2007b). The evolution can be fitted with a VOS model Eq. (6) with $\tilde{c} = 0.57$, which is 150% higher than for Nambu-Goto strings. Field theory simulations have further important applications, particularly for describing delocalised

Abelian Higgs string: $G\mu/c^2 < 3.2 \times 10^{-7}$, consistent with GUT scale strings

Gravitational Waves

Relic gravitational waves are window to very early universe; contributions from inflation, preheating & cosmic strings (see Maggiore '07, Dufaux et al '10, Hindmarsh '11); cosmological B-L breaking: prediction of GW spectrum with all contributions! Perturbations in flat FRW background:

$$ds^2 = a^2(\tau)(\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu \ , \quad \bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h^\rho_\rho$$

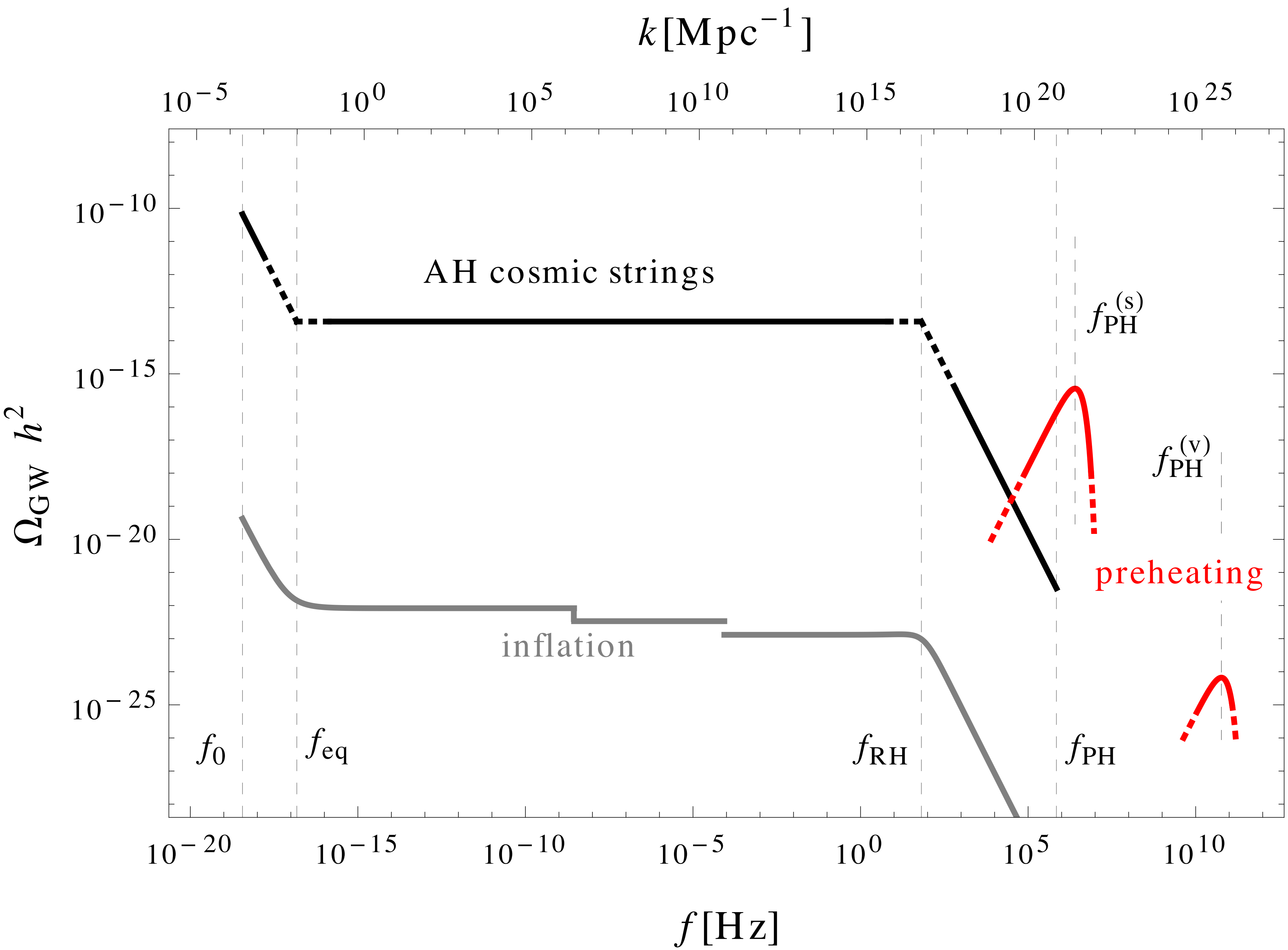
determined by linearized Einstein equations,

$$\bar{h}''_{\mu\nu}(\mathbf{x},\tau) + 2\frac{a'}{a}\bar{h}'_{\mu\nu}(\mathbf{x},\tau) - \nabla^2_{\mathbf{x}}\bar{h}_{\mu\nu}(\mathbf{x},\tau) = 16\pi G T_{\mu\nu}(\mathbf{x},\tau)$$

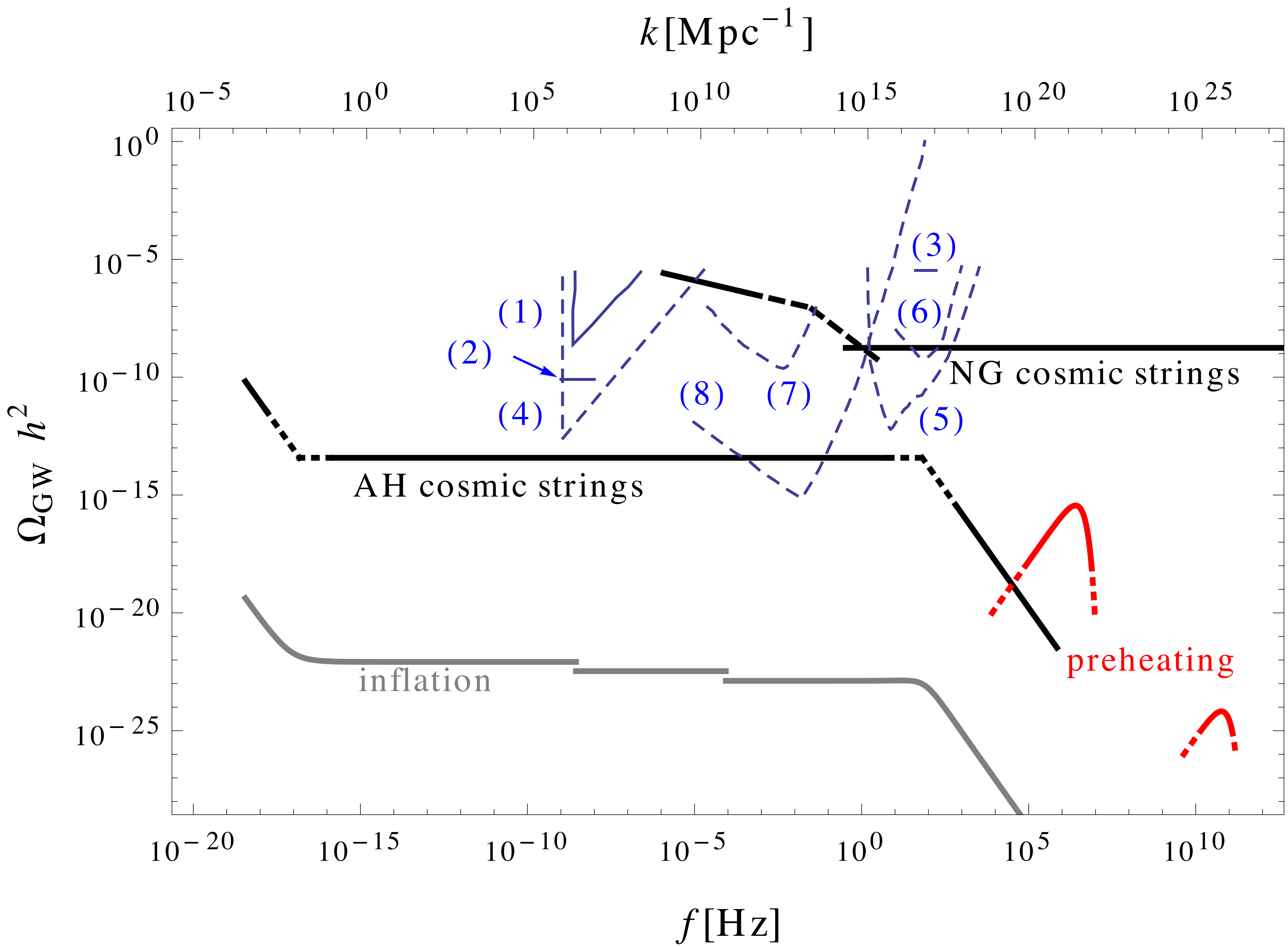
yields spectrum of GW background:

$$\Omega_{GW}(k,\tau) = \frac{1}{\rho_c}\frac{\partial\rho_{GW}(k,\tau)}{\partial\ln k} \ ,$$

$$\int_{-\infty}^{\infty} d\ln k \frac{\partial\rho_{GW}(k,\tau)}{\partial\ln k} = \frac{1}{32\pi G}\left\langle \dot{h}_{ij}(\mathbf{x},\tau)\,\dot{h}^{ij}(\mathbf{x},\tau)\right\rangle$$



Result: GWs from inflation, preheating and cosmic strings (Abelian Higgs), for typical parameters consistent with leptogenesis and dark matter; similar spectrum from inflation and cosmic strings, but different normalization!



