Towards explicit dS vacua ...

expansion speeds up
Λ > 0 ...
we live in de Sitter space!

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the general setup - type IIB on a warped CY with flux

• Kähler potential:

\[ K = -2 \ln(V + \alpha'^3 \xi) - \ln(S + \bar{S}) - \ln \left( -i \int \Omega \wedge \bar{\Omega} \right) \]

• superpotential:

- 3-form fluxes stabilize the \( U_a \)
- non-perturbative effects (ED3 or N D7's) fix the \( T_i \)

\[ W = \int_{CY3} G_3 \wedge \Omega + \sum_i A_i e^{-a_i T_i} \]

• moduli potential:

\[ V_F = e^K \left( K^{S\bar{S}} |D_S W|^2 + K^{ab} D_a W \bar{D}_b \bar{W} + K^{ij} D_i W \bar{D}_j \bar{W} - 3|W|^2 \right) \]
dS in type IIB - some relationships...

\[ |W_0| \ll 1, \alpha' - \text{correction negligible} \]
\[ W_0 \neq 0, \alpha' - \text{correction} \]
\[ \hat{V} \gg \gg \hat{\xi} \]
\[ W_0 \text{ arbitrary} \]
\[ |W_0| \sim \mathcal{O}(1 \ldots 100) \]

KKLT
[KKLT '03]
LVS
[Balasubramanian, Berglund, Conlon & Quevedo '05]
Kähler uplifting
[Balasubramanian, Berglund '04]
[Westphal '06]

- D3 branes
- F-terms from matter fields
  [Lebedev, Nilles, Ratz '06]
- F-terms from metastable vacua in gauge theories
  [Intriligator, Seiberg, Shih '07]
- D3 branes
- D-terms
  [Burgess, Kallosh, Quevedo '03, Haack, Kreifl, Lüst, Van Proyen, Zagermann '06]
- F-terms from dilaton dep. non-pert. effects
  [Cicoli, Maharana, Quevedo, Burgess '12]
- F-terms from Kähler moduli + \alpha' - correction sufficient for dS
de Sitter vacua from ‘Kahler uplifting’ at large volume

• 4d \( N=1 \) supergravity - scalar potential:

\[
V = e^K \left\{ K^{T\bar{T}} \left[ a^2 e^{-2aTr} + (-ae^{-aTr} \overline{WK}_T + c.c) \right] + 3\xi \frac{\xi^2 + 7\xi\mathcal{V} + \mathcal{V}^2}{(\mathcal{V} - \xi)(\xi + 2\mathcal{V})^2} |W|^2 \right\}
\]

• expand to leading order in \( \xi/\mathcal{V} \) and \( e^{-aT} \):

\[
V \simeq 4AW_0 \frac{ate^{-at}}{\mathcal{V}^2} \cos(at) + \frac{3\xi W_0^2}{4\mathcal{V}^3} \sim \frac{2C}{9x^{9/2}} - \frac{e^{-x}}{x^2}
\]

\[
\mathcal{V} \sim \frac{1}{\sqrt{\kappa}} (T + \bar{T})^{3/2}, \ x = at, \ T = t + i\tau, \ C \sim \frac{-W_0}{A} \xi a^{3/2} \sqrt{\kappa}
\]

[Rummel & AW ’11]
[De Alwis & Givens ’11]
de Sitter vacua from ‘Kahler uplifting’ at large volume

Figure 1. The approximate 2-term scalar potential $V(x)$ from eq. (2.14) for different values of $C$.

