Assuming the dominance of the spin-spin interaction in a diquark, we point out that the mass differences in the beauty sector $M(Z_b^\pm) - M(Z_b)$ scale with quark masses as expected in QCD, with respect to the corresponding mass difference $M(Z_c^\pm) - M(Z_c)$. Notably, we show that the decays $\Upsilon(10890) \rightarrow (h_b(1P), h_b(2P))\pi^+\pi^-$ are compatible with heavy-quark spin conservation once the contributions of $Z_b, Z_b'$ intermediate states are taken into account, $\Upsilon(10890)$ being either a $\Upsilon(5S)$ or the beauty analog of $Y_c(4260)$. We also consider the role of $Z_b, Z_b'$ in $\Upsilon(10890) \rightarrow (nS)\pi\pi$ decays and of light quark spin non-conservation in $Z_b, Z_b'$ decays into $BB^*$ and $B^*B^*$. Indications on possible signatures of the still missing $X_b$ resonance are proposed.

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Tetraquark interpretation of the hidden charm and beauty exotic resonances has been advanced and studied in considerable detail (see Refs. [1] [2], and [3]). In a recent contribution [4], a new scheme for the spin-spin quark interactions in the hidden charm resonances has been proposed, which reproduces well the mass and decay pattern of $X(3872)$, of the recently discovered [5] $Z_{c}^{\pm,0}(3900)$, $Z_{c}^{\pm,0}(4020)$, and of the lowest lying $J^{PC} = 1^{--}$ $Y$ states.

Tetraquark states in the large $N_{c}$ (color) limit of QCD have been considered in [6] and [7] (see also the review [8] and references therein). Compact tetraquark mesons may have decay widths as narrow as $1/N_{c}$, contrary to previous beliefs, and therefore they are reasonable candidates for a secondary spectroscopic meson series, in addition to the standard $q\bar{q}$ one.

In this letter we consider the extension of the scheme presented in [4] for the hidden-charm to the hidden-beauty resonances $Z_{b}^{\pm,0}(10610) = Z_{b}$ and $Z_{b}^{\pm,0}(10650) = Z_{b}'$.

These resonances are interpreted as $S$-wave $J^{PG} = 1^{++}$ states with diquark spin distribution (use the notation $|s_{[bq]},s_{[\bar{b}q]}\rangle$ for diquark spins)

\[ |Z_{b} \rangle = \frac{|1_{bq},0_{\bar{b}q}\rangle - |0_{bq},1_{\bar{b}q}\rangle}{\sqrt{2}} \]
\[ |Z_{b}' \rangle = |1_{bq},1_{\bar{b}q}\rangle \]

The $J^{P} = 1^{+}$ multiplet is completed by $X_{b}$, which is given by the $C = +1$ combination

\[ |X_{b} \rangle = \frac{|1_{bq},0_{\bar{b}q}\rangle + |0_{bq},1_{\bar{b}q}\rangle}{\sqrt{2}} \]

Assuming the spin-spin interaction inside diquarks to dominate, we expect $X_{b}$ and $Z_{b}$ to be degenerate, with $Z_{b}'$ heavier according to

\[ M(Z_{b}') - M(Z_{b}) = 2\kappa_{b} \]

where $\kappa_{b}$ is the strength of the spin-spin interaction inside the diquark. A similar analysis for the hidden-charm resonances has produced the value [4]

\[ 2\kappa_{c} = M(Z_{c}') - M(Z_{c}) \simeq 120 \text{ MeV} \]

The QCD expectation is $\kappa_{b} : \kappa_{c} = M_{c} : M_{b}$. The ratio can be estimated from the masses reported in [9]

\[ \frac{M_{c}}{M_{b}} \simeq \frac{1.27}{4.18} = 0.30 \]

which fits nicely with the observed $Z_{b}' - Z_{b}$ mass difference ($\simeq 45 \text{ MeV}$).

Next we consider another crucial prediction of QCD, namely conservation of the heavy quark spin in hadronic decays.