Observational prospects (for typical parameters); 'soon': Advanced Ligo (6) [100 Hz], eLISA [0.01 Hz]; 'later': Einstein Telescope [100 Hz], BBO/DECIGO [0.01 Hz]; eventually determination of reheating temperature (leptogenesis)?!

Hybrid Inflation in the Complex Plane

Inflationary potential can be strongly affected by supersymmetry breaking (supergravity correction [WB, Covi, Delepine '00]):

$$V(\phi) = V_0 + V_{\mathrm{CW}}(\phi) + V_{\mathrm{SUGRA}}(\phi) + V_{3/2}(\phi) \ ,$$

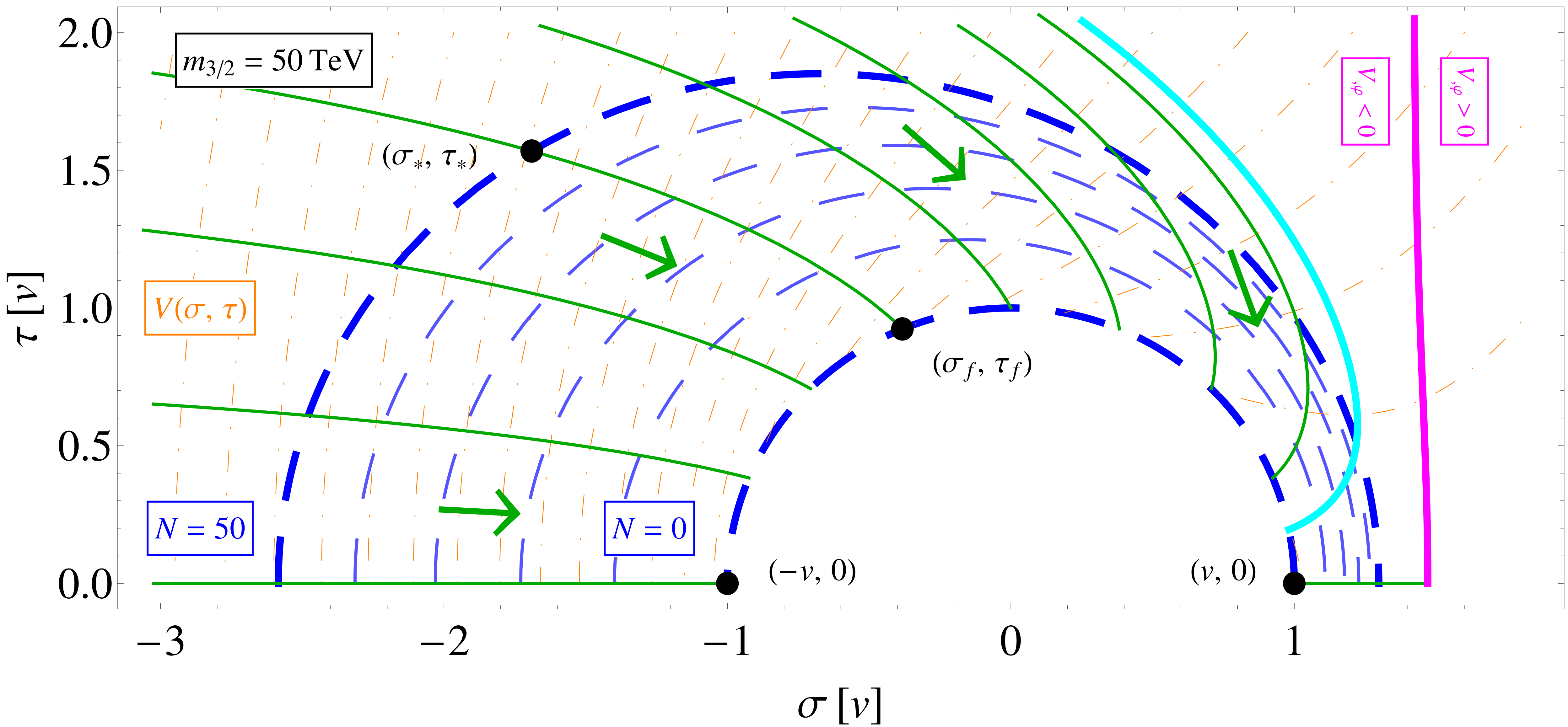
$$V_0 = \frac{\lambda^2 v^4}{4} \ ,$$

$$V_{\mathrm{CW}}(\phi) = \frac{\lambda^4 v^4}{32\pi^2} \ln\left(\frac{|\phi|}{v/\sqrt{2}}\right) + \dots \ ,$$

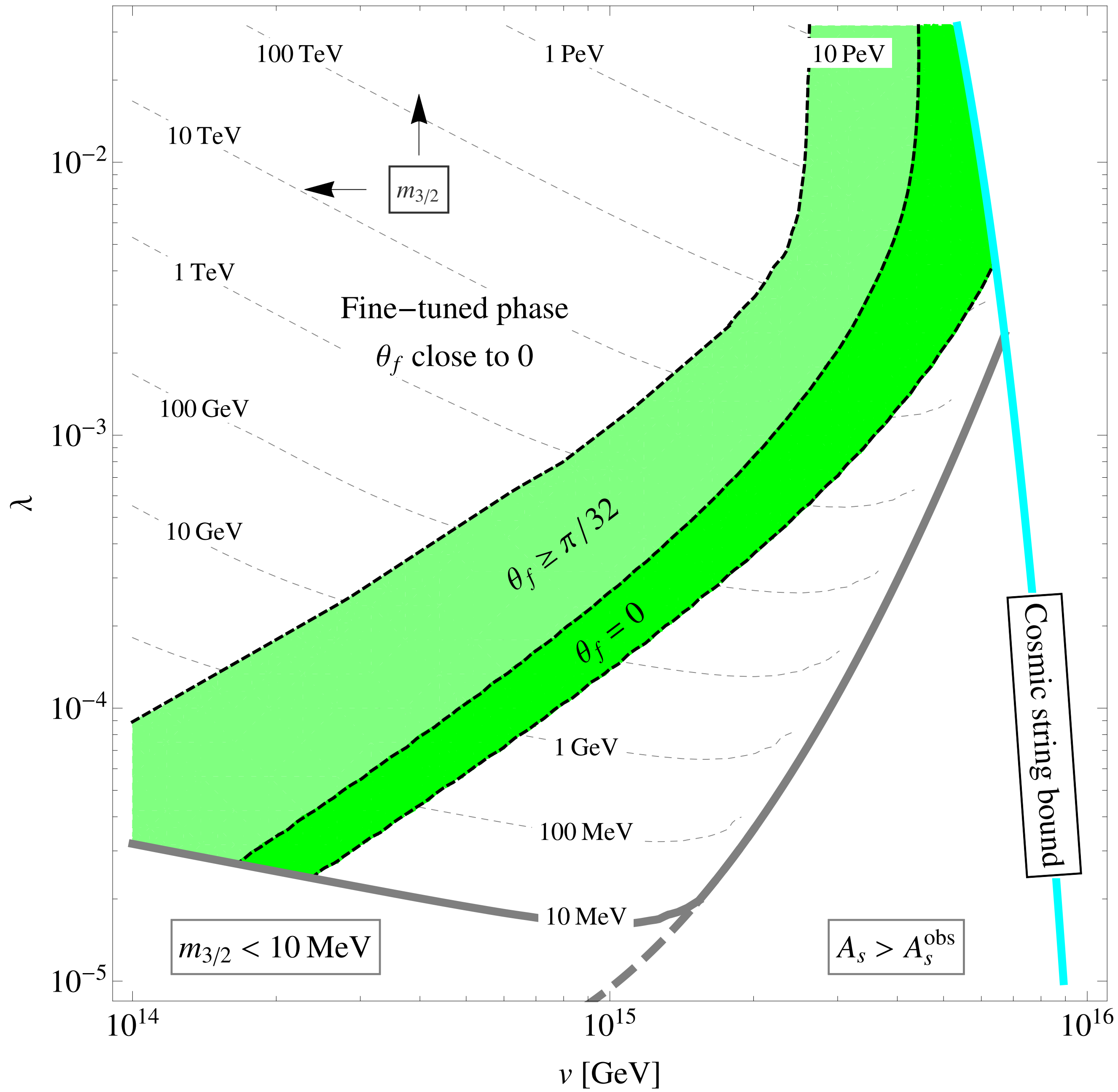
$$V_{\mathrm{SUGRA}}(\phi) = \frac{\lambda^2 v^4}{8 M_{\mathrm{P}}^4} |\phi|^4 + \dots \ ,$$

