Tensors in the Landscape ...

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Inflation...

• inflation: period quasi-exponential expansion of the very early universe

• driven by the vacuum energy of a slowly rolling light scalar field:

\[ \ddot{\phi} + 3H \dot{\phi} + V' = 0 \]

scale factor grows exponentially: \( a \sim e^{Ht} \) if: \( \dot{\phi} \ll \ddot{\phi} \)

\[ \Rightarrow \quad \epsilon \equiv -\frac{\dot{H}}{H^2} \approx \frac{1}{2} \left( \frac{V'}{V} \right)^2 \ll 1 \quad , \quad \eta \equiv \frac{\dot{\epsilon}}{\epsilon H} \approx \frac{V''}{V} \ll 1 \]

with the Hubble parameter \( H^2 = \frac{\dot{a}^2}{a^2} \approx \text{const.} \sim V \)
• inflation generates metric perturbations: scalar (us) & tensor

\[ P_S \sim \frac{H^2}{\epsilon} \sim \left( \frac{\delta \rho}{\rho} \right)^2 \]

\[ \sim k^{n_S-1} \]

\[ n_S = 1 - 6\epsilon + 2\eta \]

and

\[ P_T \sim H^2 \sim V \]

window to GUT scale &

direct measurement of scale of inflation

• **but**: if field excursion sub-Planckian, no measurable gravity waves: [Lyth '97]

\[ r \equiv \frac{P_T}{P_S} = 16\epsilon \leq 0.003 \left( \frac{50}{N_e} \right)^2 \left( \frac{\Delta \phi}{M_P} \right)^2 \]
a speculative recipe ...

Let us start by making a set of assumptions about the landscape ...

these assumptions are not taken as proven to be true for the whole of the landscape

yet they have certain support/evidence from some corners of the landscape

given the set of assumptions, we can try to figure out the consequences - valid for a landscape conforming to these assumptions
premises / assumptions ...

• large-field inflation needs shift symmetry to control UV corrections:

\[ \mathcal{O}_6 \sim V(\phi) \frac{\phi^2}{M_P^2} \Rightarrow m_\phi^2 \sim H^2, \eta \sim 1 \]

\[ \text{⇒ (i) shift symmetries only from p-form gauge fields of string theory} \]

• scalar fields in string compactifications:

\[ \text{⇒ (ii) field range is limited to } < M_P \]
scalar fields in string compactifications ...

- string(y) inflatons: typically angular moduli, e.g. (some) brane moduli or axions $\theta_a$
  - shallow potential
  - but: periodicity $\Delta \theta_a$ - limits field range

- example: axions

$$\int |F_p|^2 = \int f_a^2 (\partial_\mu \theta_a)^2 = \int (\partial_\mu \phi_a)^2$$

canonically normalized fields $\phi_a$ have periodicity $f_a \Delta \theta_a$
for compact spaces of size $R$:
[Banks, Dine, Fox & Gorbatov; Srvcek & Witten]

$$f_a \sim M_P \left( \frac{\sqrt{\alpha'}}{R} \right)^p \ll M_P$$
population of the many vacua:

(iii) only known mechanism: CdL or HM tunneling, combined with eternal inflation

basic structure of the landscape of vacua

(iv-1) exponentially many vacua in multi-dimensional moduli space

(iv-2) neighbouring vacua typically have large differences in vacuum energy:
- any vacuum with relatively small c.c. has neighbours with large c.c.
• eternal inflation

⇒ volume-weighted global measures only
   if discard eternal volume growth

⇒ in addition, there is global-local duality for:

- causal patch measure

- scale factor time measure

- light-cone time cutoff measure

⇒ use only measures without obvious pathologies (BBs, youngness paradox ...)
   - this removes, in particular, the proper time cutoff & leaves the above menu

see also for a global picture/measure emerging
from holographic many-worlds measures:
[Nomura 11; Bousso & Susskind ’11] ...
either need many fields in lockstep:

\[ \Delta \phi_{\text{diag.}} \sim \sqrt{\sum_{i=1 \ldots N} \Delta \phi_i^2} \gg 1 \quad \text{with} \quad |\Delta \phi_i| < 1 \]

called "N-flation"

