Soft gluons, large-N towers, and colour reconnection.

Simon Plätzer

DESY

arXiv:1312.2448 and work in progress

Outline.

– Colour flow bases.
– Soft anomalous dimensions.
– Approximate exponentiation.
– Colour reconnection.
– Conclusions & outlook.

Colour flow bases.

Physically transparent QCD amplitudes:
Track flow of colour charge.

Not tied to large-N limit.

Basis for highly efficient, recursive amplitude evaluation. e.g. Comix [Gleisberg, Hoeche – JHEP 0812, 039 (2008)]

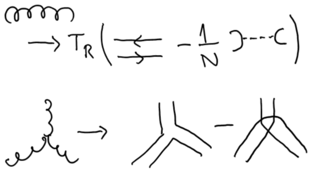
Rarely used in the context of soft-gluon evolution.

- Properties of soft anomalous dimensions?
- Insight into colour evolution dynamics?

Impacts various loose ends:
Improved showers, analytic resummation for a large number of legs, colour reconnection ...

$$A^\mu_a \to A^\mu_a (t^a)^i{}_{\bar{i}} \equiv \mathcal{A}^{\mu i}{}_{\bar{j}}$$

$$(t^a)^i{}_{\bar{i}}(t^a)^j{}_{\bar{j}} = T_R\left(\delta^i_{\bar{j}}\delta^j_{\bar{i}} - \frac{1}{N}\delta^i_{\bar{i}}\delta^j_{\bar{j}}\right)$$



Note: ‘Singlet’ gluon decouples from pure-gluon vertices.

Full set of Feynman rules e.g. in [Maltoni et al. – Phys.Rev.D 67, 014026 (2003)]

Colour flow bases.

Amplitude decomposition:

$$|\mathcal{M}_n\rangle = \sum_\sigma \mathcal{M}_{n,\sigma}|\sigma\rangle$$

'Normal ordered' numbering of (outgoing) legs α:

$$1_{\mathbf{N}}, 2_{\bar{\mathbf{N}}}, ..., n_{q,\bar{\mathbf{N}}}, (n_q+1)_{\mathbf{A}}, ..., (n_q+n_g)_{\mathbf{A}}$$

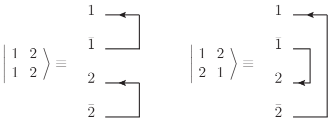
translated to (anti-)fundamental indices:

$$\begin{array}{rcl} k & \leftrightarrow & \alpha = k_{\mathbf{N}} \\ \overline{k-1} & \leftrightarrow & \alpha = k_{\bar{\mathbf{N}}} \\ \left.\begin{array}{r} \overline{k-n_q/2} \\ k-n_q/2 \end{array}\right\} & \leftrightarrow & \alpha = k_{\mathbf{A}} \end{array}$$

Basis tensors labelled by $\sigma \in S_m$, $m = n_q/2 + n_g$:

$$|\sigma\rangle = \delta^{i_1}_{\overline{i_{\sigma(1)}}} \cdots \delta^{i_m}_{\overline{i_{\sigma(m)}}}$$

Cannot restrict the basis to the tree-level subspace, even when only considering evolution of tree-level amplitudes only!



Partial amplitudes in this basis used to caluclate weights for large-N colour flow assignment.

Soft anomalous dimensions.

$$|\mathcal{M}_n'\rangle = e^{\mathbf{\Gamma}}|\mathcal{M}_n\rangle \qquad \mathbf{\Gamma} = \sum_{\alpha\neq\beta} \Gamma^{\alpha\beta}\ \mathbf{T}_\alpha \cdot \mathbf{T}_\beta$$

Translate colour correlations: Legs $\alpha \leftrightarrow$ (anti-)colour indices $\bar{i}, j$.

$$\begin{aligned}
\mathbf{T}_\alpha \cdot \mathbf{T}_\beta &= \mathbf{T}_k \cdot \mathbf{T}_l & \alpha &= k_{\mathbf{N}}, \beta = l_{\mathbf{N}} \\
\mathbf{T}_\alpha \cdot \mathbf{T}_\beta &= \mathbf{T}_k \cdot \mathbf{T}_{\bar{l}} & \alpha &= k_{\mathbf{N}}, \beta = (l+1)_{\bar{\mathbf{N}}} \\
\mathbf{T}_\alpha \cdot \mathbf{T}_\beta &= \mathbf{T}_k \cdot \mathbf{T}_{\bar{l}} + \mathbf{T}_k \cdot \mathbf{T}_l & \alpha &= k_{\mathbf{N}}, \beta = (l + n_q/2)_{\mathbf{A}} \\
\mathbf{T}_\alpha \cdot \mathbf{T}_\beta &= \mathbf{T}_k \cdot \mathbf{T}_{\bar{l}} + \mathbf{T}_k \cdot \mathbf{T}_l & \alpha &= (k + n_q/2)_{\mathbf{A}}, \\
&+\mathbf{T}_l \cdot \mathbf{T}_{\bar{k}} + \mathbf{T}_{\bar{k}} \cdot \mathbf{T}_{\bar{l}} & \beta &= (l + n_q/2)_{\mathbf{A}}
\end{aligned}$$

Express colour correlators by Fierz identities:

$$\mathbf{T}_i \cdot \mathbf{T}_j = +\frac{1}{2}\left(\delta_j^{i'}\delta_i^{j'} - \frac{1}{N}\delta_i^{i'}\delta_j^{j'}\right) \qquad \mathbf{T}_i \cdot \mathbf{T}_{\bar{j}} = -\frac{1}{2}\left(\delta^{i'}{}_{\bar{j}'}\delta^{\bar{j}}{}_i - \frac{1}{N}\delta^{i'}{}_i\delta^{\bar{j}}{}_{\bar{j}'}\right)$$

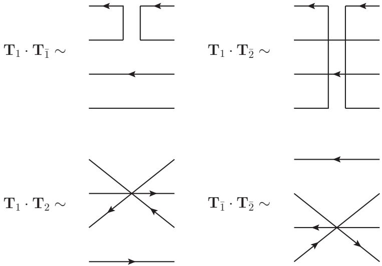
Soft anomalous dimensions.

$$\mathbf{\Gamma} = \sum_{i<j} (\gamma_{ij} \mathbf{T}_i \cdot \mathbf{T}_j + \gamma_{\bar{i}\bar{j}} \mathbf{T}_{\bar{i}} \cdot \mathbf{T}_{\bar{j}}) + \sum_{i,j} \gamma_{i\bar{j}} \mathbf{T}_i \cdot \mathbf{T}_{\bar{j}}$$

Anomalous dimension in the flow basis:

$$\mathbf{T}_i \cdot \mathbf{T}_j = \frac{1}{2} \left(\delta_j^{i'} \delta_i^{j'} - \frac{1}{N} \delta_i^{i'} \delta_j^{j'} \right)$$

$$[\tau|\mathbf{\Gamma}|\sigma] = \left(-N\Gamma_\sigma + \frac{1}{N}\rho \right) \delta_{\tau\sigma} + \Sigma_{\tau\sigma}$$



Colour 'reconnectors'
$\rightarrow$ genuinely $1/N$ supressed.

$\Sigma_{\tau\sigma} \neq 0$ *only* if $\sigma = \text{transposition}_{ij}(\tau)$

Soft anomalous dimensions.

Transparent organization of powers of N:

$$\mathbf{\Gamma} \equiv N\underline{\Gamma} + \underline{\Sigma} + \frac{1}{N}\rho\underline{1}$$

Leading-N $\rightarrow$ diagonal
Next-to-leading-N $\rightarrow$ colour reconnections, diagonal elements vanish
Next-to-next-to-leading-N $\rightarrow$ trivial

Can we learn something on exponentiation?
What is the 'leading-N' structure of the evolution matrix $e^{\mathbf{\Gamma}}$?
On a strict expansion: skip one of the pieces which is trivially to handle?!

Large-N towers.

What is the 'leading-N' structure of the evolution matrix $e^{\mathbf{\Gamma}}$?

'Never use brute force to fight an exponential!'
Perturbation theory in $1/N$ won't work.
Need resummation to get at least close to the exact result.

$$\exp \mathbf{\Gamma} \equiv \exp \left(N \underline{\Gamma} + \underline{\Sigma} + \frac{1}{N} \rho \underline{1} \right)$$

Power counting: Need to resum if $\gamma N = \mathcal{O}(1)$.
Treat remainder as higher-order effects/perturbation, but on same footing.

$$\begin{aligned} \text{at LC} &: \quad 1 + \gamma N + \gamma^2 N^2 + \ldots \\ \text{at NLC} &: \quad \left(\gamma + \frac{\gamma}{N} \right) (1 + \gamma N + \gamma^2 N^2 + \ldots) \\ \text{at NNLC} &: \quad \left(\gamma + \frac{\gamma}{N} \right)^2 (1 + \gamma N + \gamma^2 N^2 + \ldots) \end{aligned}$$

Will also consider a 'primed' resummation: Absorb ρ contribution into redifinition of Γ.

How to get there?

Work out the exponential ...

$$[\tau|e^{\Gamma}|\sigma] =$$

$$\sum_{l=0}^{\infty}\frac{(-1)^l}{N^l}\sum_{k=0}^{l}\frac{(-\rho)^k}{k!}\sum_{\sigma_0,\ldots,\sigma_{l-k}}\delta_{\tau\sigma_0}\delta_{\sigma_{l-k}\sigma}\left(\prod_{\alpha=0}^{l-k-1}\Sigma_{\sigma_\alpha\sigma_{\alpha+1}}\right)R(\{\sigma_0,\ldots,\sigma_{l-k}\},\{\Gamma_\sigma\})$$

$$R(\sigma,\Gamma) = \left[\prod_{\alpha=0}^{\#\mathrm{uniq}(\sigma)-1}\frac{1}{d_\alpha(\sigma)!}\frac{\partial^{d_\alpha(\sigma)}}{\partial^{d_\alpha(\sigma)}\Gamma_{\sigma_\alpha}}\right]\sum_{\alpha=0}^{\#\mathrm{uniq}(\sigma)-1}e^{-N\Gamma_{\sigma_\alpha}}\prod_{\beta=0,\beta\neq\alpha}^{\#\mathrm{uniq}(\sigma)-1}\frac{(\Gamma_{\sigma_\beta}/\Gamma_{\sigma_\alpha})^{d_\beta(\sigma)}}{\Gamma_{\sigma_\alpha}-\Gamma_{\sigma_\beta}}$$

(next-to-)^d-leading color is truncating the series at d.
Note that the leading colour contributions are exponentiated.

Some first observations.

Next-to-leading color:

$$[\tau|e^{\mathbf{\Gamma}}|\sigma] = \delta_{\tau\sigma} e^{-N\Gamma_\sigma}\left(1+\frac{\rho}{N}\right) - \Sigma_{\tau\sigma}\frac{1}{N}\frac{e^{-N\Gamma_\tau}-e^{-N\Gamma_\sigma}}{\Gamma_\tau-\Gamma_\sigma} + \text{NNLC}$$

At N^{d}LC need d matrix multiplications.
Or not even this: Σ is very sparse, with easily calculable non-vanishing elements.
$\to$ Think of a Monte Carlo over colour structures.

In a resummation context, to be matched to NLO:
NLC is in principle sufficient for the matching.

$$[\tau|e^{\mathbf{\Gamma}}|\sigma]\Big|_{\text{NLC}} = \delta_{\tau\sigma} + [\tau|\mathbf{\Gamma}|\sigma] + \mathcal{O}(\gamma^2)$$

Numerical results.

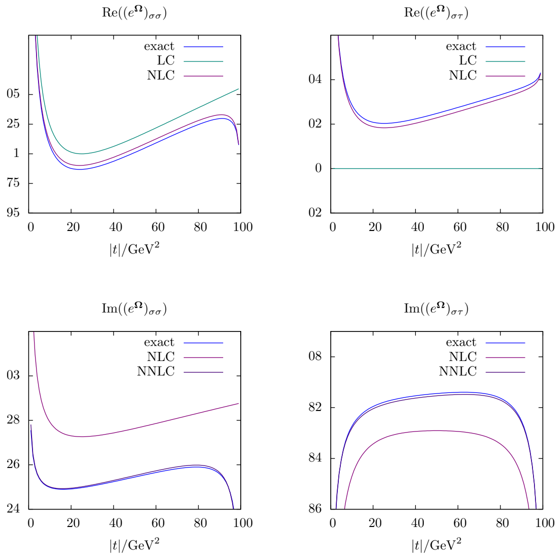
Testbed: (Dipole) shower emission veto in dijets $p_1, p_2 \to p_3, p_4$.
Each of the possible II, IF, FF dipoles can radiate with $\mu^2 < p_\perp^2 \lesssim 2 p_i \cdot p_j$.

$$
\begin{aligned}
\Gamma^{12} = \Gamma^{34} &= \frac{\alpha_s}{4\pi}\left(\frac{1}{2}\ln^2\frac{s}{\mu^2} - i\pi\ln\frac{s}{\mu^2}\right) \\
\Gamma^{13} = \Gamma^{24} &= \frac{\alpha_s}{8\pi}\ln^2\frac{|t|}{\mu^2} \\
\Gamma^{14} = \Gamma^{23} &= \frac{\alpha_s}{8\pi}\ln^2\frac{|u|}{\mu^2}
\end{aligned}
$$

Exact solution for $q\bar{q}q\bar{q}$ amplitudes can be obtained straightforwardly.
Now check convergence of the N^{d}LC approximations.

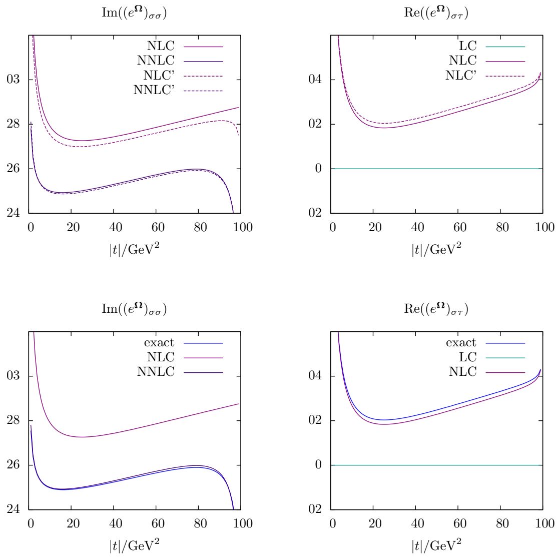
Numerical results.

C++ library CVolver will be public at some point



Numerical results: Primed resummation.

C++ library CVolver will be public at some point



Why all the fuzz?

Soft gluon evolution, exact (few legs):

- Colour structure of anomalous dimension in flow basis.
- Use powerful colour flow based ME generators.

Soft gluon evolution, approximate (many legs):

- Check convergence of successive approximations.
- No matrix exponentiation, only (sparse) matrix multiplication needed.
- Ultimately: Do this within shower algorithms.

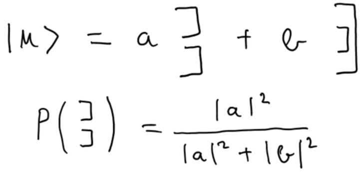
Colour reconnection. This really is what started this business ...

Colour reconnection.

Soft gluon evolution is the physics behind colour reconnection.

Understanding soft gluon evolution in the colour flow basis, and for many legs, is the key to these dynamics.

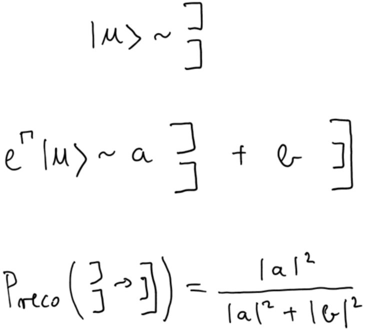
Recap how we assign colour flows to a hard process:



Colour reconnection.

Towards a model:

- Colour flow after showering: Assume amplitude is dominated by the respective partial amplitude.
- Evolve this colour flow with educated guess for soft anomalous dimension. Eventually continue to non-perturbative regime.
- Perform colour reconnection with probabilities as we would use them for a hard process.



In this case, $i\pi$ terms would be of no effect.
Also try projecting on a swapped flow: $P_{\text{reco}} \sim |\langle 21|e^{\Gamma}|12\rangle|^2$.

Would it work at all?

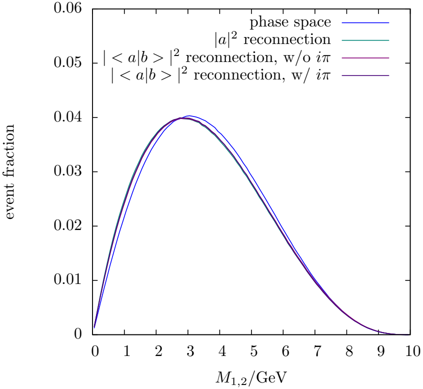
Check two colour flows as the simplest case.
Let's assume a simple veto-type anomalous dimension:

$$\Gamma^{\alpha\beta} = \frac{\alpha_s}{4\pi}\left(\frac{1}{2}\ln^2\frac{M^2_{\alpha\beta}}{\mu^2} - i\pi\ln\frac{M^2_{\alpha\beta}}{\mu^2}\right)$$

Cluster masses $M_{\alpha\beta}$ sure I'm cluster model biased ;-)

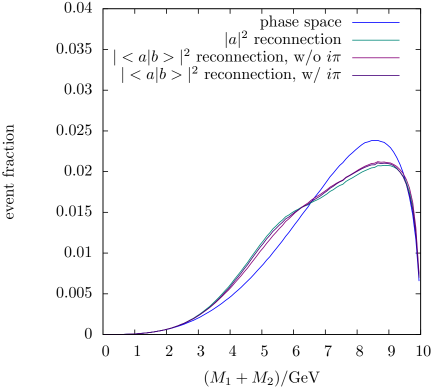
Two-cluster sandbox:

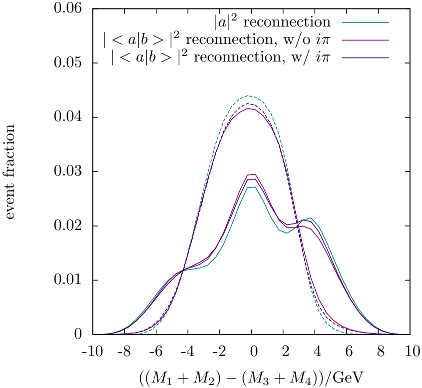
- Two $q\bar{q}$ pairs.
- Fill momenta of total mass 10 GeV by flat phase space.
- Reconnect with two-colour-flow evolution.



Some more observations.

Difference in cluster masses $\sim$ invariant.
Look at ‘string measure’ type observables, *i.e.* cluster mass sums.





Towards a full model.

Use N^dLC approximation to deal with the large number of legs.

E.g. NLC will only perform one 'swap': think of an evolution-type picture?
→ Certainly will have to iterate some N^dLC actions.

Implementation-wise can be done by a MC over possible new colour configurations.
→ Weights easily calculable due to known structure of evolution.

Open questions:

- What form of anomalous dimension to choose?
- How to continue to nonperturbative regime?
- How to calculate the reconnection probability?
- How to deal with singlet gluons?

Happy for any comments and discussion!

Conclusions & Outlook.

Soft gluon evolution in colour flow basis has a very transparent structure.

Exponentiation of the anomalous dimension can be performed approximately, by resumming large-N enhanced terms.

Checking successive approximations for convergence, evolution of a large number of legs is feasible.

This machinery allows to tackle colour reconnection models from a new perspective. Yet lot of things to sort out here ...

Thanks!