- Lower bound on $C$: 
  \[ \left\{ x_{\text{min}}, C \right\} = \left\{ 5.22, 2.25 \right\} \approx \left\{ t_1, w \right\} \]
- Upper bound on $C$: 
  \[ \left\{ x_{\text{min}}, C \right\} \approx \left\{ 3 + \sqrt{89}, u \right\} \approx \left\{ u, s \right\} \]

The region close to $\left\{ x_{\text{min}}, C \right\}$ is the one relevant for obtaining a small positive cosmological constant suitable for describing the late-time accelerated expansion of the universe. For $a = t_\pi$, the lower bound on $x$ corresponds to a volume $\hat{V} \approx s$, so we are indeed at parametrically large volume. The allowed window for $C$ to obtain meta-stable de Sitter vacua is approximately $u_x \approx \sqrt{t_W} \hat{\xi}^{3/2} x^{\gamma} A \approx u_z h_t p^{t i l l s u f f i c i e n t}$ to obtain a meta-stable minimum of the scalar potential when all the remaining moduli fields of the Calabio-Yau and the dilaton and the complex structure moduli are included in the stabilization analysis. Hence, this is truly a sufficient condition for meta-stable de Sitter vacua and no tachyonic instabilities occur by including further moduli contrary to the standard KKLT scenario.
this works for arbitrary $h^{1,1}$ on Swiss-cheese CYs!

• resulting scalar potential

\[
V = \frac{4W_0}{\mathcal{V}^2} \left( tAe^{-at} \cos(at) + \sum_{i=2}^{h^{1,1}} a_i t_i A_i e^{-a_i t_i} \cos(a_i \tau_i) \right) + \frac{3\xi W_0^2}{4\mathcal{V}^3} \\
+ \sum_{i=2}^{h^{1,1}} \frac{2\sqrt{2} a_i^2 A_i^2}{3} \frac{\sqrt{t_i}}{\gamma_i} e^{-2a_i t_i} \mathcal{V}
\]

all $V_{\tau_I \tau_I} > 0$ if $W_0 < 0$ at $\tau_I = 0$; 

or if $W_0 > 0$ at $\tau_I = \pi/a_I$

$V_{\tau_I \tau_J} = 0$ at these points, thus all axions are massive
what about the other moduli?

\[ W = W_0 + A e^{-aT} = C_1(U) - C_2(U) S + A e^{-aT} \]

\[ \Rightarrow V \sim e^K (K^{S\bar{S}} |D_S W_0|^2 + K^{T\bar{T}} |D_T W|^2 + K^{T\bar{S}} D_T W \overline{D_S W_0}) \]

\[ V_0 (K^{U\bar{U}} |D_U W_0|^2) \]

\[ \delta V \]

perturbs \( S, U \) away from \( S_0, U_0 \) ...

\[ S_0 = - \frac{C_1}{C_2} : \left. D_S W_0 \right|_{S_0} = 0 \quad \frac{\partial V}{\partial S} = 0 \]

\[ \Rightarrow \frac{\delta S}{S_0} \sim \frac{\xi}{\sqrt{V}} \]

\[ U_{a,0} : \left. D_{U_a} W_0 \right|_{U_{a,0}} = 0 \quad \frac{\partial V}{\partial U_a} = 0 \]

\[ \Rightarrow \frac{\delta U_a}{U_{a,0}} \sim \frac{\xi}{\sqrt{V}} \]
what about the other moduli?

- mass scales:

\[
\begin{align*}
    m_t^2 & \sim \frac{\xi}{V^3} \\
    m_\tau^2 & \sim \frac{\xi}{V^3} \\
    m_{\text{Re} S}^2 & \sim \frac{1}{V^2} \\
    m_{\text{Im} S}^2 & \sim \frac{1}{V^2} \\
    m_{\text{Re} U}^2 & \sim \frac{1}{V^2} \\
    m_{\text{Im} U}^2 & \sim \frac{1}{V^2} \\
    m_{3/2}^2 &= e^K |W|^2 \sim \frac{1}{V^2} \\
    m_{KK}^2 &= \frac{1}{L^2 \alpha'} \sim \frac{1}{V^{4/3}}
\end{align*}
\]