[Dimopoulos, Kachru, McGreevy & Wacker '05]

string theory embedding is challenging, due to need for large number of fields w/ instanton potentials ... see: [Grimm '07] for coming very close
or need a potential $V(\phi)$ which is monotonic:

$$V(\phi) \rightarrow V(\phi + \Delta \phi) > V(\phi) \quad \text{even if:} \quad \phi \rightarrow \phi + \Delta \phi = \phi$$

called “$V(\phi)$ has a monodromy in $\phi$“

we have seen this: think of

$$S_{5-\text{brane}} \sim \int d^4 x \sqrt{-g} \sqrt{v^2 + \phi^2} \sim \phi$$

or fluxes ...

$$\int B_2 , \int C_2$$

[Silverstein & AW '08]
[McAllister, Silverstein & AW '08][Berg, Pajer & Sjörs '09][Dong, Horn Silverstein & AW '10]
the upshot:

- the specific monodromy mechanism & field is irrelevant

- without a field in string theory with a good shift symmetry and an unbounded/non-periodic fundamental domain ...

... some monodromy in the potential energy is necessary for single-field parametrically-large-field inflation
- by the very definition of the word ‘monodromy’
axion monodromy gives effective potential:

\[ V(\phi, \chi) = V_0(\phi) + \Lambda^4(\chi) \cos\left(\frac{\phi}{2\pi f}\right) + U_{mod.}(\chi) \]

\[ V_0(\phi) \sim \phi^p \quad , \quad 0 < p < 2 \quad \text{for } \phi \gtrsim M_P \]

good shift symmetry for \( \Phi \) demands near-perfect decoupling from the moduli \( \chi \) - in particular, the minimum/minima in \( \Phi \) do not shift as a function of the moduli
consequences of (i), (ii) & (vi-2)

global measures: **highest** metastable dS is progenitor to all small-c.c dS vacua
consequences of (i), (ii) & (vi-2)

local measures: longest-lived dS - still generically of very high-scale c.c. - is progenitor to all small-c.c dS vacua
population of sufficiently many small-c.c. vacua must go via an intermediate very large c.c. vacuum because down tunneling is much more efficient

maintained by all measures free of obvious paradoxes
Up tunneling very expensive & undemocratic

\[ \Gamma_{V_+} \sim e^{-\left(\frac{1}{V_-} - \frac{1}{V_+}\right)} \]

Ratio of up tunneling rates into 2 different higher dS vacua

\[ \frac{\Gamma_{V'_+}}{\Gamma_{V_+}} \sim e^{-\frac{1}{V_+}} \quad \text{,} \quad V'_+ > V_+ \]
down tunneling less expensive & democratic

\[ \Gamma_{V_-} \sim e^{-\frac{1}{V_+}} + S_E(\phi) \]

\[ S_E(\phi) \sim \int d\xi a^3(\xi)V(\phi) \]

\[ \sim S_E^{(0)}(\phi) \left[ 1 + O\left(\frac{V_-}{V_+}\right)\right] \]

- independent from small \( V_- \)
- can average over barrier height

averaged ratio of down tunneling rates into 2 lower dS vacua

\[ \frac{\Gamma_{av.}^{V_-}}{\Gamma_{av.}^{V_-}} \sim 1 \quad , \quad V_+ \gg V_- \quad , \quad V_-' \]
• (iii) Tunneling feeds the landscape:

- proceeds via CDL instanton
  [Coleman, De Luccia '80]
- nucleates bubbles of negative spatial curvature
consequences of (iii) tunneling...

- equations of motion dictating evolution after passing through CDL tunneling:

\[
\ddot{\phi} + 3H \dot{\phi} = -V'(\phi)
\]

\[
H^2 = \frac{1}{3M_P^2} \left( \frac{\dot{\phi}^2}{2} + V(\phi) \right) + \frac{1}{a^2}
\]

- the inside of a CDL bubble is an open, negatively curved FRW universe...

- overshoot problem [Brustein, Steinhardt '93]

- resulting friction term severely slows the field on steep potentials
CDL instanton dictates very special initial conditions:

\[ a(t) = t + \mathcal{O}(t^3) \quad \dot{\phi}_0 \equiv \dot{\phi}(t = 0) = 0 \]

\[ \phi_0 \equiv \phi(t = 0) \]

\[ |\phi_0| < 1 \quad !! \]

\[ V_R(\phi) = V_- \cdot (1 - \sqrt{2\epsilon\phi}) \]

\[ V_L(\phi) = (-1)^n \frac{\lambda_n}{n} \phi^n \]
• resulting overshoot for \( n = 1 \ldots 3 \):

\[
\phi(t_c) = \frac{3}{4\sqrt{2}} \sqrt{\epsilon} O(1) \phi_0 + O\left(\frac{V_-}{V_0}\right)
\]

• on the left side - quartic potential, \( n = 4 \):

\[
\phi(t) = \frac{8\phi_0}{8 + t^2\lambda_4\phi_0^2} \rightarrow \infty \quad 0 \quad \Rightarrow \quad t_f \rightarrow \infty
\]

\[
\dot{\phi}(t) = -\frac{16t\lambda_4\phi_0^3}{(8 + t^2\lambda_4\phi_0^2)^2} \rightarrow \infty \quad 0
\]

• thus - no overshoot at all for \( n = 4 \! \)!

\( V_- > 0 \) causes vacuum energy domination for some large \( t > 0 \) already on the left for \( \Phi < 0 \) ...
consequences of (i), (ii), (iii) & (iv)

→ successful anthropic explanation of present-day small c.c. requires efficient population of a very large # of small-c.c. vacua

→ large-c.c. vacuum is effective progenitor of most inflationary valleys with exit into small c.c. vacua - because down tunneling is efficient

→ down tunneling populates small-c.c. vacua & valleys democratically

→ negative curvature inside CDL bubbles removes initial condition problem for subsequent slow-roll

→ a universal bias seems to appear: no bias ... small-field and large-field regimes appear to be seeded democratically (on the level of exponential bias)
if tunneling & measure treat small-field and large-field regimes approximately neutral ...

distribution of field-range is fully determined by number frequency of inflationary solutions

‘valley’ statistics determines $r$, as vacuum statistics (anthropically) determines late-time c.c. ! This is in principle a string theory question ...
there are exponentially many small-field models outside the
monodromy-based class -- just from accidentally fine-tuned saddle
points in the moduli potential

[Susskind ’04; Douglas ’04; Denef & Douglas ’04; Aazami & Easther ’05; Marsh, McAllister & Wrase ’11; Chen, Shiu, Sumitomo & Tye ’11; ...]

e.g. random-matrix analysis of random supergravities:
if there are \( N_H \) heavy & \( N_L \) light moduli with \( N_H \gg N_L \), with
\( \exp(c_H N_H) \) vacua from the heavy sector, then susy breaking in the
light sector will lead to

\[
\sim e^{c_H N_H} e^{-c_L N_L^p}, \quad 1 < p < 2
\]

still exponentially many positive c.c. vacua ...

[valley statistics]

[Marsh, McAllister & Wrase ’11]
[Chen, Shiu, Sumitomo & Tye ’11]
parametrize unknown counting factors - example: CY landscape ...

\[ \chi = \langle \chi(CY_3) \rangle = \mathcal{O}(100) \]

\[ N_{\text{small-field}} \sim N_{\text{CY}} \cdot e^\chi \cdot \epsilon_{\text{dS-vacua}} \cdot \beta_{\text{flat saddle}} \]

\[ \sim e^{c_H N_H} \]

\[ \sim e^{-c_L N_L^p} \]

\[ N_{\text{large-field}} \sim \beta_{h^{1,1} > 0} N_{\text{CY}} \cdot e^\chi \cdot \epsilon_{\text{dS-vacua}} \]

# of CYs supporting topological requirements of axion monodromy

\[ \rightarrow \quad \frac{N_{\text{large-field}}}{N_{\text{small-field}}} \sim \frac{\beta_{h^{1,1} > 0}}{\beta_{\text{flat saddle}}} \]

\[ = \text{??} \]
open questions ...

• all of the assumptions ... kill any one of them, and the argument fails - however, the first 3 of them at least seem to cover some swaths of the landscape ...

• Counting!! we need the ‘valley statistics’ ...

• the distribution of vacuum energies could be much steeper than the power-law from flux vacua -- e.g. \( \exp(1/V) \) from Hartle-Hawking wave function in quantum cosmology - this would change bias!

• probably many more outside my limited view ...
Welcome to Hamburg in July 2013!!

Hope to see you all there and then!

http://stringpheno2013.desy.de/