# WZ TERM IN PERMUTATION COSETS

BASED ON

A. Cagnazzo and K. Zarembo, "B-field in AdS(3)/CFT(2) Correspondence and Integrability," arXiv:1209.4049 [hep-th].

Padova 17/09/2013
Perugia 18/09/2013

GATIS NETWORK

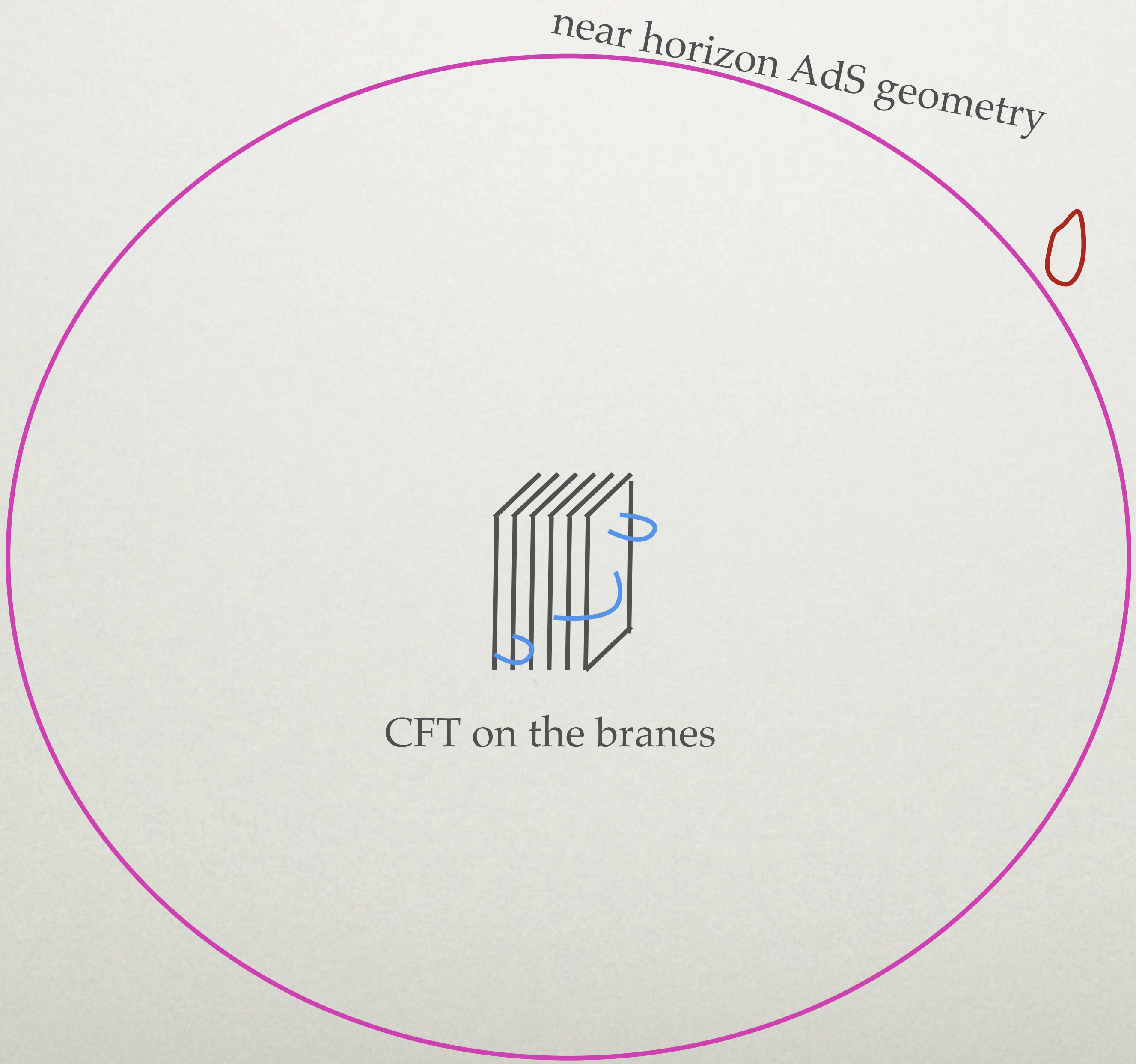
# Plan of the talk

- Introduction
- Permutation coset and WZ term
- Integrability
- Conformality and Kappa-symmetry
- BMN limit

# The aim is to study a superstring theory on an $AdS_3 \times \mathcal{M}^7$ background. Why?

## AdS /CFT

Correspondence between a d-dimensional gauge theory, the boundary theory, and a (d+1)-dimensional theory containing gravity on an Anti de Sitter space, the bulk theory.