$$V_{3/2}(\phi) = -\lambda v^2 m_{3/2}(\phi + \phi^*) + \dots$$

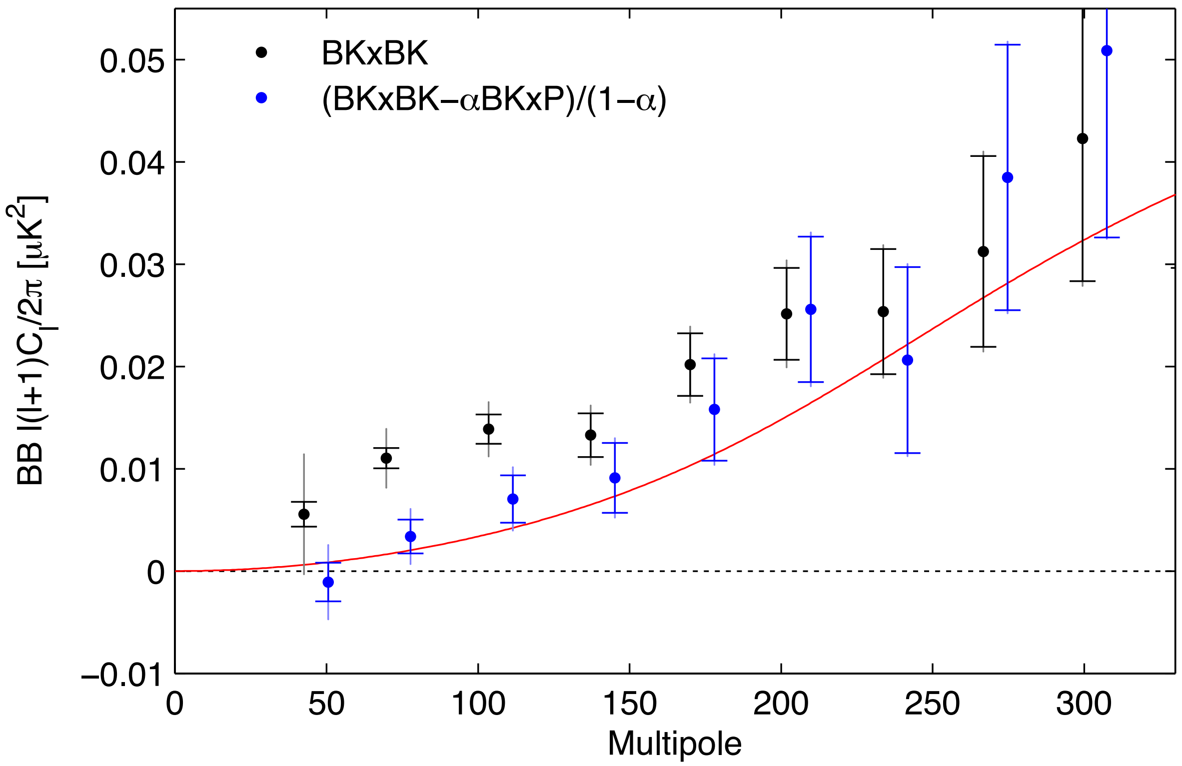
linear term turns hybrid inflation into two-field model in complex plane; strong effect on inflatonary observables, now dependent on trajectory, i.e. initial conditions! Inflation consistent with Planck data now possible (otherswise very difficult)

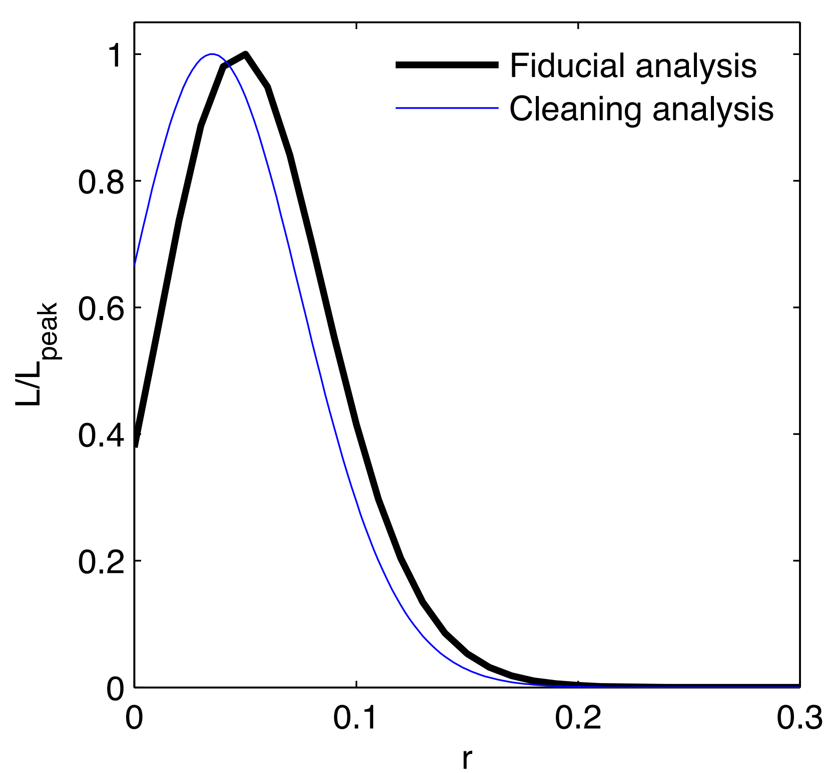


F-term hybrid inflation ($\lambda \ll 1$) with SUSY breaking: resulting linear term in inflaton potential leads to two-field dynamics of complex inflaton in field space, observables depend on inflationary trajectory, range for spectral index consistent with $n_s \simeq 0.96$



Parameter scan: relations between scale of B-L breaking, gravitino mass and scalar spectral index (correct in green band) cosmic string bound automatically fulfilled! Prediction for tensor-scalar ratio: $r \lesssim 2 \times 10^{-6}$; further predictions: characteristic spectrum of gravitational waves, connection between inflation and SUSY searches at the LHC! BUT WHAT ABOUT BICEP2?





Joint BICEP2/Keck/Planck analysis (2015)

upper: BICEP2/Keck spectrum before and after subtraction of dust contribution

lower: likelihood fit; primordial BB signal almost 2σ effect; favoured value of r about 0.5

More data this year, BB signal not yet excluded

Subcritical Hybrid Inflation

[WB, Domcke, Schmitz '14; WB, Ishiwata '14]

Version of supersymmetric D-term inflation; superpotential and Kahler potential for waterfall fields and inflaton (shift symmetry, [Yanagida et al '00]):

$$K = \frac{1}{2}(\Phi + \Phi^\dagger)^2 + |S_+|^2 + |S_-|^2 \ ,$$

$$W = \lambda \, \Phi S_+ S_- \ , \quad V_D = \frac{g^2}{2}\left(|S_+|^2 - |S_-|^2 - \xi\right)^2$$

Waterfall transition below critical point

$$\varphi_c = \frac{g}{\lambda}\sqrt{2\xi} \ ,$$

can be super-Planckian for small Yukawa couplings $\lambda/g \ll 1$ (regime used in following discussion, needed for Lyth bound); theoretical question: FI-term ξ [recent work: Wieck, Winkler '14; Domcke, Schmitz, Yanagida '14]

Inflaton potential above critical point (waterfall fields integrated out):

