Combined Inclusive Diffractive Cross Sections Measured with Forward Proton Spectrometers at HERA

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Kinematics of diffractive DIS

- $Q^2 = \text{virtuality of photon} = (4\text{-momentum exchanged at e vertex})^2$
- $W = \text{invariant mass of } \gamma^*-p \text{ system}$
- $M_X = \text{invariant mass of } \gamma^*-\text{IP system}$
- $x_{IP} = \text{fraction of proton's momentum carried by IP}$
- $\beta = \text{Bjorken's variable for the IP}$
  - $= \text{fraction of IP momentum carried by struck quark}$
  - $= x/x_{IP}$
- $t = (4\text{-momentum exchanged at p vertex})^2$
  - typically: $|t| < 1 \text{ GeV}^2$

- **Single diffraction:** $N=\text{proton}$
- **Double diffraction:** proton-dissociative system $N$
  - $\rightarrow$ represents a relevant background
Diffractive cross section & structure functions

- Diffractive cross section

\[
\frac{d\sigma^D_{\gamma p}}{dM_X} = \frac{\pi Q^2 W}{\alpha (1 + (1 - y)^2)} \cdot \frac{d^3\sigma^D_{ep \rightarrow e' Xp'}}{dQ^2 dM_X dW}
\]

\[
\frac{d\sigma}{dt} \sim e^{bt}
\]

- Diffractive structure function \( F_2^{D(4)} \)

and reduced cross sections \( \sigma_{r}^{D(4)} \) and \( \sigma_{r}^{D(3)} \)

\[
\frac{d^2\sigma_{\gamma p \rightarrow eX}}{d\beta dQ^2 dx_{ip} dt} = \frac{4\pi\alpha^2}{\beta Q^4} \left[ 1 - y + \frac{y^2}{2(1 + R^b)} \right] \cdot F_2^{D(4)}(\beta, Q^2, x_{ip}, t)
\]

\[
= \frac{4\pi\alpha^2}{\beta Q^4} \left[ 1 - y + \frac{y^2}{2} \right] \cdot \sigma_{r}^{D(4)}(\beta, Q^2, x_{ip}, t)
\]

\[
\sigma_{r}^{D(3)}(\beta, Q^2, x_{ip}) = \int \sigma_{r}^{D(4)}(\beta, Q^2, x_{ip}, t) dt
\]

- \( R^D = \frac{\sigma_L^{\gamma p \rightarrow Xp}/\sigma_T^{\gamma p \rightarrow Xp}}{\sigma_T^{\gamma p \rightarrow Xp}} \); \( \sigma_r^D = F_2^D \) when \( R^D = 0 \)
Diffractive DIS at HERA

Standard DIS

\[ e(k) \rightarrow e'(k') \]

\[ \gamma^*(q) \]

\[ P(p) \rightarrow H(p') \]

\textcolor{red}{\text{color flow}}

Diffractive DIS

\[ e(k) \rightarrow e'(k') \]

\[ \gamma^*(q) \]

\[ P(p) \rightarrow X(x) \text{ \textcolor{red}{GAP}}, \text{ \textcolor{red}{no color flow}}, \text{ \textcolor{red}{N(p')}} \]
Signatures and selection methods

Proton Spectrometer (PS) method

H1-VFPS

220

H1-FPS

90 80 64 40 24

Large Rapidity Gap (LRG) method

\[ p' \]

\[ e' \]
Signatures and selection methods

Proton Spectrometer (PS) method

H1-VFPS

\[ p \]

220

H1-FPS

\[ p' \]

90 80 64 40 24

ZEUS LPS

Large Rapidity Gap (LRG) method

near perfect acceptance at low \( x_{IP} \)

p-diss contribution no \( t \) measurement

\( \Delta \eta \)
Signatures and selection methods

Proton Spectrometer (PS) method

- **H1-VFPS**
  - $p$: 220
  - Direct measurement of $t$, $x_{IP}$
  - High $x_{IP}$ accessible
  - No $p$-diss contribution