We recall that $Z_{b}, Z_{b}'$ are observed in the decays of $\Upsilon(10890)$

\[ \Upsilon(10890) \rightarrow Z_{b}/Z_{b}' + \pi \rightarrow h_{b}(nP)\pi\pi \]
The $\Upsilon(10890)$ is usually reported as the $\Upsilon(5S)$ since its mass is close to the mass of the $5S$ state predicted by potential models. However, a different assignment was proposed in [10], namely $\Upsilon(10890) = Y_b$, the latter state being a $P$-wave tetraquark analogous to the $\Upsilon(4260)$. A reason for this assignment is the analogy of $\Upsilon(10890)$ decay (6) with $\Upsilon(4260) \rightarrow Z_c(3900) + \pi$, with $\Upsilon(4260)$ being the the first discovered $\Upsilon$ state [11]. Current experimental situation about $\Upsilon(10890)$ is still in a state of flux. In our opinion, the possibility that $\Upsilon(10890)$ is an unresolved peak involving both the $\Upsilon(5S)$ and $Y_b$, reported by Belle some time ago [12], is plausible, providing a resolution of the observed branching ratios measured at the $\Upsilon(10890)$ [13]. However, this identification is not a requirement in the considerations presented below. In fact, following the assignment of $\Upsilon(4260)$ as a $P$-wave tetraquark with $s_c\bar{c} = 1$ [4], one sees that in both cases the initial state in (6) corresponds to $s_b\bar{b} = 1$. As is well known $h_b(nP)$ has $s_b\bar{b} = 0$, pointing to a possible violation of the heavy-quark spin conservation, as suggested in [13].

We show now that the contradiction is only apparent. Expressing the states in the the basis of definite $b\bar{b}$ and $q\bar{q}$ spin, one finds

$$\left| Z_b \right\rangle = \frac{\left| 1_{q\bar{q}}, 0_{b\bar{b}} \right\rangle - \left| 0_{q\bar{q}}, 1_{b\bar{b}} \right\rangle}{\sqrt{2}}$$

$$\left| Z'_b \right\rangle = \frac{\left| 1_{q\bar{q}}, 0_{b\bar{b}} \right\rangle + \left| 0_{q\bar{q}}, 1_{b\bar{b}} \right\rangle}{\sqrt{2}} \quad (7)$$

Define

$$g_Z = g(\Upsilon \rightarrow Z_b \pi)g(Z_b \rightarrow h_b \pi) \propto \langle h_b | Z_b \rangle \langle Z_b | \Upsilon \rangle$$

$$g_{Z'} = g(\Upsilon \rightarrow Z'_b \pi)g(Z'_b \rightarrow h_b \pi) \propto \langle h_b | Z'_b \rangle \langle Z'_b | \Upsilon \rangle \quad (8)$$

where $g$ are the effective strong couplings at the vertices $\Upsilon Z_b \pi$ and $Z_b h_b \pi$. Therefore, for both assignments of $\Upsilon(10890)$, Eq. (7) and heavy quark spin conservation require

$$g_Z = -g_{Z'} \quad (9)$$

In Ref. [14] the amplitude for the decay (6) is fitted with two Breit-Wigners corresponding to the $Z_b, Z'_b$ intermediate states. Table I therein, that we transcribe here in Table 1, shows the relative normalizations and phases obtained by the fit, for decays into $h_b(1P)$ and $h_b(2P)$. Within large errors, consistency with Eq. (8), that is with the heavy-quark spin conservation, is apparent.

It is interesting that the same conclusion was drawn using a picture in which $Z_b, Z'_b$ have a "molecular" type structure [15]

$$Z_b = \frac{|B, \bar{B}^*\rangle - |\bar{B}, B^*\rangle}{\sqrt{2}}$$

$$Z'_b = |B^*, \bar{B}^*\rangle_{J=1} \quad (10)$$

It is conceivable that the subdominant spin-spin interactions may play a non negligible role in the $b$-systems, as the spin-spin dominant interaction is suppressed by the large $b$ quark mass. In
\[
\begin{array}{|c|c|c|}
\hline
 & h_b(1P)\pi^+\pi^- & h_b(2P)\pi^+\pi^- \\
\hline
\text{Relative Normalization} & 1.39 \pm 0.37_{-0.15}^{+0.05} & 1.6_{-0.4}^{+0.6_{-0.4}}_{-0.6} \pm 0.05 \\
\hline
\text{Relative Phase} & 187^{+44}_{-57} \pm 12 & 181^{+65}_{-105} \pm 74 \pm 109 \\
\hline
\end{array}
\]

Table 1: Values of \(|g_Z/g_{Z'}|\) and of the relative phases (in degrees), for \(h_b(1P), h_b(2P)\), as reported by [14].

this case the composition of \(Z_b, Z'_b\) indicated in Eq. (7) would be more general:

\[
\begin{align*}
|Z_b\rangle &= \alpha|1_{b\bar{q}}, 0_{\bar{b}q}\rangle - \beta|0_{b\bar{q}}, 1_{\bar{b}q}\rangle \\
|Z'_b\rangle &= \beta|1_{b\bar{q}}, 0_{\bar{b}q}\rangle + \alpha|0_{b\bar{q}}, 1_{\bar{b}q}\rangle \\
&= \frac{1}{\sqrt{2}} \langle \Upsilon(nS)|0_{b\bar{q}}, 1_{\bar{b}q}\rangle \langle 0_{b\bar{q}}, 1_{\bar{b}q}|\Upsilon \rangle
\end{align*}
\]

but the ratio \(g_Z/g_{Z'}\) would still be unity. To determine \(\alpha\) and \(\beta\) separately, one has to resort to \(s_{\bar{b}q} = 1 \rightarrow s_{b\bar{q}} = 1\) transitions, such as \(\Upsilon(10890) \rightarrow \Upsilon(nS)\pi\pi\) where \(n = 1, 2, 3\). The effective couplings analogous to (8) would be

\[
\begin{align*}
f_Z &= f(\Upsilon \rightarrow Z_b\pi)f(Z_b \rightarrow \Upsilon(nS)\pi) = \frac{|\beta|^2}{2} \langle \Upsilon(nS)|0_{b\bar{q}}, 1_{\bar{b}q}\rangle \langle 0_{b\bar{q}}, 1_{\bar{b}q}|\Upsilon \rangle \\
f_{Z'} &= f(\Upsilon \rightarrow Z'_b\pi)f(Z'_b \rightarrow \Upsilon(nS)\pi) = \frac{|\alpha|^2}{2} \langle \Upsilon(nS)|0_{b\bar{q}}, 1_{\bar{b}q}\rangle \langle 0_{b\bar{q}}, 1_{\bar{b}q}|\Upsilon \rangle
\end{align*}
\]

The Dalitz plot of these decays indicate indeed that a sizeable part of the transitions proceeds through \(Z_b\) and \(Z'_b\) [13, 14]. Parametrizing the amplitude in terms of two Breit-Wigner, one would determine the ratio \(\alpha/\beta\).

As a side remark, we observe that a Fierz rearrangement similar to the one used in (7) puts together \(b\bar{q}\) and \(q\bar{b}\) fields

\[
\begin{align*}
|Z_b\rangle &= |1_{b\bar{q}}, 1_{\bar{b}q}\rangle_J = 1 \\
|Z'_b\rangle &= |1_{b\bar{q}}, 0_{\bar{b}q}\rangle + |0_{b\bar{q}}, 1_{\bar{b}q}\rangle \sqrt{2}
\end{align*}
\]

The labels \(0_{b\bar{q}}\) and \(1_{b\bar{q}}\) could be viewed as indicating \(B\) and \(B^*\) mesons, respectively, leading to the prediction of the decay patterns \(Z_b \rightarrow B^* \bar{B}^*\) and \(Z'_b \rightarrow B \bar{B}^*\) [3]. This would not be in agreement with the Belle data [13].

We remark however that this argument rests on conservation of the light quark spin which, on the contrary, may change when the color octet pairs which appear in (12), evolve into pairs of...
color singlet mesons. Therefore predictions derived from (12) are not as reliable as those derived from (7).

Finally we comment on the expected positive charge conjugation state, $X_b$. On the basis of the assumed spin-spin interaction, one predicts $M(X_b) \simeq M(Z_b) \simeq 10600 \text{ MeV}$. Such a state has been searched by ATLAS [16] in the region $10500 < M < 11000 \text{ MeV}$ looking for the decay

$$X_b \rightarrow \Upsilon(1S) \pi \pi$$

so far with negative results.

In Ref. [2], it is noted that the near equality of the branching ratios for $X(3872) \rightarrow J/\psi 2\pi$ and $X(3872) \rightarrow J/\psi 3\pi$ can be understood if $X(3872)$ is predominantly isosinglet. The isospin allowed decay in $J/\psi \omega$ is phase space forbidden and the decay in the $J/\psi \rho$ mode, although isospin forbidden, is phase space favoured, leading to similar rates.

In the $X_b$ decay, both $\omega$ and $\rho$ channels are allowed by phase space, so that, if $X_b$ is isosinglet, the dominant mode would be into $\Upsilon(1S) \omega$. The suggestion therefore is to look at the decay $X_b(10600) \rightarrow \Upsilon(1S) 3\pi$ with the $3\pi$ in the $\omega$ mass band, in parallel with the search for the $X_b(10600) \rightarrow \Upsilon(1S) 2\pi$ channel with the $2\pi$ in the $\rho$ band.

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**References**