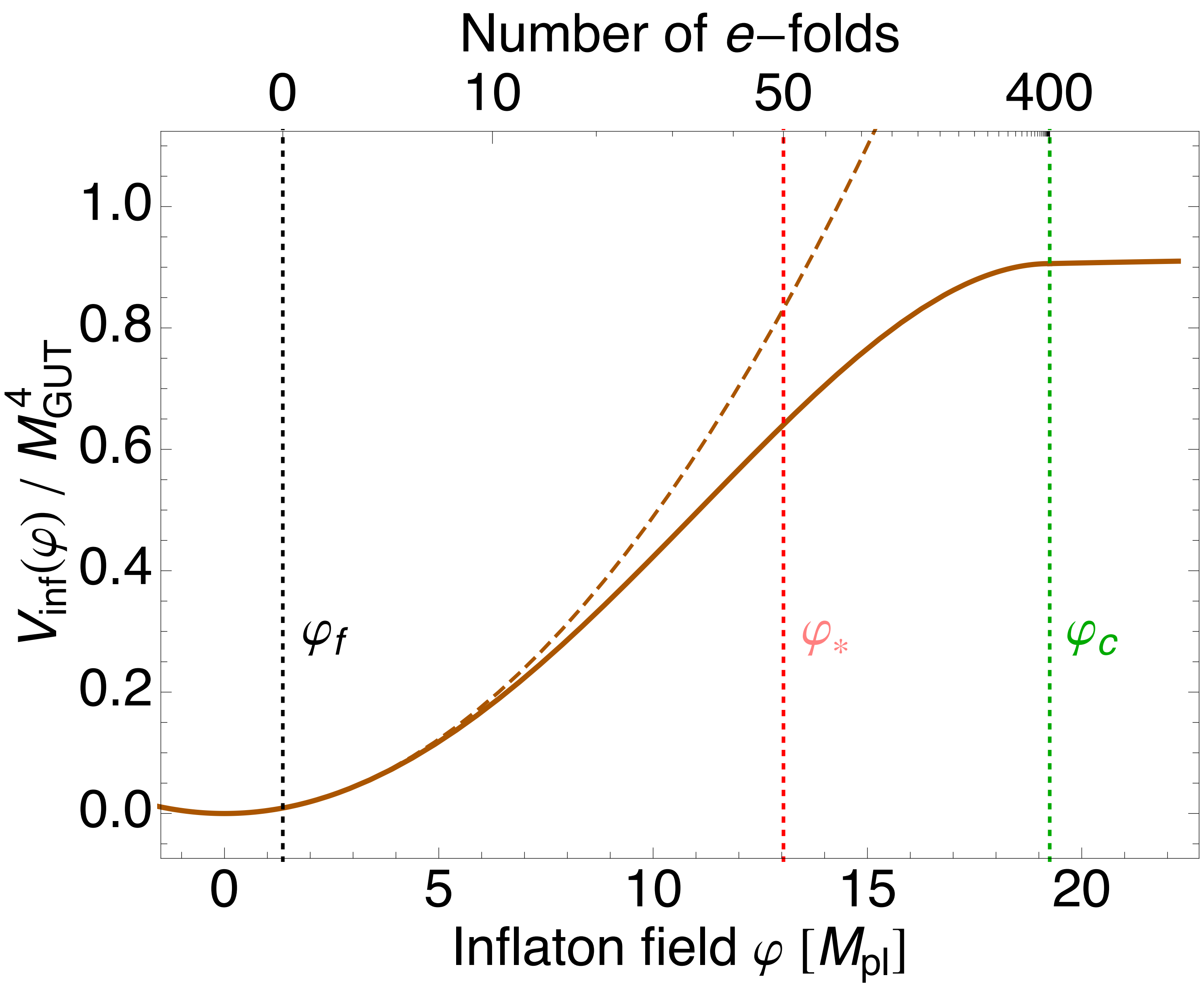
$$V(\varphi) = V_0 \left(1 + \frac{g^2}{8\pi^2} (\ln x + \dots)\right) ,$$

with $x = \lambda^2\varphi^2/(2g^2\xi)$. Below critical point scalar potential for inflaton and waterfall fields ($M_{\mathrm{P}} = 1$):

$$V(\varphi, s) = \frac{g^2}{8}(s^2 - 2\xi)^2 + \frac{\lambda^2}{4} s^2\varphi^2 + \mathcal{O}(s^4\varphi^2) .$$

Note, potential quadratic in inflaton field (consequence of shift symmetry!) Below critical point waterfall transition (spinodal decomposition; tachyonic preheating [Felder et al '00; ...]), rapid transition to global minimum. Following discussion: tachyonic growth of fluctuations for continuing exponential expansion!

Inflation can continue after critical point [Clesse '10; Clesse, Garbrecht '12], mostly models with small field inflation]; here: large field inflation after critical point, with final phase of "chaotic inflation" after tachyonic growth of fluctuations.



Below critical point φ_c inflaton potential changes from plateau to quadratic potential of "chaotic inflation"; relevant fluctuations are generated at φ_*

Tachyonic Growth of Fluctuations

Computation of quantum fluctuations beyond critical point requires mode expansion of field operators in de Sitter background:

$$\hat{\varphi}(t,\vec{x}) = \varphi(t) + e^{-\frac{3}{2}Ht}\int \frac{d^3k}{(2\pi)^{3/2}}\left(a_\varphi(\vec{k})\varphi_k(t)e^{i\vec{k}\vec{x}} + a_\varphi^\dagger(\vec{k})\varphi_k^*(t)e^{-i\vec{k}\vec{x}}\right) ,$$

$$\hat{s}(t,\vec{x}) = e^{-\frac{3}{2}Ht}\int \frac{d^3k}{(2\pi)^{3/2}}\left(a_s(\vec{k})s_k(t)e^{i\vec{k}\vec{x}} + a_s^\dagger(\vec{k})s_k^*(t)e^{-i\vec{k}\vec{x}}\right)$$

only inflaton field has classical part; close to the critical point ($t(\varphi_c) = 0$):

$$\varphi(t) \simeq \varphi_c + \dot{\varphi}_c t ,$$

$$V(s;t) \simeq \frac{1}{2}g^2\xi^2 - \frac{1}{2}D^3 t\ s^2 + \mathcal{O}(s^4;t^2) ,$$

with $D^3 = \sqrt{2\xi}g\lambda|\dot{\varphi}_c|$ given by velocity of inflaton field at critical point

Analytic estimate of variance due to growth of quantum fluctuations [Asaka, WB, Covi '01]:

$$\begin{aligned}\langle s^2(t)\rangle &= \langle s^2(t)\rangle_{\rm us} - \langle s^2(0)\rangle_{\rm us}\\ &\simeq \int_0^{k_b(t)} dk\,\frac{k^2}{2\pi^2}\,e^{-3H_c t}\,|s_k(t)|^2\\ &\simeq \frac{3^{2/3}\Gamma^2\left(\frac{1}{3}\right)}{192\pi^3} D^2(Dt)^{3/2}\exp\left(\frac{4}{3}(Dt)^{3/2}+Ht\right);\end{aligned}$$

for large times, $Dt > 1$, one has $p_b(t) > H_c$, i.e. fluctuations are generated within horizon.

For typical parameters ($g^2 = 1/2,\ \lambda = 5\times 10^{-4},\ \sqrt{\xi} = 2.8\times 10^{16}\ \mathrm{GeV}$), velocity of inflaton field and Hubble parameter

$$\dot{\varphi}_c = -\frac{\partial_\varphi V}{3H}\bigg|_{\varphi_c} = -\frac{g^2\lambda\ln 2}{4\sqrt{3}\pi^2}\sqrt{\xi}\,.$$

$$H_c \equiv H(\varphi_c) = 9.1\times 10^{13}\ \mathrm{GeV}\,,\quad \dot{\varphi}_c \simeq -21H_c^2\,,\quad D\simeq 1.5\,H_c\,.$$

Beyond decoherence time ($|s_k(t_{\rm dec})\pi_{sk}(t_{\rm dec})| \equiv R_{\rm dec} \gg \hbar \equiv 1$),

$$t_{\rm dec} \sim \frac{1}{D}\left(\tfrac{3}{4}\ln(2R_{\rm dec})\right)^{2/3} \ ,$$

soft modes ($k < k_b(t)$) generate classical waterfall field. Backreaction due to self-interaction of waterfall field can then be taken into account by means of classical field equations:

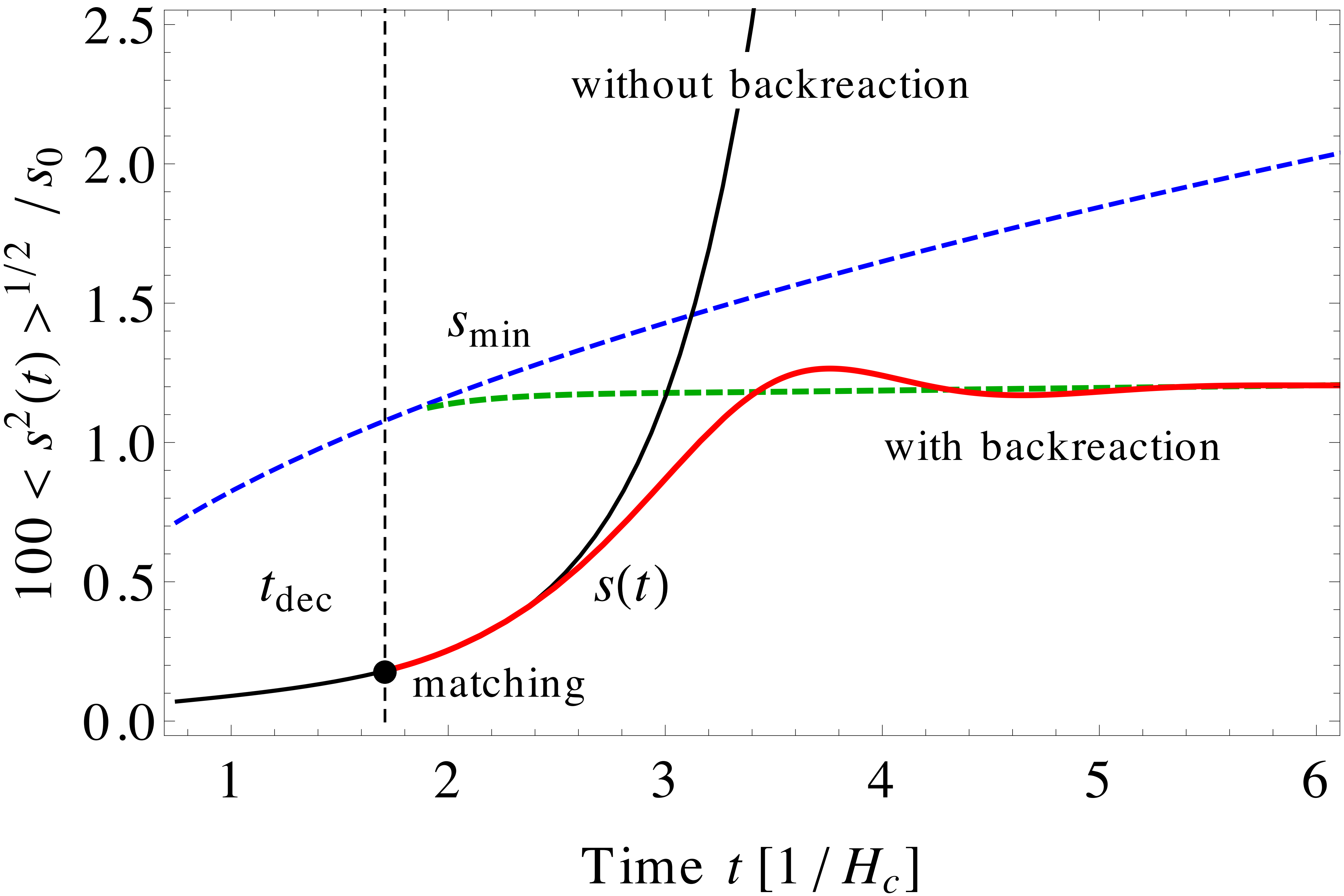
$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{1}{2}\lambda^2 s^2\varphi = 0 \ ,$$