➡️ SUSY is broken at the GUT scale, but below KK-scale!
The elliptically fibered Calabi-Yau fourfold can be defined by the Weierstrass model which is a $F_p$ theory we have to compactify on an elliptically fibered Calabi-Yau fourfold. More

2.2 D7-branes from the F-theory perspective

vacuum easily computable estimate for the largest gauge group rank one can obtain in a

We mention these points to make it clear that the indicator $N$ of an effective four dimensional $CP^3(X,Y,Z)$

The result can be expressed as

$\Delta = 27 g^2 + 4 f^3$

order of zero = rank of gauge group = # (stacked 7-branes)

order of zeros in $f,g =$ gauge group type

$A_n (SU(n))$, $D_n (SO(n))$, $E (E_6, E_7, E_8)$
Explicit models ...

'Mini' landscape: 97,036 models of the F-theory type: [Kreuzer,Skarke '00]

- Ambient toric variety described by (resolution of) weight system

\[ \mathbb{CP}_{n_1 n_2 n_3 n_4 n_\xi} : \quad 0 < n_1 \leq n_2 \leq n_3 \leq n_4 < n_\xi = \sum_{i}^4 n_i \]

\[
\begin{array}{cccccc}
  u_1 & u_2 & u_3 & u_4 & \xi \\
  n_1 & n_2 & n_3 & n_4 & n_5 \\
\end{array}
\]

with \( 0 < n_1 \leq n_2 \leq n_3 \leq n_4 \leq n_5 \)

\[ (u_1, ..., u_4, \xi) \sim (\lambda^{n_1} u_1, ..., \lambda^{n_4} u_4, \lambda^{n_5} \xi), \quad \text{with} \quad \lambda \in \mathbb{C}^* \]
Explicit models ...

- the toric divisors $D_i, D_\xi$ are hypersurfaces (4-cycles) given by holomorphic equations:

\[
D_i : \{ u_i = 0 \} \quad D_\xi : \{ \xi = 0 \}
\]

- for N=1 supersymmetry in 4D we need to orientifold - 1 possible orientifold projection:

\[
\sigma : \xi \mapsto -\xi ,
\]
Explicit models ...

- in F-theory, a CY 3-fold is then a hypersurface describing a double-cover of $B_6$ in the complex-4d toric ambient space, obtained as the Sen limit of an elliptically fibered 4-fold

\[ \xi^2 = P \left( 2 \sum_i^4 n_i, \ldots \right) \]

if its degree $= 2n_\xi$ and

\[ n_\xi \equiv n_5 = \sum_{i=1}^4 n_i \quad \Rightarrow \quad \tilde{K}(CY_3) = c_1(CY_3) = 0 \]

anti-canonical class
Explicit models ...

- can do this in F-theory with a Weierstrass model fibered over the toric base $B_6$:

\[
\begin{array}{cccc}
    u_1 & u_2 & u_3 & u_4 \\
    n_1 & n_2 & n_3 & n_4 \\
\end{array}
\]
\[\mathbb{CP}^3_{n_1 n_2 n_3 n_4}\]

- rewrite the Weierstrass equation in Tate form:

\[
Y^2 + a_1 X Y Z + a_3 Y Z^3 = X^3 + a_2 X^2 Z^2 + a_4 X Z^4 + a_6 Z^6
\]

$a_i$ are functions of the $u_i$ on the base such, that they are sections of $i\bar{K}$
Explicit models ...

• to construct the 7-brane singularity for a given gauge group on divisor $D_j$, $a_i$ have to scale in $u_j$ with certain weights (Kodaira classification):

$$a_i = u_j^{w_i} a_{i,w_i}$$

• the toric base $B_6$ of the 4-fold has anti-canonical class

$$\bar{K} = [D_\xi] = \sum_{i=1...4} [D_i] = n_\xi [D_1]$$

$$\Rightarrow n_\xi = \sum_{i=1...4} n_i$$

• because $a_i$ is in $i\bar{K}$ we have:

$$a_i = u_j^{w_i} a(i n_\xi - w_i n_j, ...)$$

, $j = 1 \ldots 4$
Explicit models ...

- because the $a_{i,w}$ must be holomorphic:

  $$N = \text{rank}(\text{gauge group}) = w_i \leq \frac{i n \xi}{n_j}$$

- strongest constraint from $a_3$, $a_6$:

  $$N_{lg} \leq \frac{3 n \xi}{n_j}$$

- Sen limit exists, engineering IIB limit gives same result
• thus, large rank needs large orientifold class $n_\xi$

• if any $n_i > 1$, then $B_6$ has singularities

• resolution (blow-up) adds new lines to weight system (GLSM) each new line is a new independent divisor, and thus new Kähler modulus

• so maximal rank will increase with $h^{1,1}$
Explicit models...

- can plot this for F-theory type Kreuzer-Skarke set:

```
\begin{center}
\includegraphics[width=\textwidth]{figure1.png}
\end{center}
```

0 50 100 150 200 0 20 40 60 80 100 120 140 160 180

$N_{lg}$ $h^{1,1}$

We conclude this section with the remark that the possibility to engineer large enough gauge groups to obtain a large volume in the framework of Kähler uplifting is a generic feature of the landscape we have analyzed.
Explicit models ...

• thus, because (naively):

\[ \mathcal{V} \sim N_{lg}^{3/2} \]

• volumes > $10^3$ are possible in a large fraction of CY space

• this ensures control & separation of Kähler and complex structure moduli mass scales → justifies treating the instanton prefactors as constant!
Explicit dS example: $\mathbb{CP}^{11169}$

Some geometric properties:

- Calabi-Yau: $0 = \xi^2 = P_{18,4}(u_i)$ in
  
  \[
  \begin{pmatrix}
  u_1 & u_2 & u_3 & u_4 & u_5 & \xi \\
  1 & 1 & 1 & 6 & 0 & 9 \\
  0 & 0 & 0 & 1 & 1 & 2 \\
  \end{pmatrix}
  \]

- $h^{1,1} = 2$, $h^{2,1} = 272$

- Divisors:
  
  \[
  \begin{array}{c|cccccc}
  \text{Divisors} & h^{0,0} & h^{1,0} & h^{2,0} & h^{1,1} & \chi_0 \\
  \hline
  D_1 & 1 & 0 & 2 & 30 & 3 \\
  D_5 & 1 & 0 & 0 & 1 & 1 \\
  \end{array}
  \]
Explicit models ...

Explicit dS example: $\mathbb{CP}_{11169}$

\[ \hat{y} = \sqrt{\frac{2}{3}} \left( \hat{y}_1 + \frac{1}{3} \hat{y}_5 \right)^{3/2} - \frac{\sqrt{2}}{9} \hat{y}_5^{3/2} \Rightarrow \text{'Approx. swiss-chesse'} \]

- Complex structure moduli: $\mathbb{Z}_6 \times \mathbb{Z}_{18}$ modding: $h_{\text{inv.}}^{2,1} = 2$
  with known prepotential. [Greene, Plesser’89], [Candelas, Font, Katz, Morrison’94]

\[
G(z) \text{ via mirror symmetry in the large complex structure limit: } \\
G(z_1, z_2) = \sum_{i+j \leq 3} c_{ij} z_1^i z_2^j + \xi + G_{\text{instanton}}(e^{-2\pi z_1}, e^{-2\pi z_2})
\]
Explicit models ...

- 1\textsuperscript{st} step - 3-form flux fixes the 2 invariant complex structure moduli supersymmetrically:

Stabilizing the $h_{\text{inv.}}^{2,1} = 2$ moduli effectively fixes all other C.S. moduli at $D_i W = 0$ since $V$ positive definite up to corrections $O(\hat{\xi}/\hat{V})$ [Giryavets, Kachru, Tripathy, Trivedi '03]

- the 270 $Z_6 \times Z_{18}$ non-invariant c.s. moduli are stabilized by invariant higher-order terms in $W$
  (from invariant higher-order terms in the 4 invariant periods) [Giryavets, Kachru, Tripathy, Trivedi '03]
A solution:

\[(f, h) = (-16, 0, 0, 0, -4, -2; 0, 0, 2, -8, -3, 0), \quad Q^{RR,NS–NS} = 66,\]

\[\langle S \rangle = 6.99, \quad \langle U_1 \rangle = 1.01, \quad \langle U_2 \rangle = 0.967, \quad \langle W_0 \rangle = 0.812,\]

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<td>(m^2_{u_1})</td>
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<td>(m^2_{\nu_1})</td>
<td>(m^2_{\nu_2})</td>
<td>(m^2_\sigma)</td>
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<td>0.24</td>
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<td>5.6 \cdot 10^{-6}</td>
<td>0.24</td>
<td>1.8 \cdot 10^{-4}</td>
<td>5.7 \cdot 10^{-6}</td>
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Explicit models ...

- **2\textsuperscript{nd} step - Kähler moduli:**
  - Brane config. ($N_{lg} = 27$): $Sp(24)$ on $D_1$ forces $SO(24)$ on $D_5$.
  - $D_5$ rigid, $D_1$ can be 'rigidified' by gauge flux $\Rightarrow Sp(24) \rightarrow SU(24)$. 
    [Martucci '06][Bianchi,Collinucci,Martucci '11]
  - Brane intersections: Switch on gauge flux $F_{1/5} + c_1(D_{1/5})/2$ to cancel Freed-Witten anomalies and tune $F_1$, $F_5$ and $B$ such that $\mathcal{F}_{1/5} = F_{1/5} - B$ is 'trivial' $\Rightarrow$ No chiral matter or D-terms.
  - D3 tadpole: $Q_{D3}^{\text{tot}} = Q_{D3}^{O7\text{h}} + Q^{\text{stacks}}_{D3} + Q^W_{D3} = \begin{cases} -104 & \text{for } Q^W_{D3} = -81 \\ -96 & \text{for } Q^W_{D3} = -73 \end{cases}$
Explicit models ...

- 2nd step - Kähler moduli:

\[ W = W_0 + A \ e^{-2\pi/24 \ T_1} + B \ e^{-2\pi/22 \ T_2}, \ A, B \neq 0. \]

- If in the complex structure sector:

\[ \langle W_0 \rangle = 0.812, \ \langle S \rangle = 6.99, \ \langle A \rangle = 1.11, \ \langle B \rangle = 1.00. \]

- Stable dS vacuum with \( \langle T_1 \rangle = 10.76, \ \langle T_2 \rangle = 12.15 \) and \( \hat{V} = 52. \)

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<tr>
<th>( m^2_{\tau_1} )</th>
<th>( m^2_{\tau_2} )</th>
<th>( m^2_{\zeta_1} )</th>
<th>( m^2_{\zeta_2} )</th>
<th>( m^2_{3/2} )</th>
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<td>( 5.24 \cdot 10^{-9} )</td>
<td>( 4.55 \cdot 10^{-8} )</td>
<td>( 1.13 \cdot 10^{-7} )</td>
<td>( 6.40 \cdot 10^{-8} )</td>
<td>( 4.08 \cdot 10^{-7} )</td>
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Conclusions

- We have explicit constructions of dS vacua in type IIB string theory/F-theory in the ‘Kähler uplifting’ scenario.

- They are fully determined by the 4-fold data:
  - the Euler characteristic
  - choice of ADE-type singularities,
  - and choice of fluxes

- They break SUSY by Kähler moduli F-terms at the GUT scale. No extra source of uplifting is needed.

- A whole ‘mini’-landscape of explicit examples (Kreuzer-Skarke) is available. One example is shown to satisfy all known F-theory/IIB consistency constraints.