- **H1-FPS**
  - $t$: 90 80 64 40 24
  - Low statistics

Large Rapidity Gap (LRG) method

- Near perfect acceptance at low $x_{IP}$
- $p$-diss contribution
- No $t$ measurement
## Available publications

<table>
<thead>
<tr>
<th>Collaboration</th>
<th>References</th>
</tr>
</thead>
</table>

Consistent results from the two methods

Comparison H1-ZEUS

Combining the measurements can provide more precise and kinematically extended data than the individual sets
Available publications

**H1 LRG**

**ZEUS LRG**

**H1 FPS**

**ZEUS LPS**

Combining the measurements can provide more precise and kinematically extended data than the individual sets.

Proton spectrometer results now combined (first combination in diffraction at HERA!)
Main H1 and ZEUS detectors used to reconstruct $Q^2$, $W$ and $x$, whereas $M_x$, $\beta$, $x_{IP}$ and $t$ derived from FPS/LPS or from combined info H1+FPS/ZEUS+LPS

- **H1 FPS HERA II**
  - Luminosity = 156.6 pb$^{-1}$
  - Visible range $|t| = 0.1 - 0.7$ GeV$^2$
  - Norm unc $\sim \pm 6\%$

- **H1 FPS HERA I**
  - Luminosity = 28.4 pb$^{-1}$
  - Visible range $|t| = 0.08 - 0.5$ GeV$^2$
  - Norm unc $\sim \pm 10\%$

- **ZEUS LPS 2**
  - Luminosity = 32.6 pb$^{-1}$
  - Visible range $|t| = 0.09 - 0.55$ GeV$^2$
  - Norm unc $\sim +11 -7\%$

- **ZEUS LPS 1**
  - Luminosity = 3.6 pb$^{-1}$
  - Visible range $|t| = 0.075 - 0.35$ GeV$^2$
  - Norm unc $\sim +12\% - 10\%$
Data sets for combination

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  Luminosity = 156.6 pb$^{-1}$
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\[\sigma_r^{D(3)}\text{ combined}\]

Combination performed in the ZEUS visible $t$ range $|t| = 0.09 - 0.55$ GeV$^2$

Prior to combining, ZEUS cross section points swam to H1 ($Q^2$, $\beta$, $x_{IP}$) grid using ZEUS DPDF SJ [Nucl.Phys. B831 (2010) 1]
$$\sigma_r^{D(3)}$$ for combination

H1 and ZEUS

\[
\begin{array}{c|c|c|c|c|c}
\beta &= 0.0018 & \beta &= 0.0056 & \beta &= 0.018 & \beta &= 0.056 & \beta &= 0.18 & \beta &= 0.56 \\
\chi^2_{IP} & = 0.0009 & = 0.0018 & = 0.0056 & = 0.0085 & = 0.016 & = 0.025 & = 0.035 & = 0.05 & = 0.075 & = 0.09 \\
0.09 < |t| < 0.55 \text{ GeV}^2
\end{array}
\]

- H1 FPS HERA II
- ZEUS LPS 2
- H1 FPS HERA I
- ZEUS LPS 1

\[Q^2 (\text{GeV}^2)\]
**Combination method**

- $\chi^2$ minimization which includes full error correlations
- Used for previous combined HERA results [JHEP 1001 (2010) 109]
- Key assumption is that H1 and ZEUS are measuring the same cross sections at the same kinematic points

→ Model independent check of the data consistency and reduction of the systematic uncertainty

For a single data set:

$\chi^2_{\text{exp}}(\bar{m}, \bar{b}) = \sum_i \frac{[m^i - \sum_j \gamma_j m^i b_j - \mu^i]^2}{\delta_{i,\text{stat}}^2} (\mu^i - \sum_j \gamma_j m^i b_j + (\delta_{i,\text{uncor}} m^i)^2) + \sum b_j^2$

- $\mu^i$ measured cross section values
- $m^i$ combined cross section values
- $b_j$ shifts of correlated systematic uncertainty sources in $\sigma$ units
- $\gamma^i_j$ relative correlated systematic unc.
- $\delta_{\text{stat}}^i$ relative statistical unc.
- $\delta_{\text{uncor}}^i$ relative uncorrelated systematic unc.