$$\ddot{s} + 3H\dot{s} - \left(g^2\xi - \frac{\lambda^2}{2}\varphi^2\right)s + \frac{g^2}{2}s^3 = 0 \ .$$

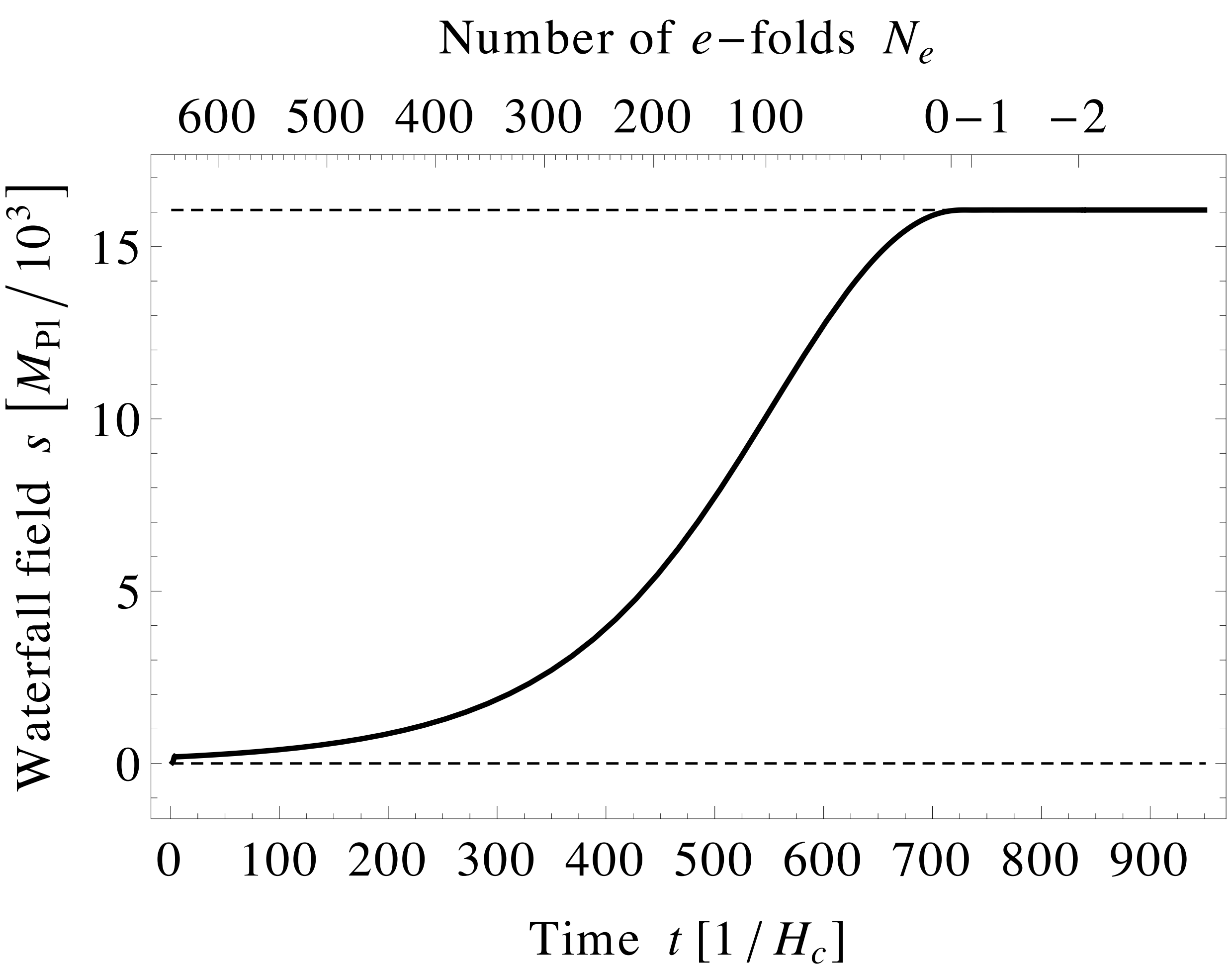
Classical waterfall field, with matching

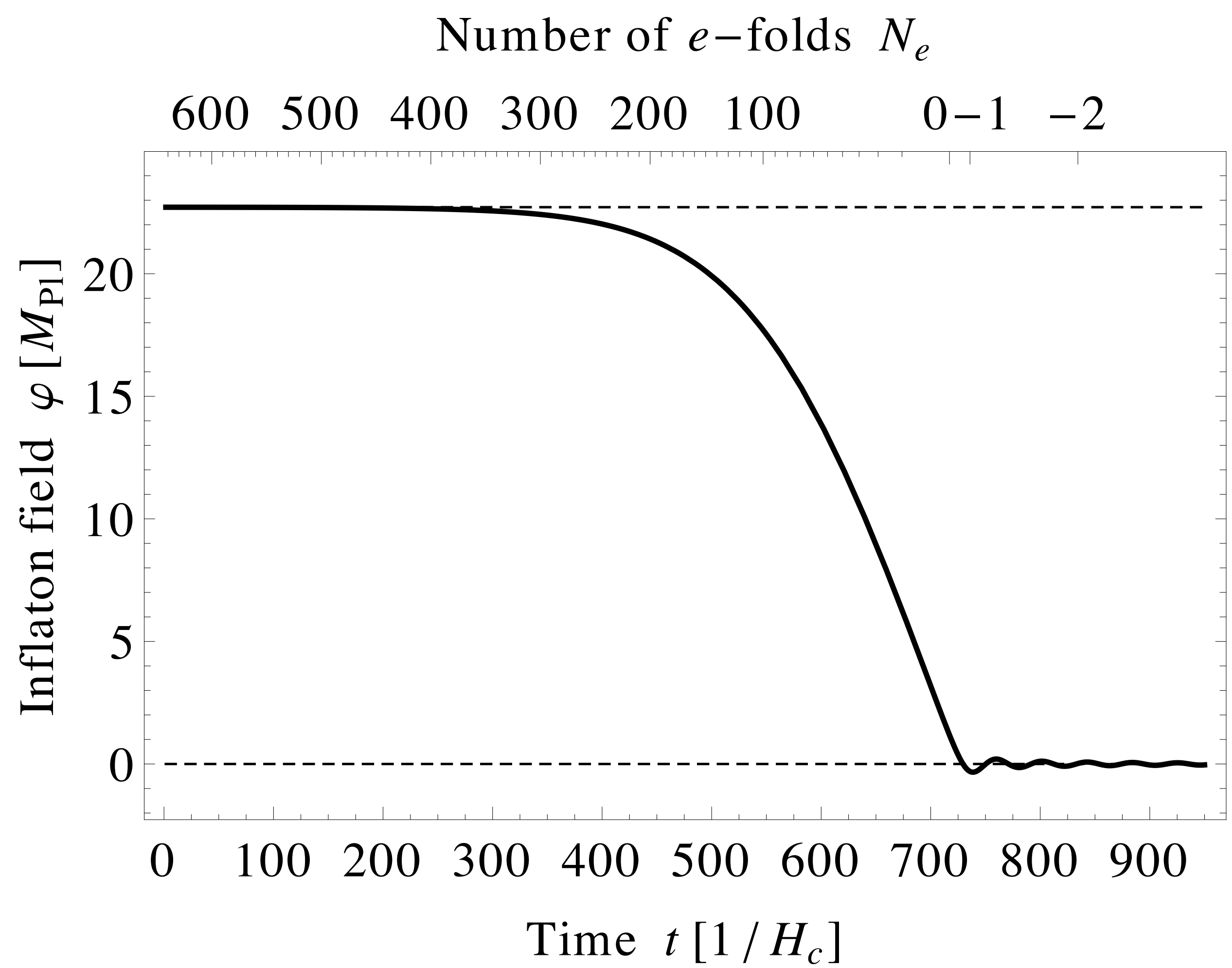
$$s(t_{\rm dec}) = \langle s^2(t_{\rm dec})\rangle^{1/2} \ ,$$

quickly reaches local minimum; approach of global minimum much later, in final stage generation of effective mass term for inflaton.



waterfall fields approaches time-dependent minimum due to backreaktion





at the end of inflation standard picture: inflaton oscillations, reheating , etc ...

