CMB power suppression at low-$l$ from string inflation

Figure 37. The Planck CMB temperature angular power spectrum. The error bars include cosmic variance. whose magnitude is indicated by the green shaded area around the best fit model. The low-$\ell$ values are plotted at 90°, 18°, 50°, 90°, 18°, 50°, 90°, 18°, 50°, 90°, 18°, 50°.

Table 8. Constraints on the basic six-parameter $\Lambda$CDM model using Planck data. The top section contains constraints on the six primary parameters included directly in the estimation process, and the bottom section contains constraints on derived parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Best fit</th>
<th>68% limits</th>
<th>Best fit</th>
<th>68% limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_b h^2$</td>
<td>$(0.015 \pm 0.004)$</td>
<td>$0.012 - 0.017$</td>
<td>$(0.015 \pm 0.004)$</td>
<td>$0.012 - 0.017$</td>
</tr>
<tr>
<td>$\Omega_c h^2$</td>
<td>$(0.116 \pm 0.007)$</td>
<td>$0.108 - 0.125$</td>
<td>$(0.116 \pm 0.007)$</td>
<td>$0.108 - 0.125$</td>
</tr>
<tr>
<td>$\theta_{MC}$</td>
<td>$(1.004 \pm 0.005)$</td>
<td>$0.994 - 1.004$</td>
<td>$(1.004 \pm 0.005)$</td>
<td>$0.994 - 1.004$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>$(0.070 \pm 0.007)$</td>
<td>$0.066 - 0.074$</td>
<td>$(0.070 \pm 0.007)$</td>
<td>$0.066 - 0.074$</td>
</tr>
<tr>
<td>$n_s$</td>
<td>$(0.967 \pm 0.011)$</td>
<td>$0.948 - 0.986$</td>
<td>$(0.967 \pm 0.011)$</td>
<td>$0.948 - 0.986$</td>
</tr>
<tr>
<td>$A_s$</td>
<td>$(1.994 \pm 0.025)$</td>
<td>$1.951 - 2.037$</td>
<td>$(1.994 \pm 0.025)$</td>
<td>$1.951 - 2.037$</td>
</tr>
<tr>
<td>$\Omega_{\Lambda}$</td>
<td>$(0.682 \pm 0.020)$</td>
<td>$0.655 - 0.709$</td>
<td>$(0.682 \pm 0.020)$</td>
<td>$0.655 - 0.709$</td>
</tr>
<tr>
<td>$\Omega_m$</td>
<td>$(0.045 \pm 0.006)$</td>
<td>$0.034 - 0.057$</td>
<td>$(0.045 \pm 0.006)$</td>
<td>$0.034 - 0.057$</td>
</tr>
<tr>
<td>$\sigma_8$</td>
<td>$(0.77 \pm 0.02)$</td>
<td>$0.73 - 0.82$</td>
<td>$(0.77 \pm 0.02)$</td>
<td>$0.73 - 0.82$</td>
</tr>
<tr>
<td>$z_{re}$</td>
<td>$(2.69 \pm 0.04)$</td>
<td>$2.62 - 2.76$</td>
<td>$(2.69 \pm 0.04)$</td>
<td>$2.62 - 2.76$</td>
</tr>
<tr>
<td>$H_0$</td>
<td>$(70.5 \pm 0.4)$</td>
<td>$69.7 - 71.3$</td>
<td>$(70.5 \pm 0.4)$</td>
<td>$69.7 - 71.3$</td>
</tr>
<tr>
<td>$z^*$</td>
<td>$(1.04 \pm 0.05)$</td>
<td>$1.00 - 1.08$</td>
<td>$(1.04 \pm 0.05)$</td>
<td>$1.00 - 1.08$</td>
</tr>
<tr>
<td>$z_{eq}$</td>
<td>$(1.47 \pm 0.06)$</td>
<td>$1.41 - 1.54$</td>
<td>$(1.47 \pm 0.06)$</td>
<td>$1.41 - 1.54$</td>
</tr>
</tbody>
</table>

arXiv: 1309.3413 (1309.3412: Cicoli, Downes, Dutta)
Francisco Pedro & AW
DESY Hamburg
March 2013:

### Observed Results:

- **Planck + WMAP7 + high-BAO + BAO**

### Constraints:

- $n_s = 0.9608 \pm 0.0054$ (68%)
- $r < 0.11$ (95%)
- $\Omega_k = -0.0004 \pm 0.00036$ (68%)
- $f_{NL}^{local} = 2.7 \pm 5.8$ (68%)
- $f_{NL}^{equil} = -42 \pm 75$ (68%)
- $f_{NL}^{orth} = -25 \pm 39$ (68%)
- $N_{eff} = 3.32^{+0.54}_{-0.52}$ (95%)
- $\sum m_\nu < 0.28$ eV (95%)

### General Notes:

- **15.5 months of temperature data**
- **no B-mode/E-mode polarization yet!**
- **full release of polarization and all 30 months of temperature data in 2014**
multipoles, the contributions to the SZ power spectrum mass clusters and groups have been studied in the literature (see Melin et al. 2010). Therefore, a plausible explanation for the lower signal at larger radii. In other words, the modeling of Nagai et al. (2007) in simulations of Nagai et al. (2007) in hydrodynamical simulations has been forced to match that from hydrodynamical simulations. The KS prediction for the projected SZ profiles is missing in low mass clusters, respectively. (We found that the scaling relation prevents us from convincingly ruling out the scaling relation. Note that if the SZ signal was at least a factor of two lower, we cut the X-ray derived pressure profile at \( r < 1 \), for \( a < 0 \).

While the KS profiles are generally in a good agreement with the pivot luminosity of the original scaling relation is a factor of two lower, we cut the X-ray derived pressure profile at \( r < 1 \), for \( a < 0 \).

If the SZ signal was at least a factor of two lower, we cut the X-ray derived pressure profile at \( r < 1 \), for \( a < 0 \).

The next generation of simulations or analytical cal-

\[ \frac{\sigma}{\sigma_0} \]

The next generation of simulations or analytical cal-

\[ \frac{\sigma}{\sigma_0} \]

The next generation of simulations or analytical cal-

\[ \frac{\sigma}{\sigma_0} \]
Inflation ...

• inflation: period quasi-exponential expansion of the very early universe

(solves horizon, flatness problems of hot big bang ...) 

• driven by the vacuum energy of a slowly rolling light scalar field:

\[ \ddot{\phi} + 3H \dot{\phi} + V' = 0 \]
**Inflation...**

- **slow-roll inflation:**

  scale factor grows exponentially: \( a \sim e^{Ht} \) if: \( \ddot{\phi} \ll \dot{\phi} \)

  \[
  \Rightarrow \quad \epsilon \equiv -\frac{\dot{H}}{H^2} \approx \frac{1}{2} \left( \frac{V'}{V} \right)^2 \ll 1, \quad \eta \equiv \frac{\dot{\epsilon}}{\epsilon H} \approx \frac{V''}{V} \ll 1
  \]

  with the Hubble parameter \( H^2 = \frac{\dot{a}^2}{a^2} \approx \text{const.} \sim V \)
Inflation...

- inflation generates metric perturbations: scalar (us) & tensor

\[ \mathcal{P}_S \sim \frac{H^2}{\epsilon} \sim \left( \frac{\delta \rho}{\rho} \right)^2 \]
\[ \sim k^{n_S - 1} \]

- scalar spectral index:
\[ n_S = 1 - 6\epsilon + 2\eta \]

\[ \mathcal{P}_T \sim H^2 \sim V \]

window to GUT scale &
direct measurement of inflation scale

but caveat: inflaton w/ pseudo-scalar couplings to light vector fields can source additional B-modes

[Barnaby, Namba & Peloso ’11; Senatore, Silverstein & Zaldarriaga ’11]
[Barnaby, Moxon, Namba, Peloso, Shiu & Zhou ’12]
**Inflation ...**