Maldacena '97

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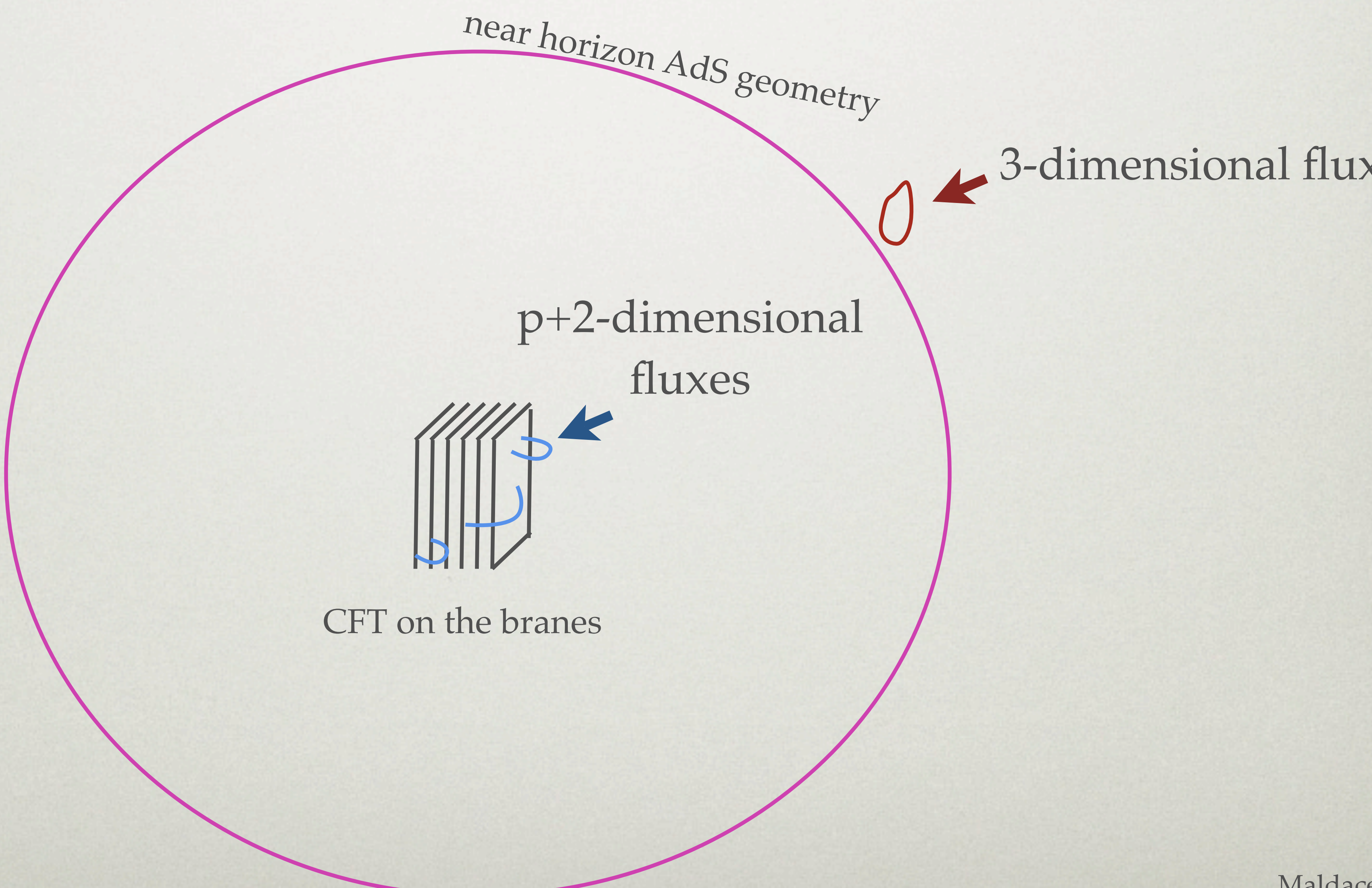


Maldacena '97

Supercoset sigma models can be used to describe a Superstring in an AdS background

# SEMI-SYMMETRIC COSETS

Supercoset sigma models can be used to describe a Superstring in an AdS background

# SEMI-SYMMETRIC COSETS

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$$\frac{G}{H_0} \quad \begin{array}{l} \longrightarrow \text{global symmetry} \\ \longrightarrow \text{subgroup of G} \end{array}$$

$$\mathfrak{g} = \mathfrak{h}_\circ \oplus \mathfrak{h}'$$

$\mathfrak{h}_\circ$ is a subalgebra

$$[\mathfrak{h}_\circ, \mathfrak{h}_\circ] \subset \mathfrak{h}_\circ$$

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Under which $\mathfrak{h}_0$ elements are invariant

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Under which $\mathfrak{h}_\circ$ elements are invariant

purely bosonic case

$\Omega$ is parity

$$\Omega^2 = \mathrm{Id}$$

$$[\mathfrak{h}', \mathfrak{h}'] \subset \mathfrak{h}_\circ$$

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Under which $\mathfrak{h}_0$ elements are invariant

## supersymmetric case

$\Omega$ is Z4 symm $\quad \Omega^4 = \mathrm{Id}$

$$\mathfrak{h}' = \mathfrak{h}_1 \oplus \mathfrak{h}_2 \oplus \mathfrak{h}_3$$

$\mathfrak{h}_0, \mathfrak{h}_2$ bosonic

$\mathfrak{h}_1, \mathfrak{h}_3$ fermionic

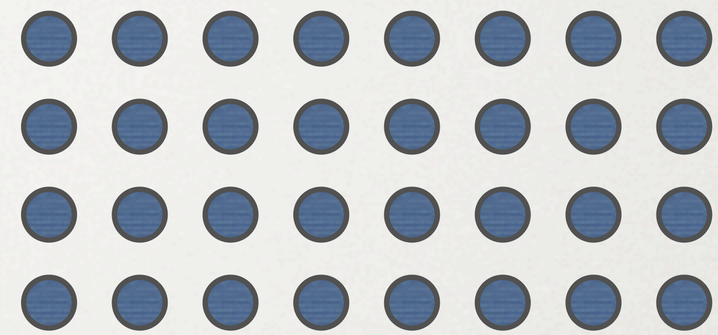
$$[\mathfrak{h}_n, \mathfrak{h}_m] \subset \mathfrak{h}_{m+n,\mathrm{mod}4}$$

# Superstring theory on AdS background and Supercoset sigma model

Metsaev,Tseytlin'98

max susy string theory

$$AdS_5 \times S^5$$

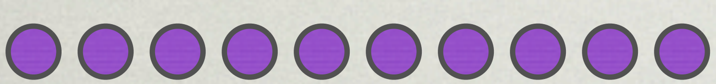


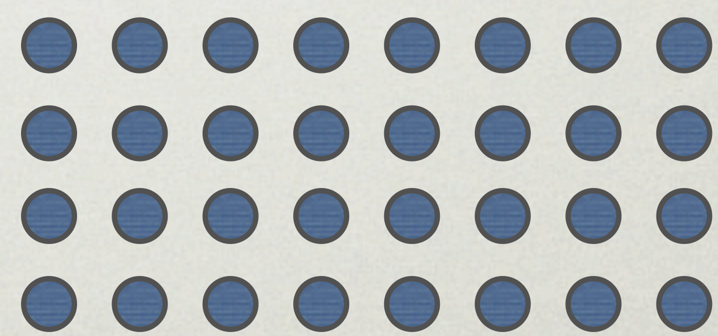
Bosons

Fermions

Supercoset

$$\frac{PSU(2,2|4)}{SO(1,4) \times SO(5)}$$





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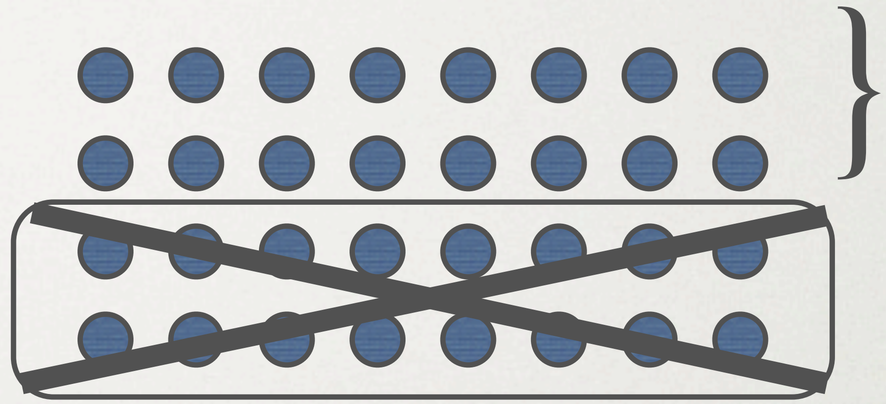


Kappa Symmetry
Local fermionic symmetry
to gauge the theory

Bosons

Fermions

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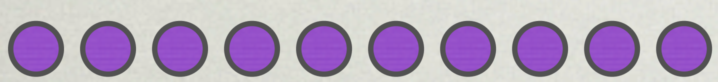
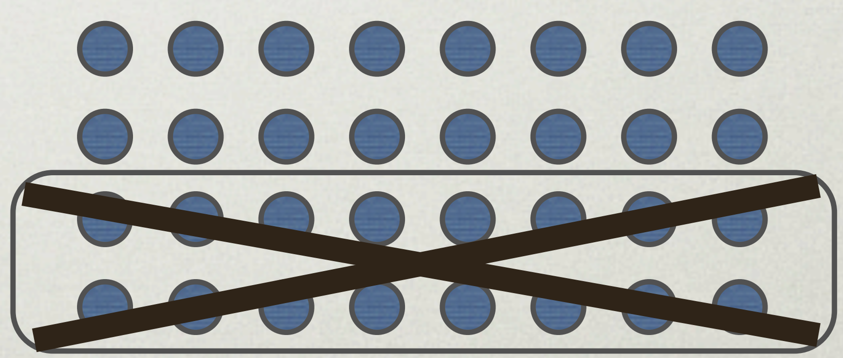
$$\frac{PSU(2,2|4)}{SO(1,4)\times SO(5)}$$





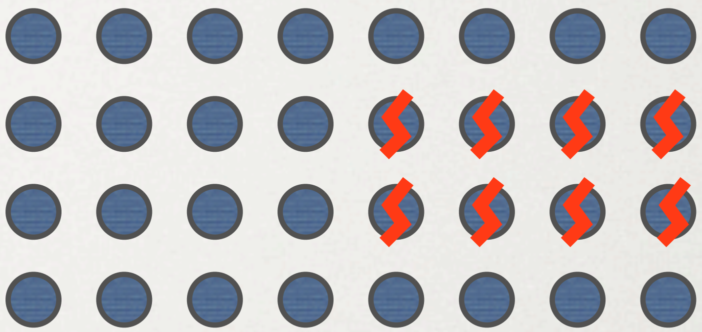
Kappa-gauge fixing

# Superstring theory on AdS background and Supercoset sigma model

Grassi, Sorokin, Wulff ‘09

non-max susy string theory

$AdS_4 \times CP^3$

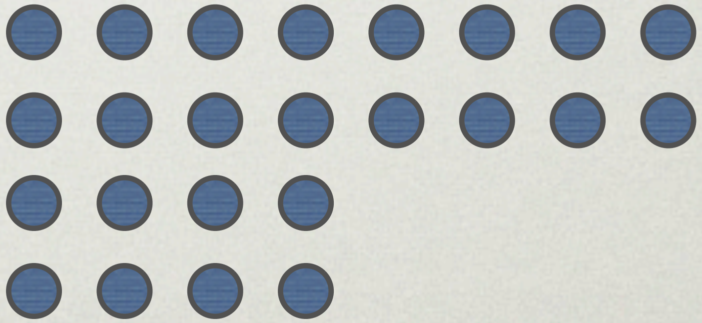


Bosons

Fermions

Supercoset

$$\frac{OSp(6|4)}{SO(1,3) \times U(3)}$$



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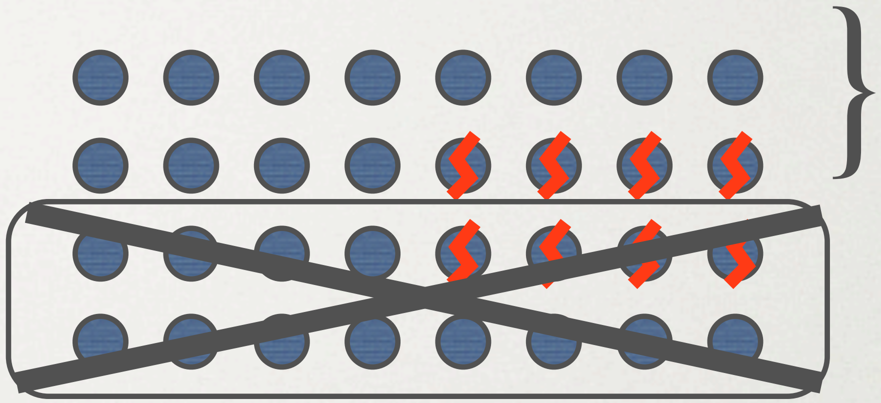