“Flattened” Chaotic Inflation

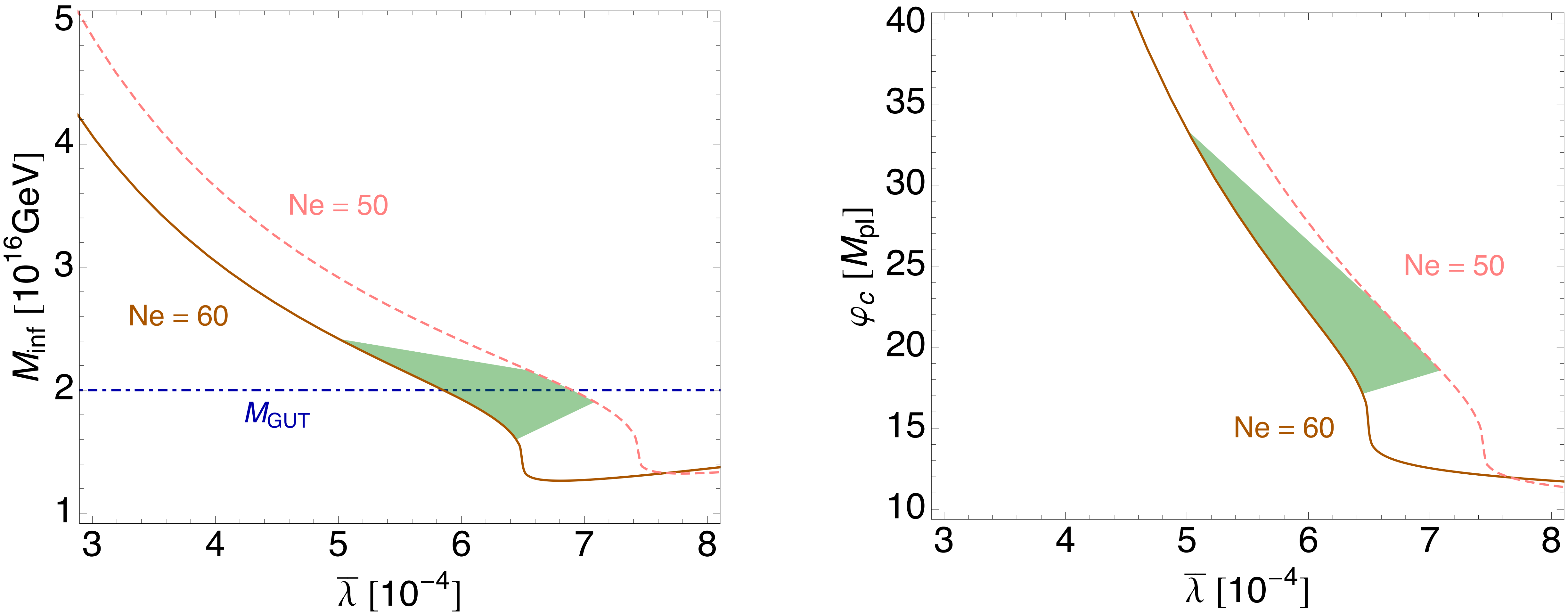
Effective inflaton potential below critical point depends on only two variables,

$$
\begin{aligned}
V_{\mathrm{inf}}(\varphi) &= V(\varphi, s_{\mathrm{min}}(\varphi)) \\
&= 2M_{\mathrm{inf}}^4 \, \frac{\varphi^2}{\varphi_c^2} \left(1 - \frac{1}{2}\frac{\varphi^2}{\varphi_c^2}\right) \,, \quad \varphi \leq \varphi_c
\end{aligned}
$$

where FI-term and Yukawa coupling are replaced by inflationary energy scale, and relative couplings strength (critical inflaton field),

$$
M_{\mathrm{inf}} = \left(\frac{g\xi}{\sqrt{2}}\right)^{1/2} \,, \quad \bar{\lambda} = \frac{\lambda}{\sqrt{g}} \;; \quad \varphi_c^2 = \frac{2\sqrt{2}M_{\mathrm{inf}}^2}{\bar{\lambda}^2}
$$

(remark: very similar to chaotic inflation in string models, corrected by moduli effects [WB, Dudas, Hurtier, Westphal, Wieck, Winkler ’15])

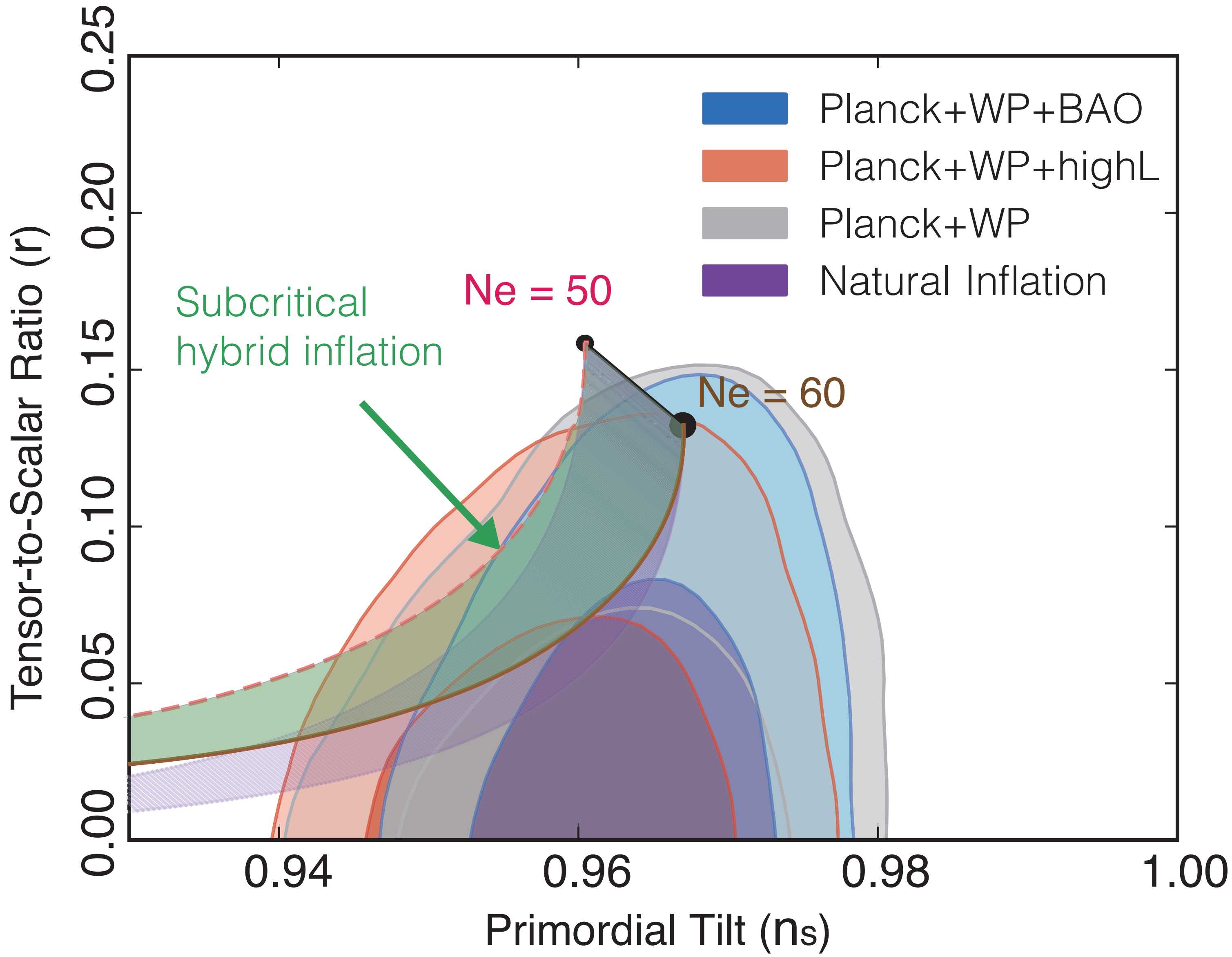


Remarkably, the Planck data

$$n_s = 0.9603 \pm 0.0073 \ , \quad r < 0.11 \, (95\% \, \mathrm{CL})$$

determine the energy scale of inflation rather precisely,

$$1.6 \ (1.9) \ \leq \left(\frac{M_{\rm inf}}{10^{16} \, \mathrm{GeV}} \right) \leq \ 2.4 \ (2.2)$$



1σ & 2σ regions for different data sets (Planck 2013) compared with "natural inflation" and "subcritical hybrid" inflation (green); prediction of subcritical hybrid inflation for bound on r for 60 (50) e-folds:

$$r > 0.049\ (0.085)$$

Models in their simplest form disfavoured by Planck 2015 data; modifications possible, more data this year ...