Full $\chi^2_{\text{tot}}$ built from the sum of the $\chi^2_{\text{exp}}$ of each data set, assuming the individual data sets to be statistically uncorrelated

$\chi^2_{\text{tot}}$ minimized wrt $m^i$ and $b_j$
Uncertainties

- Input cross sections published with their statistical and systematic uncertainties; the latter classified into point-to-point uncorrelated and correlated.
- Global normalisations included in the fit.
- H1 and ZEUS systematic uncertainties treated as independent.
- A few procedural uncertainties considered: i. additive vs multiplicative nature of the error sources ii. correlated systematic error sources ZEUS-H1 iii. swimming factors applied to ZEUS points iv. treatment of the uncertainty on the H1 hadronic energy scale.
Results

352 data points combined to 191 cross section measurements

Good consistency: $\chi^2/n_{\text{dof}} = 133/161$

<table>
<thead>
<tr>
<th>Source</th>
<th>Shift ($\sigma$ units)</th>
<th>Reduction factor %</th>
</tr>
</thead>
<tbody>
<tr>
<td>FPS HERA II hadronic energy scale $x_p &lt; 0.012$</td>
<td>-1.61</td>
<td>56.9</td>
</tr>
<tr>
<td>FPS HERA II hadronic energy scale $x_p &gt; 0.012$</td>
<td>0.13</td>
<td>99.8</td>
</tr>
<tr>
<td>FPS HERA II electromagnetic energy scale</td>
<td>0.49</td>
<td>85.9</td>
</tr>
<tr>
<td>FPS HERA II electron angle</td>
<td>0.67</td>
<td>66.6</td>
</tr>
<tr>
<td>FPS HERA II $\beta$ reweighting</td>
<td>0.15</td>
<td>90.4</td>
</tr>
<tr>
<td>FPS HERA II $x_p$ reweighting</td>
<td>0.05</td>
<td>98.3</td>
</tr>
<tr>
<td>FPS HERA II $t$ reweighting</td>
<td>-0.70</td>
<td>79.8</td>
</tr>
<tr>
<td>FPS HERA II $Q^2$ reweighting</td>
<td>0.09</td>
<td>97.6</td>
</tr>
<tr>
<td>FPS HERA II proton energy</td>
<td>0.05</td>
<td>45.6</td>
</tr>
<tr>
<td>FPS HERA II proton $p_x$</td>
<td>0.62</td>
<td>74.5</td>
</tr>
<tr>
<td>FPS HERA II proton $p_y$</td>
<td>0.27</td>
<td>86.5</td>
</tr>
<tr>
<td>FPS HERA II vertex reconstruction</td>
<td>0.07</td>
<td>97.0</td>
</tr>
<tr>
<td>FPS HERA II background subtraction</td>
<td>0.84</td>
<td>89.9</td>
</tr>
<tr>
<td>FPS HERA II bin centre corrections</td>
<td>-1.05</td>
<td>87.3</td>
</tr>
<tr>
<td>FPS HERA II global normalisation</td>
<td>-0.39</td>
<td>84.4</td>
</tr>
<tr>
<td>FPS HERA I global normalisation</td>
<td>0.81</td>
<td>48.9</td>
</tr>
<tr>
<td>LPS 2 hadronic energy scale</td>
<td>-0.02</td>
<td>55.0</td>
</tr>
<tr>
<td>LPS 2 electromagnetic energy scale</td>
<td>-0.14</td>
<td>62.4</td>
</tr>
<tr>
<td>LPS 2 $x_p$ reweighting</td>
<td>-0.32</td>
<td>98.2</td>
</tr>
<tr>
<td>LPS 2 $t$ reweighting</td>
<td>-0.26</td>
<td>86.4</td>
</tr>
<tr>
<td>LPS 2 background subtraction</td>
<td>0.40</td>
<td>94.9</td>
</tr>
<tr>
<td>LPS 2 global normalisation</td>
<td>-0.53</td>
<td>67.7</td>
</tr>
<tr>
<td>LPS 1 global normalisation</td>
<td>0.86</td>
<td>44.1</td>
</tr>
</tbody>
</table>

Table 3: Sources of point-to-point correlated systematic uncertainties considered in the combination. For each source the shifts resulting from the combination in units of the original uncertainty and the values of the final uncertainties as percentages of the original are given.
Results

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Results

352 data points combined to 191 cross section measurements

Good consistency: $\chi^2/n_{dof} = 133/161$

Influence of several correlated systematic uncertainties reduced significantly for the combined result

Cross calibration brings average improvement of experimental uncertainty of 27% wrt most precise single data set (FPS HERA II)

Correlated part of experimental uncertainty reduced from about 69% in FPS HERA II to 49%
Results

352 data points combined to 191 cross section measurements

Good consistency: $\chi^2/\text{n}_{\text{dof}} = 133/161$

Statistical uncertainty: 11%
Statistical + correlated + uncorrelated: 13.8%
Procedural uncertainty: 2.9%
Total uncertainty on cross section 14.3% on average and 6% for most precise points
Normalization uncertainty: 4%

Kinematic coverage extended wrt single input measurements

$Q^2 = 2.5 - 200 \text{ GeV}^2$
$\beta = 0.0018 - 0.816$
$x_{IP} = 0.00035 - 0.09$
$|t| = 0.09 - 0.55$

At low $x_{IP}$, where the proton spectrometer data are free from proton dissociation background, these combined data provide the most precise determination of the absolute normalisation of the diffractive cross section
Combined $\sigma_{rD(3)}$

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$Q^2$ (GeV$^2$)</th>
<th>$x_{IP}\sigma_{rD(3)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0018</td>
<td>2.5</td>
<td>0.0025</td>
</tr>
<tr>
<td>0.0056</td>
<td>5.1</td>
<td>0.025</td>
</tr>
<tr>
<td>0.018</td>
<td>8.8</td>
<td>0.025</td>
</tr>
<tr>
<td>0.056</td>
<td>15.3</td>
<td>0.025</td>
</tr>
<tr>
<td>0.18</td>
<td>26.5</td>
<td>0.025</td>
</tr>
<tr>
<td>0.56</td>
<td>46</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>0.025</td>
</tr>
</tbody>
</table>
Combined $\sigma_r^{D(3)}$

**H1 and ZEUS**

\[ x_{IP} \sigma_r^{D(3)}(t) \]

**HERA**

0.09 < |t| < 0.55 GeV$^2$

Nice and precise measurement of the scaling violation in diffraction.
Combined $\sigma_r^{D(3)}$

H1 and ZEUS

$0.09 < |t| < 0.55 \text{ GeV}^2$

$Q^2 (\text{GeV}^2)$

$\beta = 0.018 \times 6$

$\beta = 0.056 \times 3$

$\beta = 0.18$

Combined $\sigma_r^{D(3)}$
Summary

- In 15 years of running HERA provided unique diffractive data
- First combination of the H1 and ZEUS diffractive data
  - combined proton-tag results
  - consistency between datasets
  - the two experiments calibrate each other resulting in a reduction of the systematic uncertainties
  - most precise determination of the absolute normalisation of the ep \(\rightarrow\) eXp cross section
- Looking forward to combining the LRG data
Backup
Precise measurement of $\sigma_r^{D(4)}$ in bins of $|t|$

- **ZEUS**
  - $\sigma_r^{D(4)}$ from proton spectrometers
  - ZEUS LPS 33 pb$^{-1}$
  - $|t|$ range 0.09 - 0.55 GeV$^2$

- **H1 FPS**
  - $|t|$ range 0.1 - 0.7 GeV$^2$

Regression fit LPS

Regression fit IP+IR
t-slope

\[ \frac{d\sigma}{dt} \sim e^{bt} \]

**ZEUS**

- ZEUS t-slope equal to 7 GeV$^{-2}$ (constant through the kinematics)
- H1 t-slope between 5 and 6 GeV$^{-2}$ (depending on $x_{\text{IP}}$)
\[ \sigma_r^{D(3)} = \int_{-1}^{t_{\text{min}}} \sigma_r^{D(4)} \, dt \]

The measured \( b \) parameters are used to perform the integration to the range \(|t| < 1 \text{ GeV}^2\)

**Good agreement in shape between H1 and ZEUS**

**Fair agreement in normalization between H1 and ZEUS**

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( Q^2 ) (GeV^2)</th>
<th>H1 FPS HERA II norm unc</th>
<th>ZEUS LPS norm unc</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0018</td>
<td>5.1</td>
<td>( \pm 6% )</td>
<td>( +11% - 7% )</td>
</tr>
<tr>
<td>0.056</td>
<td>5.1</td>
<td>( \pm 0.09 - 0.12 ) (norm)</td>
<td></td>
</tr>
</tbody>
</table>
### Table 1: H1 and ZEUS data sets used for the combination.

<table>
<thead>
<tr>
<th>Data Set</th>
<th>$Q^2$ range [GeV$^2$]</th>
<th>$x_{p'}$ range</th>
<th>$y$ range</th>
<th>$\beta$ range</th>
<th>$t$ range [GeV$^2$]</th>
<th>Luminosity [pb$^{-1}$]</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1 FPS HERA II</td>
<td>4 – 700</td>
<td>&lt; 0.1</td>
<td>0.03 – 0.8</td>
<td>0.001 – 1</td>
<td>0.1 – 0.7</td>
<td>156.6</td>
<td>[2]</td>
</tr>
<tr>
<td>H1 FPS HERA I</td>
<td>2 – 50</td>
<td>&lt; 0.1</td>
<td>0.02 – 0.6</td>
<td>0.004 – 1</td>
<td>0.08 – 0.5</td>
<td>28.4</td>
<td>[1]</td>
</tr>
<tr>
<td>ZEUS LPS 2</td>
<td>2.5 – 120</td>
<td>0.0002 – 0.1</td>
<td>40 – 240</td>
<td>2 – 40</td>
<td>0.09 – 0.55</td>
<td>32.6</td>
<td>[4]</td>
</tr>
<tr>
<td>ZEUS LPS 1</td>
<td>2 – 100</td>
<td>&lt; 0.1</td>
<td>25 – 240</td>
<td>&gt; 1.5</td>
<td>0.075 – 0.35</td>
<td>3.6</td>
<td>[3]</td>
</tr>
</tbody>
</table>

### Table 2: Normalisation uncertainties in the full range $|t| < 1$ GeV$^2$ and in the restricted $t$ range for the data used for the combination.

| Data Set      | $|t_{min}| < |t| < 1$ GeV$^2$ | $0.09 < |t| < 0.55$ GeV$^2$ |
|---------------|--------------------------|-----------------------------|
| FPS HERA II   | ±6%                      | ±5%                         |
| FPS HERA I    | ±10%                     | ±10%                        |
| LPS 2         | +11%, -7%                | ±7%                         |
| LPS 1         | +12%, -10%               | ±11%                        |
Fig. 3: Pull distributions for the individual data sets. The root mean square gives the root mean square of the distributions.