physical dof

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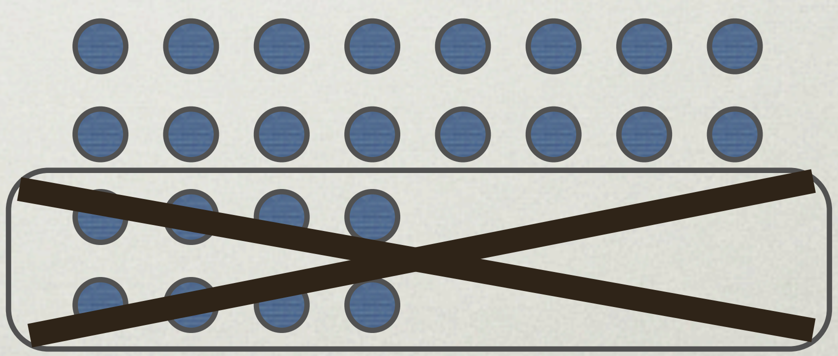

Kappa-gauge fixing

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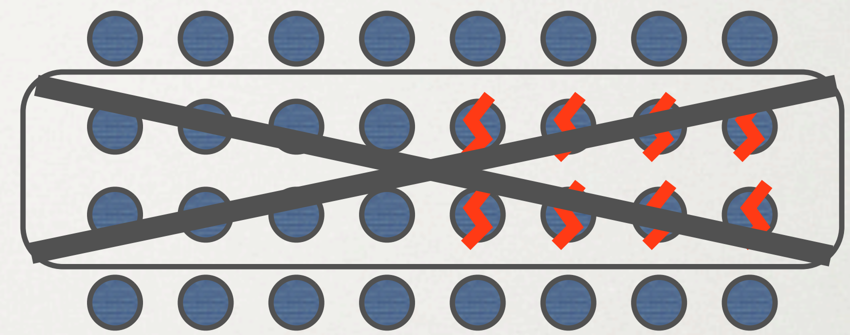


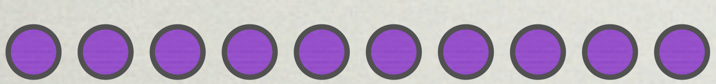
GOOD!

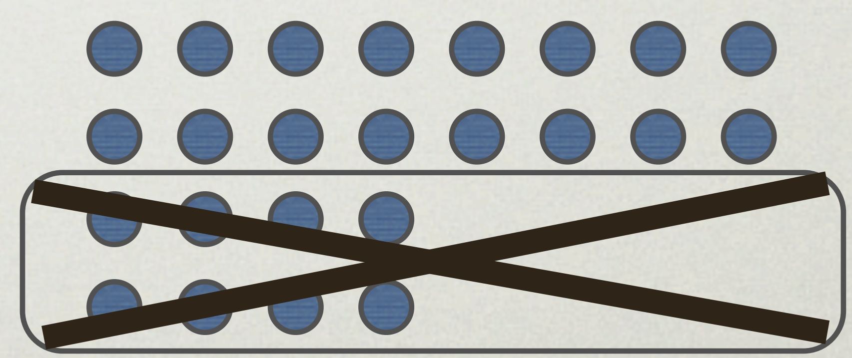
Bosons

Fermions

Supercoset

$$\frac{OSp(6|4)}{SO(1,3) \times U(3)}$$



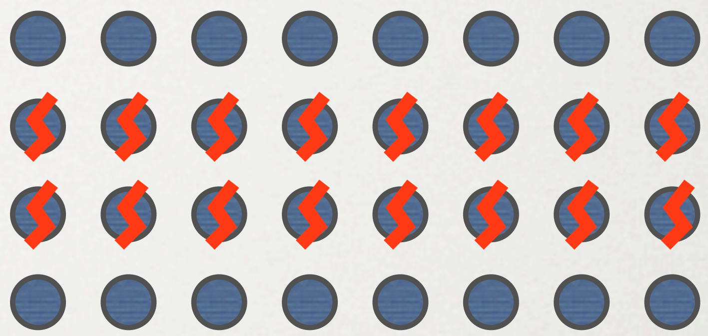


Kappa-gauge fixing

## Superstring theory on AdS background and Supercoset sigma model

non-max susy string theory

$$AdS_3 \times S^3 \times T^4$$

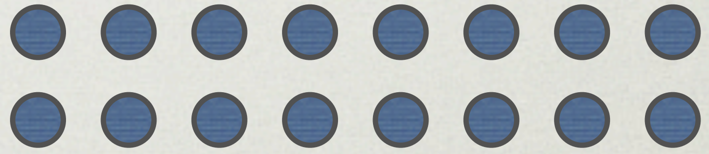


Bosons

Fermions

Supercoset

$$\frac{PSU(1,1|2)^2}{SU(1,1) \times SU(2)}$$



## Why is it useful to describe superstring with a coset sigma model?

Z4 Coset sigma models possess the property to be

INTEGRABLE

Bena,Polchinski,Roiban'03

Classical Integrability $\Longleftrightarrow$ Flat Lax connection

$$dL + LL = 0$$

To describe a string theory the sigma model has to be

CONFORMAL

zero beta function

good cosets classified by K.Zarembo in 2009

# AdS /CFT

# $AdS_3/CFT_2$

$AdS_3$ admits: NSNS
RR

# $AdS_3/CFT_2$

$AdS_3$ admits: NSNS
RR

NSNS flux is sourced by the fundamental string, RR are coupled to the D-branes.

In type IIB $AdS_3$ theory I have a NSNS three-form and a RR three-form

# $AdS_3/CFT_2$

$AdS_3$ admits: NSNS $\longrightarrow$ Worldsheet CFT methods

Maldacena and Ooguri/ Giveon, Kutasov and Seiberg/
de Boer, Feinermann, Giveon and Tsabar/....

RR $\longrightarrow$ $AdS_3 x S^3 x T^4$ and $AdS_3 x S^3 x S^3 x S^1$ can be studied with integrable supercosets.

Babichenko, Stefanski and Zarembo/...

We want to study the background supported simultaneously by NSNS and RR!

Start from the RR Background $\longrightarrow$ Z4 supercoset sigma model

\+

B field $\longrightarrow$ WZ term

# Z4 Supercoset

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Berkovits, Bershadsky, Hauer, Zhukov and Zwiebach
For AdS3: Babichenko, Stefanski and Zarembo

Semi-symmetric $\frac{\mathcal{G}}{\mathcal{H}_0}$ superspace, invariant under $\Omega$ automorphism

$$\Omega(J_n) = i^n J_n \qquad\qquad [T_n, T_m\} = T_{m+n,\mathrm{mod}4}$$

$$S = \frac{1}{2}\int_{\mathcal{M}} \mathrm{Str}\left(J_2 \wedge *J_2 + J_1 \wedge J_3\right)$$

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$$\sqrt{-h}h^{\mu\nu} J_{2\mu} J_{2\nu} \qquad\qquad \epsilon^{\mu\nu} J_{1\mu} J_{3\nu}$$

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Z4 forbidden

bosonic coset
Hull, Spence '89
Witten '92

# Permutation Supercoset

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$$\frac{G \times G}{G_B} \qquad\qquad \Omega = \begin{pmatrix} 0 & \mathrm{id} \\ (-1)^F & 0 \end{pmatrix}.$$

Babichenko, Stefanski and Zarembo

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Babichenko, Stefanski and Zarembo

$$G = PSU(1,1|2) \qquad\qquad G = D(2,1;\alpha)$$

$$AdS_3 \times S^3 \times T^4 \qquad\qquad AdS_3 \times S^3 \times S^3 \times S^1$$

$$\mathcal{G} = G_L \times G_R \longrightarrow$$

global symmetry acts by independent multiplication from the left

$$g_{L,R} \to h_{L,R} g_{L,R}$$

$$J_{L,R} = g_{L,R}^{-1} d g_{L,R} \qquad J_{L,R} = J_{L,R}^B + J_{L,R}^F,$$

$$\Omega(J_{L,R}^B) = J_{R,L}^B, \qquad \Omega(J_{L,R}^F) = \mp J_{R,L}^F$$

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Maurer-Cartan:

$$\begin{aligned} dJ^B_{L,R} + J^B_{L,R} \wedge J^B_{L,R} + J^F_{L,R} \wedge J^F_{L,R} &= 0 \\ dJ^F_{L,R} + J^B_{L,R} \wedge J^F_{L,R} + J^F_{L,R} \wedge J^B_{L,R} &= 0 \end{aligned}$$

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Maurer-Cartan:

$F = dJ_0 + J_0 \wedge J_0$

$$\begin{aligned} J_0 &= \frac{1}{2}\left(J_L^B + J_R^B\right) \\ J_1 &= \frac{1}{2}\left(J_L^F + iJ_R^F\right) \\ J_2 &= \frac{1}{2}\left(J_L^B - J_R^B\right) \\ J_3 &= \frac{1}{2}\left(J_L^F - iJ_R^F\right). \end{aligned}$$

$$\begin{aligned} F + J_2 \wedge J_2 + J_1 \wedge J_3 + J_3 \wedge J_1 &= 0 \\ DJ_2 + J_1 \wedge J_1 + J_3 \wedge J_3 &= 0 \\ DJ_1 + J_2 \wedge J_3 + J_3 \wedge J_2 &= 0 \\ DJ_3 + J_2 \wedge J_1 + J_1 \wedge J_2 &= 0. \end{aligned}$$

$DC_p = dC_p + J_0 \wedge C_p + (-1)^{p+1} C_p \wedge J_0$

# WZ term

Integral over $\mathcal{B}$, with $\partial\mathcal{B} = \mathcal{M}$. Locally a total derivative ➡ in the EoM only boundary fields.

$$\delta g_{L,R} = g_{L,R}\xi_{L,R} \qquad\qquad \xi_{L,R} \in \mathfrak{g}$$

$$\delta J_{L,R} = d\xi_{L,R} + [J_{L,R}, \xi_{L,R}]$$

$$\begin{aligned}
\delta J_0 &= D\xi_0 + [J_2, \xi_2] + [J_1, \xi_3] + [J_3, \xi_1] \\
\delta J_2 &= D\xi_2 + [J_2, \xi_0] + [J_1, \xi_1] + [J_3, \xi_3] \\
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$\delta S_{WZ}$ =total derivative

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$\delta S_{WZ}$ =total derivative

$$S_{\mathrm{WZ}} = \int_{\mathcal{B}} \mathrm{Str}\left(\frac{2}{3}\, J_2 \wedge J_2 \wedge J_2 + J_1 \wedge J_3 \wedge J_2 + J_3 \wedge J_1 \wedge J_2\right)$$

This WZ term has grading two! The manifest Z4 invariance of the action is broken by the introduction of this.

$$S = \frac{1}{2}\int_{\mathcal{M}} \mathrm{Str}\left(J_2 \wedge *J_2 + \kappa J_1 \wedge J_3\right)$$

Introduced because of integrability,
Kappa symmetry and conformality.

$$+\chi \int_{\mathcal{B}} \left(\frac{2}{3} J_2 \wedge J_2 \wedge J_2 + J_1 \wedge J_3 \wedge J_2 + J_3 \wedge J_1 \wedge J_2\right)$$

Equations of motion

$$\begin{aligned} D * J_2 - \kappa\, J_1 \wedge J_1 + \kappa\, J_3 \wedge J_3 - 2\chi J_2 \wedge J_2 - \chi J_1 \wedge J_3 - \chi J_3 \wedge J_1 &= 0 \\ (\kappa J_1 + *J_1) \wedge J_2 + J_2 \wedge (\kappa J_1 + *J_1) + \chi\,(J_2 \wedge J_3 + J_3 \wedge J_2) &= 0 \\ (\kappa J_3 - *J_3) \wedge J_2 + J_2 \wedge (\kappa J_3 - *J_3) + \chi\,(J_2 \wedge J_1 + J_1 \wedge J_2) &= 0 \end{aligned}$$

+ Maurer-Cartan equations

to find flat Lax Connection

$$dL + LL = 0$$

$$L = J_0 + \alpha_1 J_2 + \alpha_2 * J_2 + \beta_1 J_1 + \beta_2 J_3$$

$$dL + LL = 0$$

$$\kappa^2 = 1 - \chi^2$$

ensure to find a flat Lax connection

$$\begin{aligned} \alpha_2 &= \chi \pm \sqrt{-1 + \alpha_1^2 + \chi^2} \\ \beta_1 &= \pm\sqrt{\alpha_1 - \kappa\,\alpha_2} \\ \beta_2 &= \pm\sqrt{\alpha_1 + \kappa\,\alpha_2} \end{aligned}$$

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$$\alpha_1 = \kappa\,\frac{\mathrm{x}^2+1}{\mathrm{x}^2-1}$$

$$\begin{aligned} L &= J_0 + \kappa\,\frac{\mathrm{x}^2+1}{\mathrm{x}^2-1}\,J_2 + \left(\chi - \frac{2\kappa\mathrm{x}}{\mathrm{x}^2-1}\right) * J_2 \\ &\quad + \left(\mathrm{x} + \frac{\kappa}{1-\chi}\right)\sqrt{\frac{\kappa\,(1-\chi)}{\mathrm{x}^2-1}}\,J_1 + \left(\mathrm{x} - \frac{\kappa}{1+\chi}\right)\sqrt{\frac{\kappa\,(1+\chi)}{\mathrm{x}^2-1}}\,J_3 \end{aligned}$$

- $\chi = 0 \;\; \kappa = 1$ supercoset Lax connection
- $\chi = 1 \;\; \kappa = 0$ WZ point

# BETA FUNCTION

# Let's expand the action around a classical background

$$\bar{g}_{L,R} \in G^B$$

$$\bar{J}_{L,R} = A \pm K$$

Grading zero    Grading two

MC

$$\begin{aligned} F + K \wedge K &= 0 \\ DK &= 0, \end{aligned}$$

$dA + A \wedge A$

EoM

$$D * K - 2\chi K \wedge K = 0$$

$$g_{L,R} = \bar{g}_{L,R}\, \mathrm{e}^{X_{L,R}}$$

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EoM

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$$g_{L,R} = \bar{g}_{L,R}\, \mathrm{e}^{X_{L,R}}$$

Fluctuation fields

# Let's expand the action around a classical background

$$\bar{g}_{L,R} \in G^B$$

$$\bar{J}_{L,R} = A \pm K$$

Grading zero     Grading two

MC

$$\begin{aligned} F + K \wedge K &= 0 \\ DK &= 0, \end{aligned}$$

$dA + A \wedge A$

EoM

$$D * K - 2\chi K \wedge K = 0$$

$$g_{L,R} = \bar{g}_{L,R}\, \mathrm{e}^{X_{L,R}}$$

Fluctuation fields

$$J = \bar{J} + \frac{1 - \mathrm{e}^{-\operatorname{ad} X}}{\operatorname{ad} X}\, \mathcal{D}X = \bar{J} + \mathcal{D}X - \frac{1}{2}\,[X, \mathcal{D}X] + \ldots$$

$$\mathcal{D}X = dX + [\bar{J}, X]$$

## Supercoset case

Zarembo 2010

$$S = \frac{1}{2} \int_{\mathcal{M}} \mathrm{Str} \left( J_2 \wedge *J_2 + J_1 \wedge J_3 \right)$$

one-loop effective action:

# Supercoset case

Zarembo 2010

$$S = \frac{1}{2}\int_{\mathcal{M}} \mathrm{Str}\left(J_2 \wedge *J_2 + J_1 \wedge J_3\right)$$

## one-loop effective action:

$$\begin{aligned}
S^{(2)} \quad = \quad & \frac{1}{2}\int \mathrm{Str}\left(DX_2 \wedge *DX_2 - [K, X_2] \wedge * [K, X_2]\right. \\
& + X_1 D * [K, X_1] - X_1 D [K, X_1] + X_3 D * [K, X_3] + X_3 D [K, X_3] \\
& \left. -2 [K, X_1] \wedge * [K, X_3] - 2 [K, X_1] \wedge [K, X_3]\right).
\end{aligned}$$

# Supercoset case

Zarembo 2010

$$S = \frac{1}{2}\int_{\mathcal{M}} \mathrm{Str}\left(J_2 \wedge *J_2 + J_1 \wedge J_3\right)$$

# one-loop effective action:

to be kappa-gauge fixed!

$$\frac{1}{2} Sp_2 \ln(-D_+ D_- + \mathrm{ad}K_+ \mathrm{ad}K_-) - \frac{1}{2} Sp'_{1\oplus 3} \begin{pmatrix} \mathrm{ad}K_+ \mathrm{ad}K_- & D_- \mathrm{ad}K_+ \\ D_+ \mathrm{ad}K_- & \mathrm{ad}K_- \mathrm{ad}K_+ \end{pmatrix}$$

$D_\pm = D_0 \pm D_1$ and similarly for $K_\pm$

$$\begin{pmatrix} \mathrm{ad}K_+ \mathrm{ad}K_- & D_- \mathrm{ad}K_+ \\ D_+ \mathrm{ad}K_- & \mathrm{ad}K_- \mathrm{ad}K_+ \end{pmatrix} = \begin{pmatrix} D_- & \mathrm{ad}K_+ \\ \mathrm{ad}K_- & D_+ \end{pmatrix} \boxed{\begin{pmatrix} 0 & \mathrm{ad}K_+ \\ \mathrm{ad}K_- & 0 \end{pmatrix}}$$

Kappa-symmetry projector

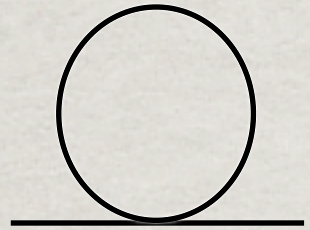
After Kappa-gauge fixing

$$S^{(ferm)}_{eff} = -\frac{1}{2} Sp_{1\oplus 3} \begin{pmatrix} D_- & \mathrm{ad}K_+ \\ \mathrm{ad}K_- & D_+ \end{pmatrix}$$

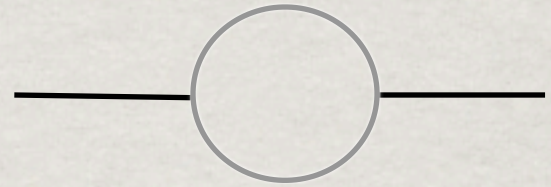
$$\mathrm{rank}_\kappa = \dim\ker \mathrm{ad}\, K_+|_{\mathfrak{h}_1} + \dim\ker \mathrm{ad}\, K_-|_{\mathfrak{h}_3}$$

# Beta-function: conformality?

Log-divergent contribution to the effective action, responsible for the renormalization of the dimension two operator $K \wedge *K$.



bosonic



fermionic

$$I = -\frac{1}{8\pi} \ln \Lambda \int \mathrm{Str}\, \mathrm{ad}K \wedge * \mathrm{ad}K$$

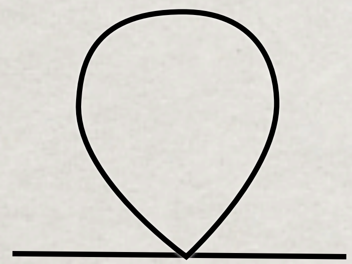
For a generic coset, the beta function is proportional to the Killing form. When the Killing form vanishes the theory is conformal. This in particular happens for

$PSU(n|n)$ and $OSp(2n+2|2n)$

$D(2,1;\alpha)$

# Introducing the WZ term:

Bosonic contribution



$$S_B^{(2)} = \frac{1}{2}\int \mathrm{Str}\left\{ D_{\chi * K} X_2 \wedge * D_{\chi * K} X_2 - \left(1-\chi^2\right)[K, X_2] \wedge * [K, X_2] \right\}$$

enters in dim 4 operators

$$I = -\frac{1-\chi^2}{8\pi} \ln \Lambda \int \mathrm{tr}_B \, \mathrm{ad}K \wedge * \mathrm{ad}K$$

$$S_F^{(2)} = \frac{1}{2}\int \mathrm{Str}\, X_I \underbrace{\left(D + \sigma_1\, \mathrm{ad}\, K\wedge\right)^{IJ} \left(* - \kappa\sigma_3 - \chi\sigma_1\right)^{JL}}_{\text{Hermitian Dirac Operator}} \mathrm{ad}\, K\, X_L$$

$I = (1, 3)$

$$S_F^{(2)} = \frac{1}{2}\int \mathrm{Str}\, X_I \underbrace{(D + \sigma_1\, \mathrm{ad}\, K\wedge)^{IJ} (* - \kappa\sigma_3 - \chi\sigma_1)^{JL}}_{\text{Hermitian Dirac Operator}}\, \mathrm{ad}\, K\, X_L$$

$I = (1,3)$

# Kappa-symmetry

A consistent WZW has not to spoil the Kappa-symmetry, so we expect that its introduction do not affect the rank of the Kappa-symmetry.

$$\mathrm{rank}_\kappa = \dim \ker \mathrm{ad}\, K_+|_{\mathfrak{h}_1} + \dim \ker \mathrm{ad}\, K_-|_{\mathfrak{h}_3}$$

$$[K_\pm, \epsilon^\pm] = 0 \qquad \delta X_I = C_I^\pm \epsilon^\pm$$

$$\delta S_F^{(2)} = \int \mathrm{Str}\, X\, (D + \sigma_1\, \mathrm{ad}\, K\wedge)\, (\pm 1 - \kappa\sigma_3 - \chi\sigma_1)\, C^\pm\, \mathrm{ad}\, K\, \epsilon^\pm$$

$$\Downarrow$$

$$(\pm 1 - \kappa\sigma_3 - \chi\sigma_1)\, C^\pm = 0$$

$$S_F^{(2)} = \frac{1}{2}\int \mathrm{Str}\, X_I \underbrace{\left(D + \sigma_1\,\mathrm{ad}\,K\wedge\right)^{IJ}\left(* - \kappa\sigma_3 - \chi\sigma_1\right)^{JL}}_{\text{Hermitian Dirac Operator}}\,\mathrm{ad}\,K\, X_L$$

$I = (1,3)$

# Kappa-symmetry

A consistent WZW has not to spoil the Kappa-symmetry, so we expect that its introduction do not affect the rank of the Kappa-symmetry.

$$\mathrm{rank}_\kappa = \dim\ker\mathrm{ad}\,K_+\big|_{\mathfrak{h}_1} + \dim\ker\mathrm{ad}\,K_-\big|_{\mathfrak{h}_3}$$

$$[K_\pm, \epsilon^\pm] = 0 \qquad\qquad \delta X_I = C_I^\pm\epsilon^\pm$$

$$\delta S_F^{(2)} = \int \mathrm{Str}\, X\left(D + \sigma_1\,\mathrm{ad}\,K\wedge\right)\left(\pm1 - \kappa\sigma_3 - \chi\sigma_1\right)C^\pm\,\mathrm{ad}\,K\,\epsilon^\pm$$

$$\Downarrow$$

$$\left(\pm1 - \kappa\sigma_3 - \chi\sigma_1\right)C^\pm = 0$$

has solutions if:

$$1 - \kappa^2 - \chi^2 = 0$$

SAME CONSTRAINT
FROM INTEGRABILITY
AND KAPPA-SYMMETRY

The fermionic action can be written as:

$$S_F^{(2)} \quad = \quad = \frac{1}{2}\int \mathrm{Str}\, X_I \left\{\left(-D_+\bar{\gamma} + D_-\gamma - \mathrm{ad}\, K_+\sigma_1\bar{\gamma} + \mathrm{ad}\, K_-\sigma_1\gamma\right)\sigma_1\Gamma\right\}^{IJ}\, X_J$$

$$\Gamma = \sigma_1(\bar{\gamma}\,\mathrm{ad}\, K_- + \gamma\,\mathrm{ad}\, K_+)$$

$$\gamma = \frac{1+\kappa\sigma_3+\chi\sigma_1}{2} \qquad \bar{\gamma} = \frac{1-\kappa\sigma_3-\chi\sigma_1}{2}$$

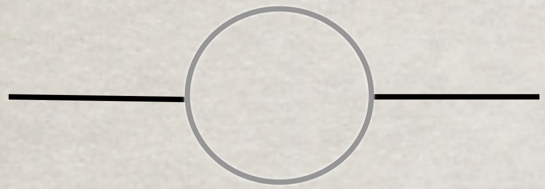
$$\gamma^2 = \gamma \quad \bar{\gamma}^2 = \bar{\gamma} \quad \gamma\bar{\gamma} = \bar{\gamma}\gamma = 0$$

Kappa-gauge fixing:

$$X_I = (\Gamma\, X)_I$$

This means that the Dirac operator reduces to:

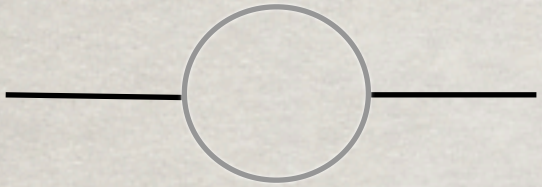
$$D_F = -D_+\sigma_1\bar{\gamma}\sigma_1 + D_-\sigma_1\gamma\sigma_1 - \mathrm{ad}\, K_+\bar{\gamma}\sigma_1 + \mathrm{ad}\, K_-\gamma\sigma_1$$



$$D_F = \underbrace{-D_+\sigma_1\bar{\gamma}\sigma_1 + D_-\sigma_1\gamma\sigma_1}_{\not{D}} \underbrace{- \mathrm{ad}\,K_+\bar{\gamma}\sigma_1 + \mathrm{ad}\,K_-\gamma\sigma_1}_{M}$$

$$I_{fer} = -\frac{1}{2}\mathrm{tr}'\ln(\not{D} + M)$$

$$-\frac{\kappa^2}{8\pi}\ln\Lambda\int\;\mathrm{tr}_F\,\mathrm{ad}K\wedge *\mathrm{ad}K$$



$$D_F = \underbrace{-D_+\sigma_1\bar{\gamma}\sigma_1 + D_-\sigma_1\gamma\sigma_1}_{\not{D}} \underbrace{- \mathrm{ad}\, K_+\bar{\gamma}\sigma_1 + \mathrm{ad}\, K_-\gamma\sigma_1}_{M}$$

$$I_{fer} = -\frac{1}{2}\mathrm{tr}'\ln(\not{D} + M)$$

$$-\frac{\kappa^2}{8\pi}\ln\Lambda\int \mathrm{tr}_F\, \mathrm{ad}K \wedge *\mathrm{ad}K$$

Summing the two contributions

$$I = -\frac{\kappa^2}{8\pi}\ln\Lambda\int \mathrm{Str}\,\mathrm{ad}K \wedge *\mathrm{ad}K$$

differs from the supercoset case by a factor $\kappa^2$

coset vanishing conditions+ a fixed point in $\chi = 1$ $\kappa = 0$

# BMN limit

*Without WZW: Babichenko, Stefanski and Zarembo*

Background: point-like string moving along a light-like geodesic

$$\bar{g}_{L,R} = e^{i(D+J)\tau}$$

Dilaton generator Rotation generator Worldsheet time

$$A = 0, \;\; K = i(D+J)d\tau$$

$$S_B^{(2)} = \frac{1}{2}\int \mathrm{Str}\left\{D_{\chi * K}X_2 \wedge *D_{\chi * K}X_2 - \boxed{\left(1-\chi^2\right)[K, X_2] \wedge *[K, X_2]}\right\}$$

After a rotation

$$e^{-\frac{i}{2}s\sigma_2}$$

$\cos s = \kappa$ $\sin s = \chi$

$$S_F^{(2)} = \frac{1}{4}\int \mathrm{Str}\, X\left\{-\partial_+(1-\sigma_3) + \partial_-(1+\sigma_3) + 2(\chi - i\kappa\sigma_2)\,\mathrm{ad}\,K\right\}[K, X].$$

Field redefinition

$$X \longrightarrow \mathrm{e}^{\,i\chi(D+J)\sigma}\, X\, \mathrm{e}^{\,-i\chi(D+J)\sigma}$$

$$\mathcal{M}_B^2 = -\kappa^2(\mathrm{ad}K)^2 \qquad \mathcal{M}_F = i\kappa\mathrm{ad}K$$

# Conclusions

- We have built the WZ term for a particular class of cosets, the Permutation cosets in order to Study superstring theory on $AdS_3$ backgrounds, supported by both type of fluxes RR and NSNS.
- We have shown that this does not spoil Integrability, Kappa Symmetry and Conformality, though we have to introduce a relation between the coupling constants of the theory.
- We have computed the BMN spectrum for the theory.

- S-matrix with deformation: proposal by Tsytlin and Hoare.
- Direct string worldsheet calculation by Wulff and Sundin
- To study the case $\chi = 1$ $\kappa = 0$ could provide a link between integrability technique and CFT worldsheet methods.
- Study of the finite gap equation to understand how to modify the receipt in absence of $Z4$ symmetry
- Integrability of the full theory: non supersymmetric and MASSLESS MODES in the light cone gauge.

Thank
you