Conclusions

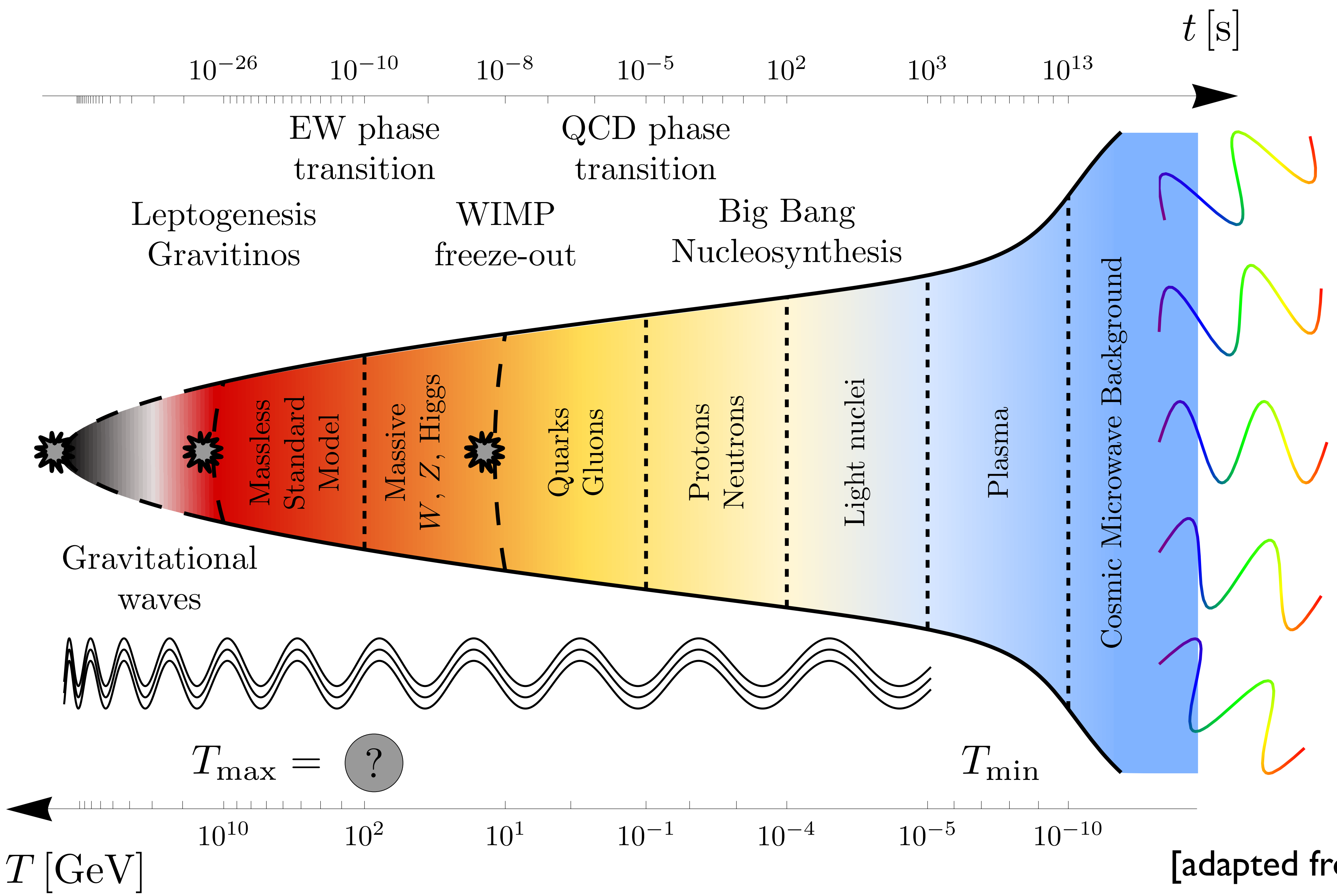
Strong theoretical motivation for unification of forces beyond current microscopic theory (SM)

Possible cosmic signatures of grand unification: cosmic strings, gravitational waves, B-modes in CMB; several indirect hints: leptogenesis, nonthermal dark matter, ...

Hybrid inflation attractive framework for describing very early universe (production of entropy, dark matter, leptogenesis,...)

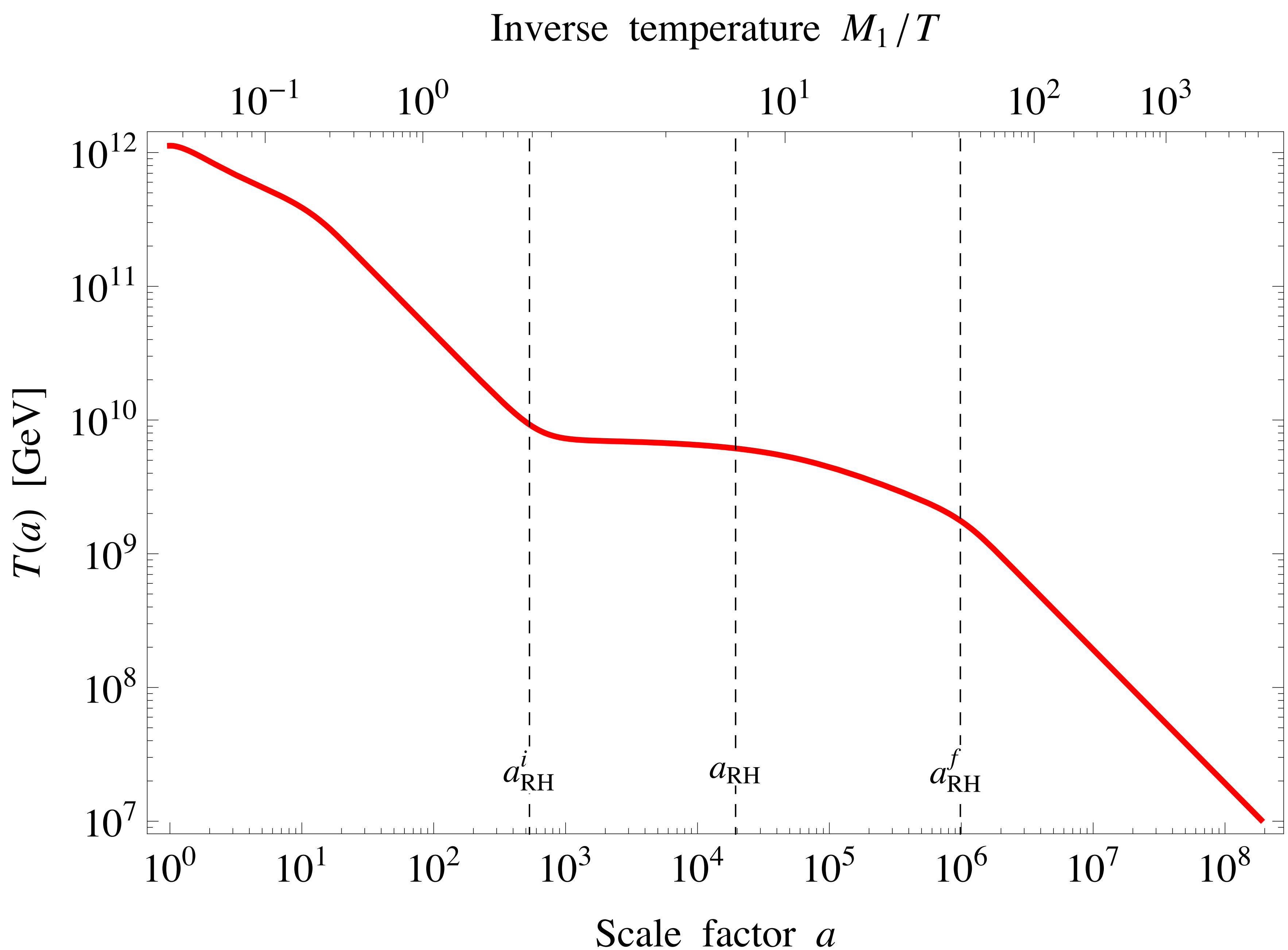
Hints for GUT scale already in subcritical hybrid inflation together with Planck data; prediction: lower bound on tensor-to-scalar ratio, in reach of upcoming experiments!