- **but:** if field excursion sub-Planckian, no measurable gravity waves: [Lyth ’97]

\[ r \equiv \frac{P_T}{P_S} = 16\epsilon \leq 0.003 \left( \frac{50}{N_e} \right)^2 \left( \frac{\Delta \phi}{M_P} \right)^2 \]

- **alternative:** [Hebecker, Kraus & AW ’13] also: [Ben-Dayan & Brustein ’09] [Hotchkiss, Mazumdar & Nadathur ’11]

  - have \( \epsilon \) **decreasing** during inflation ...  
  - then \( r \) can be \(~10\) x larger at 60 e-fold point than for typical small-field model  
  - automatic in "hybrid natural inflation" from an axion
Intermediate models fl
Inverse power law potential
r
logical perturbations unmodified
5 this class of models predicts
such a mechanism exists and leaves predictions for cosmological end without an additional mechanism to stop it
Assuming a
because the exact solution for the scale factor is given by
Inflation with an exponential potential
reheating priors allowing
the theoretical predictions of selected inflationary models
Fig. 1.
The constraints are given at the pivot scale
Table 4.

Planck Collaboration+ Constraints on inflation

Tensor\textsubscript{vto}\textsubscript{Scalar Ratio $r$

0.00 0.05 0.10 0.15 0.20 0.25

Primordial Tilt ($n_s$)

Convex
Concave

Planck+WP
Planck+WP+highL
Planck+WP+BAO
Natural Inflation
Power law inflation
Low Scale SSB SUSY
$R^2$ Inflation
$V \propto \phi^{2/3}$
$V \propto \phi$
$V \propto \phi^2$
$V \propto \phi^3$

$N_*=50$
$N_*=60$
single field models ...

- $R+R^2$ / Higgs inflation / fibre inflation in LVS string scenarios:

$$S = \int d^4x \sqrt{-g} \frac{M_{\text{pl}}^2}{2} \left( R + \frac{R^2}{6M^2} \right) \quad \Rightarrow \quad V(\phi) = \frac{3M^2}{4} \left( 1 - e^{-\sqrt{\frac{2}{3}} \phi} \right)^2$$

or fibre inflation: $V(\phi) \sim \left( 1 - \frac{4}{3} e^{-\sqrt{\frac{1}{3}} \phi} \right)$

$$n_s = 1 - 8 \frac{4N_e + 9}{(4N_e + 3)^2} \quad , \quad r = \frac{192}{(4N_e + 3)^2}$$
shades of difficulty ...

• observable tensors link levels of difficulty:

\[ r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_R} = 16\epsilon \leq 0.003 \left( \frac{50}{N_e} \right)^2 \left( \frac{\Delta \phi}{M_P} \right)^2 \] [Lyth ’97]

• \( r \ll O(1/N_e^2) \) models:

\[ \Delta \phi \ll O(M_P) \implies \text{Small-Field inflation ... needs control of leading dim-6 operators} \]
\[ \implies \text{enumeration & fine-tuning reasonable} \]

• \( r = O(1/N_e^2) \) models:

\[ \Delta \phi \sim O(M_P) \implies \text{needs severe fine-tuning of all dim-6 operators, or accidental cancellations} \]

• \( r = O(1/N_e) \) models:

\[ \Delta \phi \sim \sqrt{N_e} M_P \gg M_P \implies \text{Large-Field inflation ... needs suppression of all-order corrections} \]
\[ \implies \text{symmetry is essential!} \]
shades of difficulty ...

- Observable tensors link levels of difficulty:

\[ r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_R} = 16\epsilon \leq 0.003 \left( \frac{50}{N_e} \right)^2 \left( \frac{\Delta \phi}{M_P} \right)^2 \] [Lyth ’97]

- \( r \ll O(1/N_e^2) \) models:

observable tensors:

\[ \Delta \phi \ll O(M_P) \Rightarrow \]

- \( r = O(1/N_e^2) \) models:

\[ \Delta \phi \sim O(M_P) \Rightarrow \]

- \( r = O(1/N_e) \) models:

\[ \Delta \phi \sim \sqrt{N_e} M_P \gg M_P \Rightarrow \]

warped D-brane inflation & DBI;
varieties of Kahler moduli inflation

\[ r > 0.01 \]

fibre inflation in LARGE volume scenarios (LVS)

axion monodromy inflation
• hemispheric asymmetry of mean power and temperature $\sim 3\sigma$

• quadrupole - octopole alignment

• cold spot $\sim 3\sigma$

• fit Planck data from high-precision data at $l > 50$, then predict from that power at $l < 30$: *too low* power at low-$l$, 10% deficit, $\sim 2.5\sigma$

**theory task: explain!**
PLANCK anomaly - a lack of power at large scales !!

\[ q = \frac{C_\ell^{\ell=2...50}}{C_\ell^{\ell>50}} \]

\[ q = 0.9 \text{ at } 2 - 2.5\sigma \]
• significance of the power suppression -- now:

\[ \frac{\sigma^2_\ell}{C^2_\ell} = \frac{2}{N_\ell} \]

\[ N^{(\text{CMB})}_\ell = 2\ell + 1 \]

\[ \Rightarrow \text{significance of suppression measurement}: \]

\[ 2 \ldots 3 \sigma \quad \text{CMB} + \text{a bit of LSS} \]

• the future:

\[ N^{(\text{CMB}+\text{LSS}+21\text{cm})}_\ell = 2\ell + 1 + 4\pi \frac{\ell^2}{\ell_\zeta^3} \]

\[ \Rightarrow \text{significance of suppression measurement}: \]

\[ 3 \ldots 4 \sigma \quad \text{CMB} + \text{LSS from EUCLID} \]

\[ 5 \ldots 6 \sigma \quad \text{CMB} + \text{LSS from EUCLID} + \text{21cm data} \]
an idea: rapid steeping potential can suppress power ...

[Contaldi, Peloso, Kofman, Linde ’03]

• rapidly growing $V'$ such that $\varepsilon$ grows much faster than $V$ in a narrow interval $\Delta \phi$

  if $V \to \alpha V$, $\varepsilon \to \beta \varepsilon$, $\beta > \alpha \simeq 1$

  while $\phi \to \phi + \Delta \phi$

  then $\Delta_R^2 \sim \frac{H^4}{\dot{\phi}^2} \sim \frac{V}{\varepsilon} \to \frac{1}{\beta} \Delta_R^2 < \Delta_R^2$

• our claim: there is a model of string inflation - fibre inflation - which can do this!

[Pedro, AW ’13; Cicoli, Downs, Dutta ’13]

also: [Bousso, Harlow, Senatore ’13]
fibre inflation in type IIB string theory...

[Cholesi, Burgess & Quevedo '08]

**set up**

\[ K = -2 \log \left( \mathcal{V} + \frac{\xi}{2} \right) \quad \text{and} \quad W = W_0 + Ae^{-a\tau_3} \]

\[ \mathcal{V} = \lambda_1 t_1 t_2^2 + \lambda_2 t_3^3 = \alpha \left( \sqrt{\tau_1 \tau_2} - \gamma \tau_3^{3/2} \right) \]

\( t_i \) denote 2-cycle volumes, \( \tau_i = \partial \mathcal{V}/\partial t_i \) 4-cycle volumes

\[ \alpha = 1/(2\sqrt{\lambda_1}) \quad \text{and} \quad \gamma = \frac{2}{3} \sqrt{\lambda_1/(3\lambda_2)} \]
• **\( N = 1 \) eff. supergravity - F-term scalar potential**

\[
V_{\text{LVS}} = \frac{8\sqrt{\tau_3}A^2a^2e^{-2a\tau_3}}{\mathcal{V}} - \frac{4W_0\tau_3Aae^{-a\tau_3}}{\mathcal{V}^2} + \frac{3\xi}{4g_s^{3/2}\mathcal{V}^3}
\]

\[
\langle \mathcal{V} \rangle \sim e^{1/g_s} \quad \text{and} \quad \langle \tau_3 \rangle \sim \frac{1}{g_s}
\]

**SUSY breaking LVS (large volume) minimum**
• string loop corrections from KK- & winding modes

\[ \delta K_g^{KK} = \sum_{l=1}^{h(1,1)} \frac{C_i^{KK} a_{il} t^l}{Re(S)V} \]

\[ \delta K_g^W = \sum_{l=1}^{h(1,1)} \frac{C_i^W}{a_{il} t^l V} \]

• correct moduli potential

\[ \delta V_g = \left( \frac{(g s C_1^{KK})^2}{\tau_1^2} - 2 \frac{C_{12}^W}{V \sqrt{\tau_1}} + 2 \left( \frac{\alpha g s C_2^{KK}}{V^2} \right)^2 \tau_1 \right) \frac{W_0^2}{V^2} \]

• stabilizes fibre modulus \( \tau_1 \)

\[ \frac{1}{\langle \tau_1 \rangle^{3/2}} = \frac{4 \alpha C_{12}^W}{(g s C_1^{KK})^2 V} \left( 1 + \text{sign}(C_{12}^W) \sqrt{1 + 4 g_s^4 \left( \frac{C_1^{KK} C_2^{KK}}{C_{12}^W} \right)^2} \right) \]
• fibre modulus kinetic term - canonically normalize

\[ L_{\text{kin}} = \frac{3}{8\tau_1^2} \partial_\mu \tau_1 \partial^\mu \tau_1 \]

\[ \phi \equiv \frac{\sqrt{3}}{2} \ln \tau_1 \quad \text{or} \quad \tau_1 \equiv e^{\kappa \phi} \quad \text{with} \quad \kappa = \frac{2}{\sqrt{3}} \]

• fibre modulus scalar potential

\[ \delta V_{gs} = \frac{W_0^2}{V^2} \left( (g_s C_{1}^{KK})^2 e^{-2\kappa \phi} - 2 C_{12}^{W} \frac{C_{12}^{KK}}{V} e^{-\frac{1}{2} \kappa \phi} + 2 \left( \kappa \phi \right) \frac{\alpha g_s C_{2}^{KK}}{V^2} \right) \]

\[ V = V_0 \left( 1 - C_{1/2} e^{-\kappa \phi/2} + C_2 e^{-2\kappa \phi} + C_1 e^{\kappa \phi} \right) \]

\text{slow-roll flat & asymmetric plateau - and sudden steep wall}!!

\[ n_s \approx 0.97 \quad r \approx 0.006 \]
tune a Minkowski minimum for $\tau_1$ at zero VEV:

$$C_{1/2} = \frac{4}{3} , \quad C_2 = \frac{1}{3}$$

$$C_1 \sim g_S^\#, \# > 0 \quad \text{with} \quad C_1 \lesssim 10^{-5} \quad \text{tunes plateau}$$

there's always an inflection point on the plateau:

$$e^{-\kappa\phi_{ip}/2} = 3C_1 e^{\kappa\phi_{ip}}$$

at $\phi > \phi_{ip}$ we have $\epsilon$ monotonically increasing

best chance to get rapid increase in $\epsilon$ at large scales:

$$\phi_{60} = \phi_{ip}$$
• above the inflection point we have:

\[ V \simeq V_{ip} (1 + C_1 e^{\kappa \phi}) \]

\[ \epsilon = \frac{\kappa^2 C_1^2}{2} e^{2\kappa \phi} = \frac{3}{8} \eta^2 \quad \text{for} \quad \phi > \phi_{ip} \]

• field range of steepening:

find the point \( \phi_\delta > \phi_{ip} \) where \( \epsilon_\delta > \epsilon_{ip} \)

has a value such that \( \Delta^2_R(\phi_\delta) = \frac{\delta}{100} \Delta^2_R(\phi_{ip}) \)

\[ e^{-\kappa(\phi_\delta - \phi_{ip})} = \sqrt{\frac{\epsilon_{ip}}{\epsilon_\delta}} = \sqrt{\frac{\Delta^2_R(\phi_\delta)}{\Delta^2_R(\phi_{ip})}} = \frac{\sqrt{\delta}}{10} \]

• for moderate power suppression (\( \delta = 10...90 \)) implies:

\[ C_1 e^{\kappa(\phi_\delta - \phi_{ip})} \ll 1 \]

• \( V \) essentially unchanged while \( \epsilon \) increasing ...
• compute e-folds for field range of given power suppression:

\[
\Delta N_e^{(\delta)} = \int_{\phi_{ip}}^{\phi_{ip}} \frac{d\phi}{\sqrt{2\epsilon}} = \frac{1}{\kappa C_1} \int_{\phi_{ip}}^{\phi_{ip}} d\phi e^{-\kappa \phi} = \frac{1}{\kappa \sqrt{2\epsilon_{ip}}} \left( 1 - \frac{\sqrt{\delta}}{10} \right)
\]

\[
\Delta N_e^{(50\%)} \gtrsim 3 \sqrt{\frac{0.06}{1 - n_s}}
\]

while

\[
\Delta N_e (\ell = 2 \ldots 30) = \ln \frac{\ell = 30}{\ell = 2} < 3
\]

• positive exponential from loop term not steep enough but close ...
• Assume modified string loop corrections:

replace \[ \delta V \sim \frac{\tau_1}{V^4} \]

with \[ \delta V \sim \frac{\tau_1}{V^p} \quad \text{with} \quad p > 4 , \quad \frac{\tilde{k}}{k} \gtrsim 3 \]

• Modifies scalar potential:

\[ V = V_0 \left( 1 - C_{1/2}e^{-\kappa \phi/2} - C_2 e^{-2\kappa \phi} + \tilde{C}_1 e^{\tilde{k} \phi} \right) \]

\[ \Rightarrow \quad \Delta N^{(\delta)}_e = \frac{1}{\tilde{k}\tilde{C}_1} \int_{\phi_{ip}}^{\phi_\delta} d\phi e^{-\tilde{k} \phi} = \frac{1}{\tilde{k}\sqrt{2\epsilon_{ip}}} \left( 1 - \frac{\sqrt{\delta}}{10} \right) \]
- numerical check - solve e.o.m. for scalar field:

\[ \phi'' + 3 \left( 1 - \frac{1}{6} \phi'^2 \right) \left( \phi' + \frac{1}{V} \frac{\partial V}{\partial \phi} \right) = 0 \quad \text{with} \quad (\cdot)' \equiv \frac{d}{dN_e}(\cdot) \]
compute \( \Delta^2_R = \frac{1}{4\pi^2} \frac{H^4}{\dot{\phi}^2} \) on the solution

![Graphs showing power spectrum \( \Delta^2_R \) vs. \( l \)]

Figure 4: Power spectrum \( \Delta^2_R \) computed on the numerical solution \( \phi(N_e) \). Left: The original fibre inflation setup with \( \tilde{\kappa} = \kappa = 2/\sqrt{3} \) and \( C_1 = 10^{-5} \). Right: The modified setup with \( \tilde{\kappa} = 10\kappa \) and \( \tilde{C}_1 = 7 \times 10^{-33} \). The extra steepening leads to a clear suppression of curvature perturbation power at low \( \ell \).
open questions ...

• fibre inflation in type IIB string theory provides an asymmetric inflationary plateau with steep wall, which can explain the low-l power suppression hinted at by PLANCK

• here: conservative analysis, power suppression arises entirely within slow-roll -- if using fast-roll further up the steep wall, can use less steepness for the loop correction [Cicoli, Downs, Dutta ’13]

• how rare in the string model landscape? / change of the quantum vacuum between steep wall & plateau? / inflaton quanta or particle production? ... work in progress ... [Cicoli, Downs, Dutta, Pedro, AW]
The kinetic part of the Lagrangian is:

\[ \mathcal{L}_{\text{kin}} = K_{ij} \partial_{\mu} T_i \partial_{\nu} T_j = \frac{1}{4} \frac{\partial^2 K}{\partial \tau_i \partial \tau_j} \left( \partial_{\mu} \tau_i \partial_{\nu} \tau_j + \partial_{\mu} b_i \partial_{\nu} b_j \right) \]

\[
K^0_{ij} = \begin{pmatrix}
\frac{1}{4\tau_1^2} & \frac{\tau_3^{3/2}}{4\tau_1^{3/2} \tau_2} & -\frac{3\sqrt{3}}{8\tau_1^{3/2} \tau_2} \\
\frac{\tau_3^{3/2}}{4\tau_1^{3/2} \tau_2} & \frac{1}{2\tau_2} & -\frac{3\sqrt{3}}{4\sqrt{\tau_1 \tau_2}} \\
-\frac{3\sqrt{3}}{8\tau_1^{3/2} \tau_2} & -\frac{3\sqrt{3}}{4\sqrt{\tau_1 \tau_2}} & \frac{3}{8\sqrt{\tau_1 \tau_2} \tau_3} \\
\end{pmatrix}
\]

\[ \mathcal{L}_{\text{kin}} = \frac{3}{8\tau_1^2} \partial_{\mu} \tau_1 \partial_{\nu} \tau_1 \quad \phi \equiv \frac{\sqrt{3}}{2} \ln \tau_1 \quad \text{or} \quad \tau_1 \equiv e^{\kappa \phi} \quad \text{with} \quad \kappa = \frac{2}{\sqrt{3}} \]

\[ \delta V_{gs} = \frac{W_0^2}{V^2} \left( (g_s C_1^{KK})^2 e^{-2\kappa \phi} - 2 \frac{C_1^{W}}{V} e^{-\frac{1}{2} \kappa \phi} + 2 \left( \alpha g_s C_2^{KK} \right)^2 e^{\kappa \phi} \right) \]

\[ V = V_0 \left( 1 - C_1/2 e^{-\kappa \phi/2} + C_2 e^{-2\kappa \phi} + C_1 e^{\kappa \phi} \right) \]
result of a (nearly) scale invariant spectrum

To determine how strongly the data favour a primordial spectrum of the initial spectrum in Eq. 3. Thus it is difficult to constrain on the lowest multipoles in considering the angular fit to an arbitrary cut-off of cosmological parameters, an exercise which should be done with the exact evolution of the grid values are regular in the parameters and have corresponding to these cut-off.

The authors of [49] erroneously compare our analysis to theirs. Other parameters that are not expected to affect claims which strongly depend on their use.

In Fig. 2 we show the resulting best-fit models we obtained with the exact numerical evolution of the CMBFAST [50] code to compute CMB spectra with various parameters that have been kept fixed such as the energy density of the Universe.

In Fig. 3 we marginalize over the range in values giving one, two, and three standard deviations. The energy density of the Universe is 5.6% too strong conclusions by making use of the E-type power spectrum, as obtained from an exact evolution of the model without cut-off used in the two cuto-FF code and the WMAP data only. We believe that it is still premature to draw any conclusions concerning the problem of low values as a function of time, and we concentrate on the lowest multipoles in considering the angular value of the amplitude of the power spectrum.

The bottom panel shows the two primordial power spectra in the initial spectrum will not produce a step-function in the CMB spectrum at the lowest multipoles, namely the spectral position between the two regimes in the initial spectrum.

FIG. 2: CMB anisotropies with cut-off used in the WMAP results.

In Fig. 1: Power spectrum for $P_\ell$ of Eq. 16, a new model derived from the simple model discussed above, has fit the WMAP data with two template cut-offs, Eqs. (15) and (16). This is to be compared with the contribution to equation (14)