The Time-Flow Approach as a Tool for Large-Scale Structure
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Abstract
We discuss how the time-flow approach of cosmological perturbation theory can be used as a tool for large-scale structure. In particular, we show that the flow equations allow to derive straightforwardly consistency relations for equal-time correlators involving both density and velocity fields and underlying different background cosmologies.

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Motivation
Time-flow approach in cosmological perturbation theory
Tool for large-scale structure:
- Consistency relations:
  - Relate $(n+1)$- to $n$-point correlation functions, e.g., bispectrum vs. power spectrum
  - Analytical result in the squeezed limit $\eta \to 0$
  - Comparison to numerical simulations/observations

The time-flow approach
Flow equations
- Central quantities of cosmological perturbation theory $\delta$ and $\theta$ evolve $\nabla \psi$
as a doublet
$\phi_i(\kappa, \theta) \equiv e^{i\kappa \psi}(\delta, -\theta/\kappa)$
- Time variable $\eta \equiv \ln \nu(\eta)$, expansion rate $N \equiv 3H\left(\frac{a}{c}\right)/\nu$
Fluid equations (continuity, Euler and Poisson equation) [1]:
$\frac{\partial \rho_0}{\partial \tau} = -\nabla \cdot \rho_0 \phi_0 + \mu^2 \nabla \cdot \phi_0$
$\rho_0 \phi_0$ depends on cosmological model, vertex functions $\gamma_{av}$
Perturbative solution
- Iteration of correlation functions:
$\delta_i(\phi_0, \phi, \phi) = -\nabla_i \phi_0 \nabla_i \phi + \phi_0 \phi_0$
$+ \mu^2 \nabla \cdot \phi_0 \phi_0 + \mu^2 \nabla \cdot \phi_0 \phi_0$
$= \cdots$
- Flow equations
- Correlation functions:
$\left\{ \phi_0 \phi_0 \right\} \sim \rho_0$
$\left\{ \phi_0 \phi, \phi \right\} \sim \delta_i$
$\left\{ \phi_0 \phi_0, \phi_0 \phi_0 \right\} \sim \rho_0 \rho_0 + \rho_0 \rho_0 + \rho_0 \rho_0 + Q_{00}$
- Closure approximation $Q_{00} = 0$ → Solution of flow equations

Consistency relations from flow equations
Bispectrum consistency relations
For general cosmological models
- Derivation:
  - Gaussian initial conditions $Q_{00}(0, 0) = 0$
  - Perturbative expansion of $g_{ab}, \rho_0, \gamma_{av}$ up to $O(\nu^3)$
  - Squeezed limit $\eta \to 0$
  - Angular average $\langle \ldots \rangle \equiv \int d\Omega_4 / (4\pi)$
$B_{ab0}(k_1, k_2) = \frac{1}{2} \left( \delta_i^a \delta_i^b \phi \right)$
$g_{ab0} = \int \left[ 3 g_{ab0} P_{\phi}(\kappa) P_{\phi}(\kappa) + P_{\phi}(\kappa) P_{\phi}(\kappa) \right]$
$+ 2 g_{ab0} P_{\phi}(\kappa) P_{\phi}(\kappa) + P_{\phi}(\kappa) P_{\phi}(\kappa) + Q_{00}$
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$\langle H^4 \rangle^2 = \frac{1}{2} \mu^2 \frac{\partial^2}{\partial \mu^2} \langle H^4 \rangle^2$
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- For $a, b, c = 1$
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- In the EdS/ΛCDM case: result in literature only valid at linear order

For a flat universe (EdS, ΛCDM)
- Derivation:
  - Gaussian initial conditions $Q_{00}(0, 0) = 0$
  - Perturbative expansion of $g_{ab}, \rho_0, \gamma_{av}$ up to $O(\nu^3)$
  - Squeezed limit $\eta \to 0$
  - Angular average $\langle \ldots \rangle \equiv \int d\Omega_4 / (4\pi)$
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Conclusions
Summary
- Time-flow approach as tool for large-scale structure
  → Bispectrum consistency relations
- General equation:
  → For all correlations of $\delta$ and $\theta$
- For all cosmological models
- In the EdS/ΛCDM case: result in literature only valid at linear order

Open issues
- Analytical/numerical justification of closure approximation $Q_{00} = 0$
- Higher-order contributions to consistency relations?

References