BACKUP SLIDES

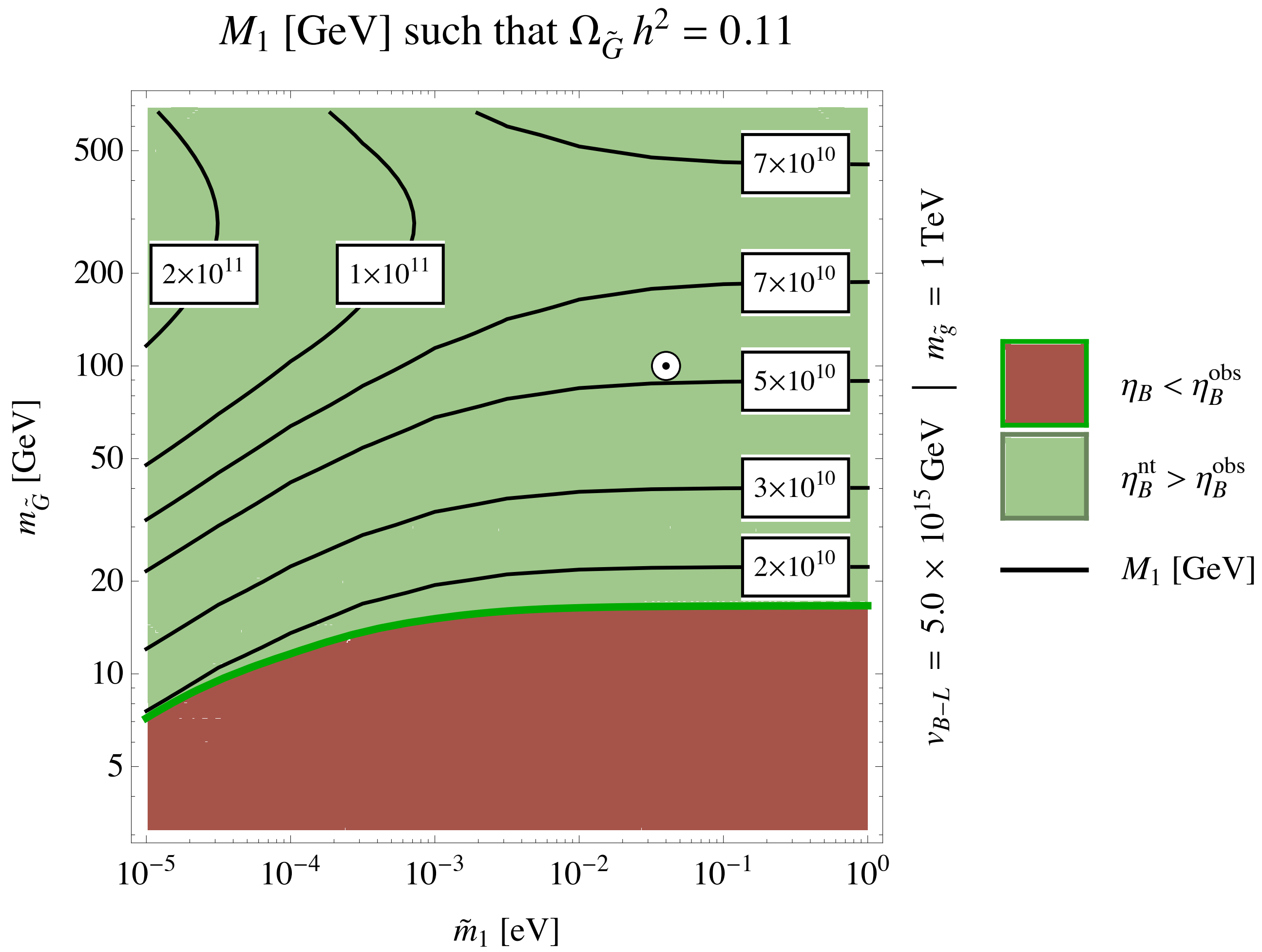


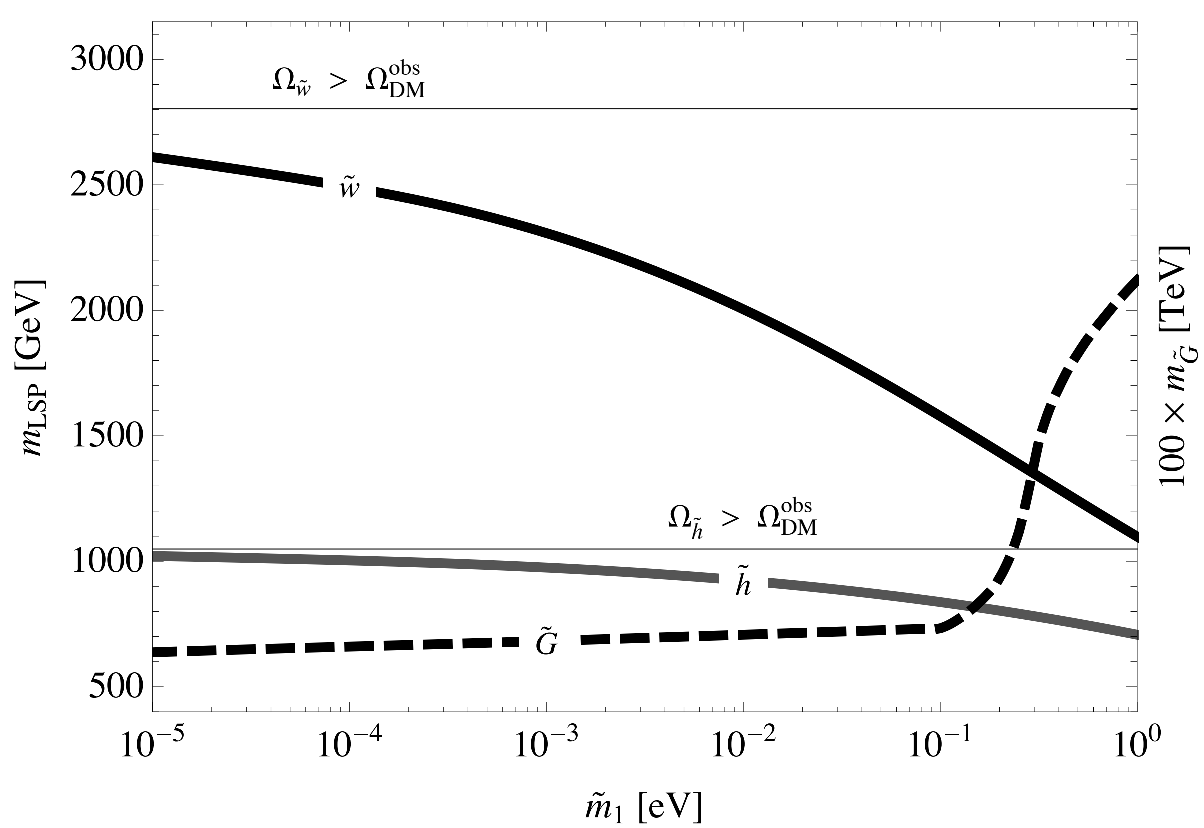
[adapted from Schmitz '12]

Connection to particle physics in very early universe, before formation of CMB: nucleosynthesis, formation of hadrons, generation of mass (electroweak phase transition), dark matter, leptogenesis ... preceeded by inflation (?) which generates initial conditions of hot early universe; important: global picture of very early universe!



Time evolution of temperature: intermediate plateau ("maximal temperature"), determined by neutrino properties! Yields gravitino abundance when combined with 'standard formula'





parameter scans: successful leptogenesis and gravitino DM (left) or neutralino DM (right) [non-thermally produced in decays of thermally produced gravitinos] constrains neutrino and superparticle masses; can be tested at LHC!

Slow-Roll Inflation

Inflation is driven by slowly varying potential energy $V(\phi)$ of inflaton field; small slow-roll parameters determine spectral indices:

$$\epsilon = \frac{M_{\rm pl}^2}{2}\left(\frac{\partial_\phi V}{V}\right)^2 , \quad \eta = M_{\rm pl}^2 \frac{\partial_\phi^2 V}{V}$$

$$n_s - 1 = 2\eta_* - 6\epsilon_* \,, \quad n_t = -2\epsilon_*$$

Tensor-to-scalar ratio yields energy density during inflation:

$$r = \frac{\Delta_s^2}{\Delta_t^2} = \frac{2}{3\pi^2\Delta_s}\frac{V(\phi_*)}{M_{\rm pl}^4} , \quad V^{1/4} \approx \left(\frac{r}{0.01}\right)^{1/4} 10^{16}\ {\rm GeV}$$

Tensor-to-scalar ratios $r \gtrsim 0.005$ require super-Planckian field values [Lyth '97; Antusch, Nolde '14]; theoretical challenge, ongoing theory activities...

Beyond local spinodal time waterfall field tracks local minimum; asymmetric trajectory in field space: